

THERMODYNAMICS

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FIFTH EDITION

THIRD IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.

NEW YORK AND LONDON

1943

THERMODYNAMICS

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THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE TO THE FIFTH EDITION

Earlier editions of "Thermodynamics" by the late Professor J. E. Emswiler of the University of Michigan were received with such general approval it seemed unadvisable not to offer a new edition with some additions and revision necessitated by recent developments in the presentation of the subject.

The lucid explanations, numerous illustrative examples, free use of diagrams and graphical representations in earlier editions have been retained. Some rearrangement of the material has been made. The general case has been presented before the specific. The several phases of the working substance are presented in general and vapors and gases are considered subsequently. The polytropic process for gases is presented in detail and followed by the adiabatic, isothermal, constant volume and constant pressure processes as special applications of the polytropic. More emphasis on gases and the general energy equation for steady flow processes appears in the new edition than in former editions.

Throughout this edition American Standards Association symbols and abbreviations have been used.

New material on absorption refrigeration, gas turbines, gas cycles, adiabatic saturation of air-water vapor mixtures, and supersaturation has been added.

The author of the revision wishes to thank his associates in the department of Mechanical Engineering in the University of Michigan for their interest and constructive criticism of the revised edition.

F. L. SCHWARTZ.

ANN ARBOR, MICHIGAN,
October, 1943.

PREFACE TO THE FIRST EDITION

The subject of thermodynamics has already been covered by so many able writers that the author has been conscious of a feeling somewhat akin to presumption in the preparation of this book. On the other hand, an experience of several years in teaching thermodynamics and allied subjects has convinced him that there is still something to be accomplished in the manner of its presentation. It is a regrettable fact that, in too many of our engineering schools, thermodynamics is the bugbear of the curriculum. It is approached with dread by the student, and laid aside with a sigh of relief at its conclusion, without his ever having been touched by the real beauty and fascination of the subject.

An attempt is made here to present the subject matter with especial regard to the student's viewpoint. It is the aim in the various developments to proceed progressively so that the reader may easily recognize the relation of each new demonstration that he faces to the whole, and thus avoid that most discouraging of all situations for the student, of working in the dark on what appears to him to be an isolated section of the subject with no connection to the main body.

Two principal features of the book that appear in the endeavor to accomplish the object in view may be mentioned.

In the first place, the order of presentation of the various subdivisions of the subject is somewhat different from that usually employed in texts on thermodynamics. Steam is placed first inasmuch as this division of the work always appears to be easiest for the student to grasp, probably partly because the steam heat engine is more familiar to

him as a piece of mechanism; and certainly because the steam heat engine lends itself more easily to study, on the part of the student, because of the fact that its mechanical elements are separated, each one performing but a single function that can be quite well understood. After steam comes naturally vapor refrigeration; then the study of permanent gases, mixtures, and air heat engines. Formal discussion of the rather abstract phases of the subject—the laws of thermodynamics and the kinetic theory of heat—is reserved until the student has acquired, in some measure, an experience of his own. It is believed that his better appreciation and understanding of these generalities will far more than offset any temporary need of a knowledge of them in a few places in the book preceding their presentation.

In the second place, wherever possible, the student is presented, at the beginning of a section of the subject, with a schematic diagram of an actual heat engine whose principle is to be exemplified. This plan seems to establish a thread of relationship among the several articles that follow, and tends to keep the student anchored to a rather definite conception, and to prevent the development from becoming, for him, a mere abstraction.

A few problems are furnished at the end of several of the chapters. It will be found that these correspond very closely to the matter presented in the chapter. They are arranged in the order of the development, so that, in general, it will be found that as soon as a lesson has been covered in a chapter, a problem can be assigned to exemplify the principles. Of course, after all, the problems of the book are chiefly suggestive, inasmuch as every progressive instructor will wish to vary his examples from semester to semester, and also because every different class seems to necessitate problems designed especially for its needs.

The author is deeply indebted to his colleague, Professor C. H. Fessenden, who has very kindly assisted by offering many valuable suggestions, and by reading proof; to all his

associates in the Department of Mechanical Engineering in the University of Michigan for their sympathetic interest in the progress of the work; and to many of his former students whose frank constructive criticisms have served to indicate the value of methods of presentation in the class room that have been incorporated in this book.

J. E. EMSWILER.

ANN ARBOR, MICH.,
May, 1921.

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PROPERTIES OF SUPERHEATED STEAM		

NOMENCLATURE

	area
<i>a</i>	area
	pressure ratio
<i>a</i>	refers to one constituent of a mixture
<i>b</i>	refers to one constituent of a mixture
<i>C</i>	clearance
<i>c</i>	specific heat
<i>c</i>	refers to one constituent of a mixture
<i>d</i>	cutoff ratio
η	efficiency
<i>e</i>	natural logarithm base
	refers to exit
<i>F</i>	force
<i>f</i>	refers to liquid state
<i>f_g</i>	refers to change by evaporation
<i>g</i>	acceleration of gravity, 32.2 ft per sec ²
<i>g</i>	refers to vapor state
<i>H</i>	total enthalpy
<i>h</i>	enthalpy per pound
<i>i</i>	refers to solid state
<i>J</i>	mechanical equivalent of heat, 778 ft-lb per Btu
<i>k</i>	constant
	ratio of specific heat = c_p/c_v
<i>l</i>	distance
<i>M</i>	molecular weight
	rate of flow of weak aqua ammonia solution
<i>m</i>	mass
<i>m</i>	refers to mixture
<i>n</i>	number of molecules
<i>n</i>	exponent for a polytropic

	indicates a polytropic
P	power
p	pressure
p	indicates constant pressure
ρ	radius
	density
Q	heat
R	gas constant
	distance
	compression ratio
	critical pressure ratio
	radius
	refers to Rankine cycle
S	entropy
s	entropy per lb
Σ	summation
	sigma heat content
T	temperature, Fahrenheit absolute
t	temperature, Fahrenheit
t	refers to throat
U	internal energy
u	internal energy per lb
V	volume
\bar{V}	velocity
v	specific volume
v	indicates constant volume
W	work
\bar{W}	weight
\bar{w}	weight rate of flow
X	concentration
	distance
	quality
y	distance
Z	distance, head
	distance

THERMODYNAMICS

CHAPTER I

THERMODYNAMICS

1. Definition of Thermodynamics.—The Standard Dictionary gives this definition of thermodynamics: "Thermodynamics is that branch of the theory of heat that treats of the relations between heat and mechanical work, especially as acting in a heat engine."

Two outstanding facts are to be noted:

a. Thermodynamics is a science that deals primarily with energy and energy transformations.

b. Energy appears in many forms.

The energy interchanges that accompany all phenomena of nature are of great interest and importance to scientists in general. But this book is intended to cover only those phases of the subject that deal with the development of energy in relatively large quantities, or upon what may be termed an engineering or commercial scale. Such a treatment of the subject is sometimes alluded to as "engineering thermodynamics."

2. Sources of Energy.—In primitive times, man depended upon his own muscles to secure for himself the things necessary to his existence. Later, he widened his field of conquest by domesticating animals stronger than himself. In the course of time, he invented the water wheel, which made available to him another source of energy. But the great epoch of man's utilization of energy from natural stores began with the improvement of the steam engine, about one hundred and eighty years ago. Since that beginning, man's ingenuity has successively per-

fect energy-transformation devices, until today we are calling forth from Nature's stores quantities of energy so vast that serious-minded men begin to look with apprehension toward a distant date when those reservoirs shall have become exhausted. Mechanical power has been adapted to one purpose after another so rapidly that what was a luxury or convenience yesterday in everyday life has by its commonness become a necessity today.

Energy is applied in the active or mechanical form. What is the source of active or mechanical energy? With the exception of muscular activity, the derivation of which is not yet clearly understood, and which constitutes in magnitude but a small amount of the total energy now employed by civilized peoples, the source of all useful energy is *heat*. Heat appears in the combustion of coal for the production of steam; in the burning of oil or gasoline in the internal-combustion engine; and in the use of the gas, which is derived from coal, in the gas burner or the gas engine. When the windmill is considered, it is found that the currents of air that furnish the energy are set in motion by the heat of the sun. And even the hydraulic turbine is actuated by water which has been elevated from the surface of the land or ocean by the sun's rays.

3. The Study of Thermodynamics.—The subject of thermodynamics, as presented in this book, includes a study of the properties of gases and vapors and mixtures that serve as energy vehicles; the manner in which heat is turned into mechanical work through the agency of these energy vehicles, and the mechanisms—heat engines—for accomplishing this purpose; the reverse process wherein thermal effects are produced by the expenditure of mechanical work upon gases, as in refrigeration and air compression; and the relation between heat and work as expressed by the general laws of thermodynamics.

CHAPTER II

ENERGY

4. **The Kinetic Theory of Gases.**¹—What is designated as matter or substance is not a solid homogeneous mass but is, in fact, a structure composed of structural elements, the molecules, the smallest division into which a substance can be broken up without losing its identity. The molecules in turn are structures composed of atoms; and the atoms in their turn are structures made up of still smaller units called *electrons*. Presumably all electrons are alike. Electrons may arrange themselves in something more than a hundred different ways, each different arrangement constituting a different kind of atom, the fundamental unit of a different substance known as a chemical element. The atoms of different elements may unite in an infinite variety of combinations to form the molecules of all known substances. Matter, according to this theory, is therefore structural in nature, consisting of systems within systems.

Considering a molecular system, what are the forces that hold the atoms in place to form a definite configuration? It is believed that permanency is maintained by the average balance of two opposing forces: one an attraction which tends to concentrate the elements of the system; the other a centrifugal force resulting from orbital velocities, which tends to separate the constituent elements to a greater distance. The molecule is, in fact, a miniature solar

¹ The laws of thermodynamics do not depend upon any theory relating to the structure of matter. This article and a few succeeding references to the kinetic theory may be omitted without detracting from the validity of thermodynamic equations. The kinetic theory is briefly presented here to afford a picture of why gases may exert pressure, why several gases may simultaneously occupy the same volume, and to explain at least one way in which energy may be stored in substances.

system, although invisibly small in dimensions. The general relation between the sizes of the solidified masses of the sun and planets to the dimensions of the solar system is similar to those of the atomic masses as compared with molecular dimensions. Furthermore, an entire molecular system, while its component parts are involved in mutual relative motions, may be participating as a unit, in motions relative to surrounding molecular systems. While the magnitudes of the dimensions of molecules are so small as to be far below the reach of the most powerful microscope, the velocities of their motions and the corresponding kinetic energies are enormous.

The kinetic theory of gases offers an explanation for many of the thermodynamic properties of matter and is very helpful in acquiring a mental picture of the behavior of molecules when subjected to thermodynamic processes. Although the theory cannot be relied upon to explain all thermodynamic phenomena, it nevertheless will be found useful in many instances. The kinetic theory as applied to perfect gases is based on the following assumptions:

- a. Molecules of a gas behave like perfectly elastic spheres, *i.e.*, there is no loss of energy upon collision.
- b. The time of collision is instantaneous.
- c. The number of molecules is enormous so that average quantities hold implicitly.
- d. Molecules move with high velocities, and the distance between molecules is large compared with the diameters of the molecules; *i.e.*, there are no attractive forces between molecules.

Newton's law of gravitation is

$$F = k \frac{m_1 m_2}{r^2}$$

where F is the force of attraction, m_1 and m_2 are the masses of any two bodies, r is the distance between their centers of gravity, and k is a constant. When one of the masses is the earth, having a very large mass, and another is an

ENERGY

object on the surface of the earth, the force is appreciable and we call it the force of attraction, the force of gravity or weight. Regardless of size, however, the force of attraction between any two bodies is real. If the two bodies are two molecules very close together, the force of attraction is existent, but as the distance between molecules increases, the force of attraction decreases. In the case of a perfect gas, the force of attraction is negligible.

The characteristic equation of a perfect gas is

$$pV = \bar{W}RT \quad (1)$$

where \bar{W} is the weight of a gas in pounds under pressure p lb per sq ft abs at an absolute temperature T F abs and having a volume V cu ft. R is a constant.

Imagine a rectangular box, of dimensions x , y , and z , containing a perfect gas. Let the total number of molecules be n . Let \bar{V} represent the average velocity of the molecules. Each time a molecule impinges upon a wall, the change of momentum will be $2m\bar{V}$, if m is used to designate the mass of the molecule. If two faces on the box are considered, of which x and y are the dimensions, the molecule may be imagined to bounce back and forth from face to face, traveling a distance z between. The number of times the particle will impinge upon either face per second will be $\bar{V}/2z$. The total force produced by the one molecule is

$$2m\bar{V} \times \frac{\bar{V}}{2z} = \frac{m\bar{V}^2}{z}$$

By the law of averages, one-third of the total number of molecules, n , in the box will be traveling in the direction z . The total force exerted upon the face xy will be

$$\frac{n}{3} \times \frac{m\bar{V}^2}{z} \quad \text{or} \quad \frac{2n}{3z} \left(\frac{m\bar{V}^2}{2} \right)$$

Let p = the pressure or force per unit of area.

Then

$$p = \frac{2n}{3xyz} \left(\frac{m\bar{V}^2}{2} \right)$$

But

$$xyz = \text{the volume, } V$$

hence

$$pV = \frac{2n}{3} \left(\frac{m\bar{V}^2}{2} \right) \quad (2)$$

The weight \bar{V} of the gas in the box is proportional to the number of molecules n . $m\bar{V}^2/2$ is the kinetic energy of a molecule. Equation (1) is based on data determined by experiments with gases thought to be very nearly perfect gases. Equation (2) is deduced purely on the basis of the kinetic theory. Therefore, comparing Eq. (1) with Eq. (2), we may conclude that the temperature of a perfect gas is proportional to the kinetic energy of the molecules. In deriving Eq. (2), the force exerted upon the walls of the box was the force caused by the change of momentum of the molecules as they impinged on the faces of the box. The pressure of the gas may be pictured as an average force caused by multitudinous impacts of molecules. If the temperature of the gas were to be increased, the kinetic energy of the molecules would increase, *i.e.*, \bar{V} would increase, and hence the change of momentum would become greater and we would expect the pressure to rise. If the volume of the box were suddenly increased and the temperature remained the same, the distance between molecules would increase, there would be fewer impacts per second, and thus the pressure would drop. Furthermore, it is understood that the space occupied by the molecules is very small compared with the volume of the box, so the volume may be changed over large limits without destroying our conception of a perfect gas. Many kinds of molecules may be contained in the box at the same time, each kind with a different mass, but it should be noted that since all molecules are at the same temperature and, therefore,

each contains the same average kinetic energy, different kinds of molecules will have different average velocities. If the volume of the box remained fixed and additional gas of the same kind is added at the same temperature, the impact force per molecule is the same, but there now being more molecules, there will be more impacts per second and hence a higher pressure. This picture of a perfect gas is thus seen to be useful in explaining the thermodynamic properties—pressure, volume, and temperature.

5. Forms of Energy.—Energy appears in many forms. Some energy is stored energy and remains stored in a given quantity of matter until such time when changes are permitted to take place and the amount of stored energy may be increased or decreased. This change of stored energy may be brought about by adding or taking away heat or by doing work on the substance or allowing the substance to perform work. Other forms of energy (heat and mechanical work) are evident only as energy in transition. As an example, mechanical work is a form of energy, but it can exist only while some other form of energy is being transformed into work. When the transformation ceases, the work ceases.

6. Heat.—Heat energy is that form of energy which is transferred from one body to another by reason of a temperature difference. Heat energy is therefore not a stored form of energy but occurs only in transition and when present, energy of some other form is being transferred from one place to another.

Heat energy may be transferred in three ways, by conduction, convection, and radiation.

If one end of a metal rod insulated against heat transfer to the air is heated to a high temperature, heat energy will flow from the hot end to the cold end and the temperature of the cold end will rise. The heat is transferred from one cross section to the next by molecular activity. The molecular activity at the hot end causes the molecules in an adjacent section to increase their molecular activity.

Thus, energy is carried along the entire length of the bar, and the temperature of the cold end rises. If the two ends of the bar are at the same temperature, the molecules of the two ends contain the same molecular activity and there is no tendency for energy to be transferred from one cross section to the next, *i.e.*, there is no heat transfer. This mode of heat transfer is called *conduction*.

Let 1 lb of air at 70 F pass over a hot plate and be heated to a temperature of 80 F. Then this air might be circulated to some other place *P* where the temperature had previously been 70 F. The energy of the pound of air at the place *P* has been increased, for the temperature has been raised from 70 F to 80 F. Heat energy has been transferred from the plate to the place *P* by a mass motion of the circulating air. This mode of heat transfer is called *convection*.

Heat energy may be transferred by way of electromagnetic waves, great distances or short distances, through vacuated space or through gases, and to a lesser extent, through liquids and solids. Consider the sun at a very high temperature. The molecules contain enormous activity and are able to radiate electromagnetic waves of various wave lengths. The electromagnetic wave energy is transferred to the earth, and as the electromagnetic waves impinge upon molecules the molecules increase their activity, their temperature rises, and there has been a heat transfer from the sun to the earth. This mode of heat transfer is called *radiation*.

In the three modes of heat transfer, there must be a temperature difference, and the direction of the heat transfer is in the direction of decreasing temperatures.

The symbol *Q* will be used to denote transferred heat, and heat will be measured in Btu (British thermal units). The amount of heat received by a body is proportional to the weight of the body and the change of temperature that occurs while heat is being transferred to the body.

$$Q \propto W(t_2 - t_1)$$

or written as an equality

$$Q = c\bar{W}(t_2 - t_1) = c\bar{W}(T_2 - T_1) = \bar{W} \int_1^2 c dT \quad (3)$$

where Q = heat added or withdrawn, Btu

\bar{W} = weight of body, lb

t_1 = original temperature, F

T_1 = original temperature, F abs

t_2 = final temperature, F

T_2 = final temperature, F abs

c = proportionality constant or specific heat

The numerical value of the constant c depends upon the units employed in the equation and the material being heated. Equation (3) may be used to define a Btu. Let \bar{W} be one pound of water and let the temperature rise $(t_2 - t_1)$ be a one degree Fahrenheit rise. Then if we choose to let the value of the constant c equal one for water, the amount of heat Q added will be one Btu. Although the Btu has been defined by Eq. (3) and is a unit of heat energy, it may be used as a unit for any other form of energy. Having defined the Btu, if the same equation is applied to a pound of mercury and the temperature rise is one degree when heat has been transferred, the value of the constant c will be different from one ($c = 0.032$ Btu/F-lb for mercury). The constant c is called the *specific heat* of the material. The value of c varies with temperature. For a small temperature difference, t_1 to t_2 , the variation is small and c is substantially the same for the ranges t_1 to $(t_1 + 1)$ and $(t_2 - 1)$ to t_2 . If t_2 is considerably higher than t_1 , the specific heat for the range t_1 to $(t_1 + 1)$ is noticeably different from the specific heat for the range $(t_2 - 1)$ to t_2 . For example, the specific heat of air at constant pressure for the range 60 to 61 F is 0.241, whereas for the range 1599 to 1600 F it is 0.277 Btu per F-lb. The specific heat of a substance at temperature T F abs may be represented by empirical equations, such as Eqs. (4a-f). c_p is the specific heat at constant pressure, Btu per pound-degree Fahrenheit (see Art. 100).

$$\text{Oxygen} \quad c_p = 0.3599 - \frac{5.}{T} \quad \text{range 540} - 5000 \text{ F abs} \quad (4a)^1$$

$$\text{Nitrogen} \quad c_p = 0.3382 - \frac{123.9}{T} + \frac{41430}{T^2} \quad \text{range 540} - 9000 \text{ F abs} \quad (4b)^1$$

$$\text{Carbon monoxide} \quad c_p = 0.3379 - \frac{123.9}{T} + \frac{38210}{T^2} \quad \text{range 540} - 9000 \text{ F abs} \quad (4c)^1$$

$$\text{Hydrogen} \quad c_p = 2.858 + 0.0002868T + \frac{9.923}{T} \quad \text{range 540} - 4000 \text{ F abs}$$

$$\text{Water vapor} \quad c_p = 1 \quad \text{range 540} - 5400 \text{ F abs} \quad (4e)^1$$

$$\text{Carbon dioxide} \quad c_p = 0.3682 - \frac{148.4}{T} + \frac{32050}{T^2} \quad \text{range 540} - 6300 \text{ F abs} \quad (4f)^1$$

When a quantity of heat is to be calculated for a temperature range in which the specific heat is not constant, the integral form of Eq. (3) should be used with c expressed as a function of T . Equations (4a-f) are typical of equations in which c is expressed as a function of T .

7. Work.—Work is defined as the product of a force by a distance wherein the direction of the force is that of the distance being traversed. Work, therefore, always involves movement and is a transferred form of energy. If there is no movement, there is no work performed. Work is transitory, and it is incorrect to speak of work contained in a body. The work form of energy might be the means whereby the stored energy of a body is increased, but the energy will be stored in some other form. Consider a piston of area A sq ft in a cylinder, Fig. 1. Initially, the

¹ From SWEIGERT, R. L. and M. W. BEARDSLEY, "Empirical Specific Heat Equations Based upon Spectroscopic Data," *Georgia State Eng. Exp. Sta. Bull.* 2, 1938. Equations (4a-f) are for atmospheric pressure.

piston is at position 1 and is then moved to position 2, traveling a distance l by a pressure p lb per sq ft. For the present, assume the pressure to be maintained constant by

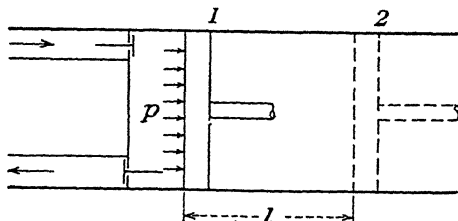


FIG. 1.

admitting air as the piston moves. Then the work W done by the air is

$$\begin{aligned} {}_1W_2 &= \text{force} \times \text{distance} \\ &= pAl \end{aligned}$$

but

$$Al = \text{the change in volume } V_2 - V_1$$

where V_2 denotes the volume in the cylinder when the piston is in position 2, V_1 the volume when the piston is in position 1, and ${}_1W_2$ the work done as the piston moves from position 1 to 2.

Then

$${}_1W_2 = p(V_2 - V_1) \text{ ft-lb} \quad (5)$$

If p is a varying pressure

$$(6)^1$$

and it becomes necessary to know how p varies as the volume is increased. Work produced by a varying pressure will be considered in detail in succeeding chapters. The term *work* will be used to indicate mechanical work transferred through the shaft of an engine or the work transferred through the shaft of a machine compressing a gas, pumping a liquid, etc., unless otherwise specified.

¹ Equation (6) is valid for evaluating the mechanical or shaft work produced only for a reversible process.

8. Power.—Power is the time rate of performing work. It is not energy, but it is a rate of transferring the work form of energy. A common unit for power is the horsepower, hp.

$$1 \text{ hp} = 33,000 \frac{\text{ft-lb}}{\text{min}} = 550 \frac{\text{ft-lb}}{\text{sec}}$$

Another common unit for power is the kilowatt, kw, more generally used for the electrical equivalent of mechanical power

$$1 \text{ hp} = 0.746 \text{ kw}, \quad \text{or} \quad 1 \text{ kw} = 1.34 \text{ hp}$$

9. Work and Power Units.—Work may be measured in ft-lb or equivalent Btu. Power may be measured in horsepower or equivalent kilowatts. The following relations should be memorized:

Power,

$$\begin{aligned} 1 \text{ hp} &= 33,000 \frac{\text{ft-lb}}{\text{min}} \\ &= 550 \frac{\text{ft-lb}}{\text{sec}} \\ &= 0.746 \text{ kw} \\ 1 \text{ kw} &= 1.34 \text{ hp} \end{aligned}$$

Energy,

$$\begin{aligned} 1 \text{ Btu} &= 778 \text{ ft-lb} = \\ 1 \text{ hp-hr} &= 2545 \text{ Btu}^1 \\ 1 \text{ kw-hr} &= 3413 \text{ Btu}^2 \end{aligned}$$

10. Flow Work.—Flow work is a form of energy which is associated with the flow of a fluid. Consider the flow of a

$$\begin{aligned} 1 \text{ hp} &= 33,000 \frac{\text{ft-lb}}{\text{min}} \\ &= \frac{33,000 \text{ Btu}}{778 \text{ min}} \\ &= 2545 \frac{\text{Btu}}{\text{hr}} \end{aligned} \qquad \begin{aligned} &^2 \quad 1 \text{ kw} = 1.34 \text{ hp} \\ 1 \text{ kw-hr} &= 1.34 \text{ hp-hr} \\ &= 1.34 \times 2545 \text{ Btu} \\ &= 3413 \text{ Btu} \end{aligned}$$

$$1 \text{ hp-hr} = 2545 \text{ Btu}$$

column of fluid through a pipe of cross section A into a large container in which the pressure p remains constant, Fig. 2. Let sections 1 and 2 be a distance v/A apart where v is the specific volume, *i.e.*, the volume of 1 lb of the fluid



FIG. 2.

flowing. The fluid in front of section 1 is pushing this column of fluid with a force pA just as though a piston were pushing the column. Likewise the column between 1 and 2 is pushing a similar column beyond it. The energy required to move the column the distance v/A is then

$$pA \frac{v}{A} = pv$$

Some place ahead of section 1, work by a pump or its equivalent is being performed upon the column of moving fluid. The column of fluid may be thought of as a connecting link between a pump or its equivalent and the container, through which a moving force is transmitted. When flow ceases, the pressure and specific volume may remain the same, and consequently the product $p v$ does not change, but no energy in the form of flow work now enters the container since no fluid enters the container. The pump may continue to produce a pressure, but it will have ceased to do work on the column of fluid. Flow work is always evaluated by the product $p v$, yet under conditions when no flow is taking place this quantity has no physical significance. The column of fluid may be thought of as a means of transmission of energy rather than a medium containing this flow work form of energy. Flow work would be expressed in foot-pounds if p is measured in pounds per

square foot and v in cubic feet per pound. pv/J would be expressed in Btu.

11. Kinetic Energy.—Kinetic energy is energy of motion. It is a form of stored energy. It is measured by the velocity and mass of a body in motion according to the equation

$$\text{Kinetic energy} = \frac{1}{2} m \bar{V}^2 = \frac{1}{2} \frac{\bar{W}}{g} \bar{V}^2 \text{ ft-lb} = \frac{\bar{W} \bar{V}^2}{2g} \text{ Btu} \quad (7)$$

where m = mass of the body = \bar{W}/g

\bar{W} = weight of the body, lb

g = acceleration of gravity = 32.2 ft/sec²

\bar{V} = velocity of the body, fps

As long as the velocity of a body does not change, the amount of kinetic energy stored in the body remains constant. In most engineering applications, the mass of the body in motion does not change.

12. Potential Energy.—Potential energy is energy of position. It is a stored form of energy, and the magnitude of the potential energy stored in a body is measured with reference to some accepted zero level or position. Let the floor of a room be the accepted level of zero potential energy. A pound of metal suspended at the ceiling possesses potential energy with reference to the floor, for if the pound of metal is released it will fall because of the attraction between the earth and the piece of metal and be capable of doing work upon a machine situated on the floor. The material of the falling pound of matter is of no consequence. If 1 lb of water falls the same distance, the same amount of work could be accomplished. If the pound of water happens to be flowing down through a pipe, the work would again be the same, neglecting friction. The pound of water might be used to turn a water wheel. If 1 lb of air or steam flows in the pipe leading from the ceiling to the floor, again the work would be the same, neglecting friction. If the ceiling is made twice as high,

twice as much work could be performed. Potential energy is expressed by the equation

$$\text{Potential energy} = \bar{W}Z \text{ ft-lb} = \frac{\bar{W}Z}{J} \text{ Btu} \quad (8)$$

where \bar{W} = total weight of substance, lb

Z = the height of the center of gravity of the substance above the zero level for measuring potential energy, ft

J = mechanical equivalent of heat = 778 ft-lb/Btu

If a body is situated below the zero level for measuring potential energy, Z will be negative and the potential energy will be negative. Negative potential energy is to be interpreted as meaning that work would have to be done by the machine instead of on the machine as the \bar{W} lb of substance approached the zero level. If two molecules are close enough together to produce an appreciable attractive force on each other, there will be an amount of potential energy stored in the system of two molecules just as there is in the case of the two bodies, the earth and a 1-lb piece of metal. In either case, work must be done to separate farther the two objects.

13. Internal Energy.—Internal energy is a stored form of energy and is the energy contained in a substance by virtue of the molecular activity and intermolecular forces present. We have seen that as the temperature of a gas is increased the molecular activity increases. Therefore, the internal energy has increased. The internal energy of the molecules may be the sum of several kinds of energy. It will consist of some kinetic energy of the molecules. In vapors and imperfect gases, there are attractive forces between molecules which are associated with a form of internal potential energy. Also there may be magnetic and rotational energies existing in molecules. Just how the internal energy is stored in the substance is immaterial; the fact is that in a given condition of a substance there is a certain amount of internal energy. If energy is added to the con-

fined substance by doing work on it such as that done by a piston moving in an insulated cylinder, so that no heat can be added or removed from the gas, the internal energy will increase. The internal energy form of energy plays a very important part in the science of thermodynamics, and the conception of molecules containing stored energy by virtue of the motions and forces among them, ready to be converted into other forms of energy, is an essential concept. Internal energy will be represented by the symbol U and is measured in Btu. u represents the internal energy per pound.

14. Other Forms of Energy.—Energy may be found in other forms, such as electrical, magnetic, and light energy. These forms do not appear frequently in the science of engineering thermodynamics. Chemical energy plays an exceedingly important part in the production of power, for the most abundant source of energy which is readily transformed into work for the generation of power comes from the chemical energy stored in fuels. The combustion of fuels is accompanied by a transformation of chemical energy into heat energy which is transformed to some other form of energy. Chemical energy is sometimes included with the form of energy we have called internal energy. Although chemical energy is energy stored in molecules, it is not associated with the kinetic energy of molecules and potential energy accompanying the various attractive forces of molecules. When a chemical reaction takes place, a heat of reaction, the magnitude of which depends upon the stored chemical energy, is involved. The stored chemical energy is associated with the arrangement of the atoms in the molecules taking part in the reaction. When chemical energy is included as part of the internal energy, the total internal energy may be subdivided into two parts, chemical energy and internal energy associated with the motions and attractions of molecules. In studying the thermodynamics of the combustion process, total internal energy is usually considered. Engineering thermodynamics

deals more with the conversion of heat into work than with the conversion of chemical energy into heat energy. That branch of thermodynamics dealing with the chemical energy of substances is considered in physical chemistry.

15. Properties or Point Functions, Energy Transfers or Line Functions.—All substances may appear in three phases, solid, liquid, gaseous. When a substance appears in a given phase or combination of phases, it is characterized by certain variables. Variables that determine the state of a substance are called *properties*. Properties that have been discussed include pressure, specific volume, temperature, and internal energy. Other properties will be introduced later. Two independent properties are necessary to define the state of a pure substance, *i.e.*, a substance that is homogeneous and invariant in chemical composition. When values of two independent properties have been arbitrarily chosen, thereby defining a particular state, the values of all other properties for that state become fixed. Any one of a group of interdependent properties may be chosen as an independent property. The others then become dependent properties. A property may be independent in one state and dependent in another state. Consider several examples. Pressure and temperature of nitrogen in a gaseous state may be selected as the two independent properties. The value of p and T then may be used to determine the specific volume, internal energy, and all other properties. Pressure and volume might equally well have been chosen as the independent properties, in which case the temperature and remaining properties may be determined. Now if the nitrogen is cooled down to a state where it begins to liquefy and two phases are present, liquid and gaseous, pressure and temperature are no longer independent of each other. In this state the temperature is a function of the pressure. Pressure or temperature may be selected as an independent variable but not both. If the pressure is selected as the independent variable, the temperature becomes a dependent variable.

Specific volume may be selected as the second independent variable. For a given state, only one value of each variable *i.e.*, property, may exist. Single-valued variables for a given state are called *point functions* in mathematics and may be represented by a point on a graph, the coordinates of which are properties, Fig. 3.

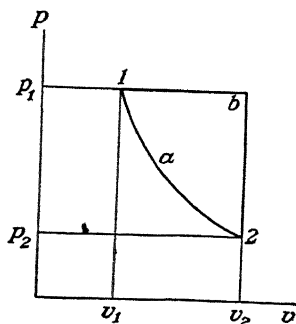


FIG. 3.

For example, suppose air in the gaseous state in a container has a specific volume v_1 and is under a pressure p_1 . This could be represented on a graph by point 1, *i.e.*, state 1, as shown in Fig. 3. The temperature t_1 for state 1 can then be calculated, and since internal energy of a gas is a function of temperature the internal energy u_1 may be determined. Now let the air expand to some other pressure p_2 and specific volume v_2 . The new state will be represented by point 2, and a definite temperature t_2 may be determined, and a given quantity of internal energy u_2 will be associated with the air in state 2.

Work and heat are two forms of energy that may appear only while the state of a substance is changing. The magnitude of the energy being transferred depends on how the properties p , v , and t vary in going from state 1 ($p_1v_1t_1$) to state 2 ($p_2v_2t_2$). If the change is effected by successive changes of state from 1 to 2 along path a , Fig. 3, the work and heat transfers will have one set of values. If the change is effected along path b , a different set of values for the work and heat transferred will exist. Thus work and heat are energy transfers and depend on the path of the change of properties in going from one state to another. Mathematically, such functions are called *line functions*.

Problems

1. If 1 kw-hr of electrical energy can be produced from 10,000 Btu of chemical energy in coal in a modern steam power plant, what is the over-all efficiency of the plant?

2. Calculate the heat required to raise the temperature of 5 lb of water from 40 F to 100 F. How does this compare with the heat required to raise the temperature of 5 lb of mercury from 40 F to 100 F?

3. *a.* Calculate the specific heats of oxygen and nitrogen at constant pressure when the temperature is 3000 F.

b. Considering air to be made up of 77 per cent N_2 and 23 per cent O_2 by weight, what is the specific heat of air at constant pressure when the temperature is 3000 F?

4. Calculate the heat transferred when 10 lb of carbon dioxide are heated from 100 F to 3000 F. What is the percentage error involved if a constant specific heat of 0.200 (c_p at 14.7 psia and 60 F) is used?

5. Calculate the heat transferred when 10 lb of hydrogen are heated from 100 F to 3000 F.

CHAPTER III

FIRST LAW OF THERMODYNAMICS

16. First Law of Thermodynamics.—The first law of thermodynamics is a special case of the general law of conservation of energy. The law of conservation of energy states that energy is neither created nor destroyed, although energy may be transformed from one form to another. As applied to engineering thermodynamics, only those forms of energy which occur in the production of mechanical work from heat energy are considered.

17. General Energy Equation.—The general energy equation is a mathematical statement of the first law of thermodynamics and includes, here, all the energy terms that may be useful in the science of engineering thermodynamics. Using the terms and symbols described in Chap. II, we have

$$\begin{array}{cccccc}
 \text{Transferred} & & \text{Flow} & \text{Internal} & \text{Kinetic} & \text{Potential} \\
 \text{heat} & \text{Work} & \text{work} & \text{energy} & \text{energy} & \text{energy} \\
 \\
 Q_{\text{in}} & + \frac{W_{\text{in}}}{J} + \frac{p_1 v_1}{J} + u_1 + \frac{\bar{V}_1^2}{2gJ} + \frac{Z_1}{J} \\
 \\
 \text{Transferred} & & \text{Flow} & \text{Internal} & \text{Kinetic} & \text{Potential} \\
 \text{heat} & \text{Work} & \text{work} & \text{energy} & \text{energy} & \text{energy} \\
 \\
 = Q_{\text{out}} & + \frac{W_{\text{out}}}{J} + \frac{p_2 v_2}{J} + u_2 + \frac{\bar{V}_2^2}{2gJ} + \frac{Z_2}{J} \frac{\text{Btu}}{\text{lb}} \quad (9)
 \end{array}$$

Each energy term is that amount of energy associated with 1 lb of fluid. For \bar{W} lb of fluid each term should be multiplied by \bar{W} . The terms on the left side of Eq. (9) represent the energies carried into a machine or added to a machine, and those on the right side represent energies carried away from or discharged from that machine. Equation (9) is a general equation and applies to any

process in which one or more of the forms of energy contained in the equation may appear.

18. Enthalpy.—The two energy quantities, flow work and internal energy, always appear in the general energy equation when there is a steady flow through any device in which thermodynamic properties may be changing. The sum of these two energy quantities is a property since internal energy is a property and flow work is a function of the properties p and v .

The sum is given the name *enthalpy* (en-thal'py).¹ The symbol h will be used to represent the enthalpy per pound.

$$h = \frac{pv}{\gamma} \quad , \quad \frac{\text{Btu}}{\text{lb}} \quad (10)$$

Then the general energy equation may be written:

$$Q_{\text{in}} + W_{\text{in}} + \bar{V}_1 \frac{Z_1}{J} + h_1 = \bar{V}_2 \frac{Z_2}{J} + h_2 \quad (11)$$

19. Applications of the General Energy Equation.—

Assume that a hydraulic pump is to be tested and that the various instruments are located as shown in Fig. 4. The general energy equation may be written for any two cross sections. Choose cross sections through points 1 and 2 where the pressure gages are connected. The fluid flowing

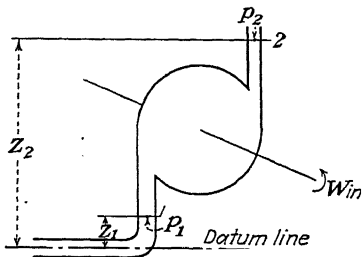


FIG. 4.

has a velocity \bar{V}_1 at cross section 1, and \bar{V}_2 at cross section 2. Heat would be transferred if the fluid should be at a temperature different from room temperature. In such a case, the pump would be insulated to reduce heat transfer to a negligible amount. Because of the inefficiency of the

¹ Heat content, total heat, and thermal potential have been used for this term in the past.

pump, some of the energy entering the pump would be converted into internal energy, thereby raising the temperature of the fluid, and heat would be transferred to the surrounding air, but the amount of heat Q lost would probably be negligible compared with other energy quantities. Work is done on the pump, but no mechanical work is obtained from the pump. If the temperature of the fluid entering and leaving is the same, $u_1 = u_2$.¹ The datum line for measuring potential energy may be chosen wherever convenient.

Then Eq. (9) as applied to a pump becomes

$$\frac{W_{\text{in}}}{J} + \frac{p_1 v_1}{J} + \frac{\bar{V}_1^2}{2gJ} + \frac{Z_1}{J} = \frac{p_2 v_2}{J} + \frac{\bar{V}_2^2}{2gJ} + \frac{Z_2}{J}$$

or

$$W_{\text{in}} = (p_2 v_2 - p_1 v_1) + \frac{(\bar{V}_2^2 - \bar{V}_1^2)}{2g} + (Z_2 - Z_1) \frac{\text{ft-lb}}{\text{lb fluid}} \quad (12)$$

W_{in} is the work in foot-pounds done on each pound of fluid flowing. The quantity v is the specific volume of the fluid. Density ρ of a fluid is the reciprocal of specific volume, $\rho = 1/v$ or $v = 1/\rho$ and

$$W_{\text{in}} = \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} + \frac{\bar{V}_2^2 - \bar{V}_1^2}{2g} + (Z_2 - Z_1) \frac{\text{ft-lb}}{\text{lb fluid}} \quad (13)$$

If q represents the quantity of fluid flowing per unit time, q lb per min, then the power P supplied to the fluid is

$$P = q W_{\text{in}} \frac{\text{ft-lb}}{\text{min}}$$

and the horsepower is

$$HP = \frac{q W_{\text{in}}}{33,000}$$

If Eq. (9) is applied to the flow of fluid in a system and

¹ Pressure has a negligible effect on the internal energy of a

viscous friction is neglected, we have

$$\frac{p_1}{\rho_1} + \frac{\bar{V}_1^2}{2g} + Z_1 = \frac{p_2}{\rho_2} + \frac{\bar{V}_2^2}{2g} + Z_2 \quad (14)$$

which is Bernoulli's equation for an incompressible fluid.

The general energy equation may be applied to the transfer of heat in a steam generator. Heat Q_{in} is transferred from the hot gases to the water. No work transfers take place in the generator. The differences in the kinetic energy and potential energy entering and leaving are small compared with other energy changes and are usually neglected, *i.e.*, $(\bar{V}_2^2 - \bar{V}_1^2)/2gJ$ and $(Z_2 - Z_1)/J$ are negligible. Then Eq. (11) reduces to

$$Q_{in} = h_2 - h_1 \quad (15)$$

All the heat transferred to the water increases the enthalpy of the water.

The function of a nozzle is to convert enthalpy in a fluid into kinetic energy. The general energy equation for a nozzle becomes

$$\frac{\bar{V}_2^2}{2gJ} = (h_1 - h_2) + \frac{\bar{V}_1^2}{2gJ} \quad (16)$$

There is no transfer of heat or work as the fluid flows through the nozzle, and if the nozzle is horizontal $Z_1 = Z_2$ or for any other position of the nozzle $(Z_1 - Z_2)/J$ is negligible since the nozzle is usually short. If the velocity \bar{V}_1 of the fluid flowing into the nozzle is low, Eq. (16) becomes

$$\bar{V}_2 = \sqrt{2gJ(h_1 - h_2)} \quad (17)$$

The velocity of the fluid leaving the nozzle is proportional to the square root of the decrease of enthalpy.

20. Steady-flow and Nonflow Processes.—In some devices in which thermodynamic processes take place, there is a steady flow of fluid carrying energy into and out of the device. In the device, energy transformations take place

and the fluid leaving may have more or less energy than the fluid had as it entered. Such devices might be called *steady-flow devices*. Many of them are machines. Steady-flow devices would include nozzles, throttle valves, turbines, centrifugal pumps, and steam generators. If a reciprocating internal-combustion engine is considered as a whole, it is a steady-flow machine. There is a flow of air and fuel into the machine carrying stored energy with it (some of it in the form of chemical energy) and a flow of exhaust gas leaving the machine. Likewise a reciprocating steam engine becomes a steady-flow machine with steam flowing in and out.

If one considers the flow of gas or steam through a particular cylinder of a reciprocating engine, the flow is intermittent and unidirectional. There is a period of time when no flow takes place into or out of the cylinder. Energy transformations take place during this period of no flow, and what happens in this interval is called a *nonflow process*. Examples of nonflow processes include the heating of a substance in a closed container, the expansion or compression stroke of a reciprocating engine, and changes in the properties of atmospheric air during a given time interval.

21. General Energy Equation for Nonflow Processes.—

If there is no flow into a space where energy transformations are taking place, there are no flow work or kinetic energy changes taking place in the space. Each of these forms of energy depends upon a motion of the fluid. Also the center of gravity of the mass of fluid remains at a fixed point or nearly so, and the potential energy Z/J does not change as the state changes. For nonflow processes, the general energy equation becomes

$$Q_{\text{in}} + \frac{W_{\text{in}}}{J} + u_1 = Q_{\text{out}} + \frac{W_{\text{out}}}{J} + u_2 \frac{\text{Btu}}{\text{lb}}$$

or

$$(Q_{\text{in}} - Q_{\text{out}}) = (u_2 - u_1) + \frac{W_{\text{out}} - W_{\text{in}}}{J} \frac{\text{Btu}}{\text{lb}} \quad (18)$$

Let Q represent the net amount of heat added. If Q_{in} is greater than Q_{out} , Q is positive and heat is *added*. If Q_{in} is less than Q_{out} , Q is negative and heat has been *withdrawn* from the enclosed fluid. Let W represent the net amount of work done by the fluid in the machine. If W_{out} is greater than W_{in} , W is positive and work is done *by* the fluid in the machine. If W_{out} is less than W_{in} , W is negative and work is done *on* the fluid in the machine. Thus

$${}_1Q_2 = u_2 - u_1 + \frac{{}_1W_2}{J} \frac{\text{Btu}}{\text{lb}} \quad (19)$$

It is to be understood that ${}_1Q_2$ and ${}_1W_2$ are the heat and work transferred per pound as the state changes from 1 to 2. During this change of state, the internal energy has changed from u_1 to u_2 .

There are four nonflow processes which are given special names. (1) In the constant-pressure process, a substance may be heated or cooled while the pressure remains constant. An example would be that of a free piston moving vertically in a cylinder as heat is added to the substance contained in the cylinder. (2) In the constant-volume process, a substance may be heated or cooled at constant volume. (3) In the isothermal process, heat transfers and work transfers may take place while the temperature of the substance remains fixed. (4) The adiabatic process is one in which no transferred heat is added to or withdrawn from the substance.

Problems

1. A flywheel has a radius of gyration of 4 ft, weighs 4 tons, and revolves at 120 rpm. If it is brought to rest while a piston attached by a connecting rod and crank compresses 5 lb of air, how much internal energy has been added to the air? Neglect friction and heat losses.

2. Using the general energy equation, prove that for a freely falling body and also for a jet of fluid leaving an orifice the velocity \bar{V} equals $\sqrt{2gZ}$ where Z is the initial height of a freely falling body or the head of fluid on the orifice.

3. One pound of gas is compressed at a constant pressure of 100 psia in a cylinder 10 in. in diameter. The piston moves 24 in., and 25 Btu of heat are transferred from the cylinder to a water jacket during the compression.

What is the change of internal energy? Would you expect the temperature to rise or fall during the compression? If 20.2 Btu of heat had been radiated from the cylinder during the compression, what would be the change of internal energy?

4. A centrifugal compressor takes in air at 14 psia, 40 F, and has an internal energy of 10 Btu/lb. Air is discharged at 30 psia, 150 F, and has an internal energy of 29 Btu/lb. No heat transfers take place. Neglect change of potential and kinetic energies. Find the work done on the air per pound.

5. In a given state, steam has the following properties: pressure, 250 psia; specific volume, 1.8438 cu ft/lb; internal energy, 1115.8 Btu/lb with reference to a certain datum for measuring internal energy. The steam has a velocity of 120 fps and is 20 ft above a certain datum for measuring potential energy. Find the enthalpy and the total energy of 1 lb of steam with reference to the datum levels mentioned.

6. An oil having a specific gravity of 0.8 flows through a horizontal Venturi tube which has a throat $1\frac{5}{8}$ in. in diameter and an entrance 4 in. in diameter. A vertical U tube containing mercury has its two ends attached to the Venturi tube at the entrance and throat. If the difference of levels of the mercury columns is 6.28 in., what is the flow of oil in gpm?

7. Steam enters a nozzle at 140 psia and has an enthalpy of 1250 Btu/lb. The steam expands to a pressure of 14 psia and has an enthalpy of 1100 Btu/lb. Considering the velocity of the steam as it enters the nozzle to be negligible, what is the velocity of the steam in fps and mph leaving the nozzle?

8. Brine having a specific gravity of 1.25 is pumped from a tank located 10 ft below the ground level and under a pressure of 20 psia to a point 80 ft above the surface of the ground and at a pressure of 100 psia. The brine pump is located 20 ft above the ground, the inlet pipe is 4 in. in diameter, and the outlet pipe 2 in. in diameter. The brine is pumped at a rate of 120 gpm. What is the horsepower output of the pump?

9. Air enters a compressor at 14.5 psia and a specific volume of 13.3 cu ft/lb. The air is discharged at 90 psia and with a specific volume of 3.3 cu ft/lb. The internal energy is increased by 124 Btu/lb. The horsepower input is 24 hp, and 6 lb of air are compressed per minute. Find the energy carried away by the jacket cooling water and radiated from the bearings. Neglect the kinetic energy of the air entering and leaving.

10. A 50,000-kw. steam turbine uses 500,000 lb. of steam per hour. Steam enters at 1200 psia pressure; enthalpy, 1311 Btu/lb; specific volume, 0.4909 cu ft/lb and leaves at 1.5 in Hg pressure; enthalpy, 950 Btu per lb; specific volume, 380 cu ft/lb. The steam enters at an elevation 6 ft above the exhaust outlet. The inlet pipe diameter is 10 in., the exhaust outlet diameter is 10 ft. Find the change of potential energy, kinetic energy, internal energy, flow work, radiation, and work output expressed in Btu per pound of steam and as a percentage of the work output.

CHAPTER IV

POWER PLANTS

22. The Steam Power Plant.—One of the most familiar combinations of mechanisms employed for the conversion of heat into mechanical work is the steam power plant. Figure 5 exhibits the essential apparatus of a typical modern steam power plant. Let it be supposed that the

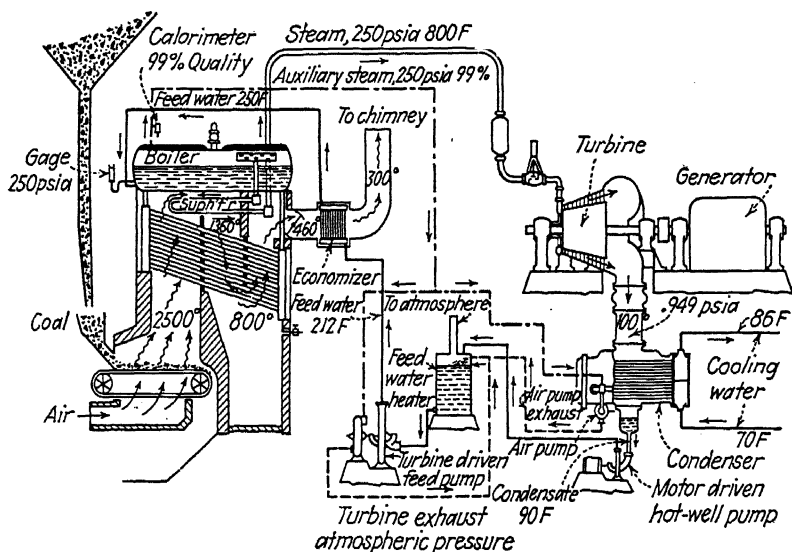


FIG. 5.

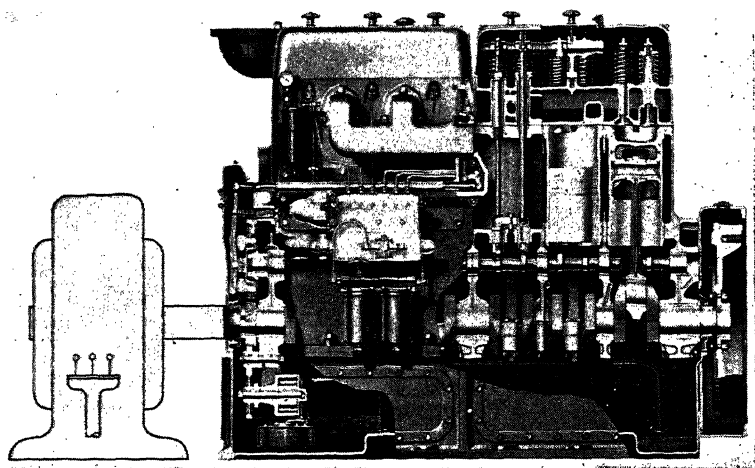
reader steps into this plant with a view to making an extended study of the operations and thermodynamic processes.

The first requirement is that he shall thoroughly familiarize himself with the relation of each piece of apparatus to all the others. He finds the steam, or water, or mixture of the two, to be confined within certain channels, which constitute a circuit or circuits, and upon these circuits

are threaded the various pieces of apparatus, each performing its own definite function.

The sole purpose of the whole assemblage of mechanisms that constitutes the power plant is to derive mechanical energy from heat. The power plant as a whole is therefore a heat engine in the thermodynamic sense.

23. The Internal Combustion Engine Power Plant.—The internal combustion engine power plant, Fig. 6, differs



(Courtesy of Waukesha Motor Company.)

FIG. 6.

from the steam power plant in several important respects. The working substance flowing through the plant is at all times in the gaseous state, whereas the working substance in the steam plant is liquid at some points of the plant and a vapor at other points. The entire flow of working substance takes place in one machine in the internal combustion engine plant, whereas in the steam power plant a separate piece of equipment is provided for the various operations. As the name implies, combustion takes place in the engine in which power is developed. Combustion in the steam plant takes place in the furnace of the steam generator. Usually the working substance in the steam plant is recirculated, whereas air and fuel are admitted to

the internal combustion engine and rejected to the atmosphere after performing useful work. In general, the internal combustion engine plant operates with higher temperature of the working substance than the temperature of the steam in the steam power plant.

24. The Essential Elements of a Heat Engine.—An inspection of either Fig. 5 or 6 reveals the fact that fluids flow through the systems and change their properties from place to place and from time to time. Heat is received at some points and rejected at other points. These fluids constitute the energy vehicle or *working substance*. The original *source* of the heat supplied to the working substance is the chemical energy of the fuel. Some energy is abstracted from the working substance in the turbine or engine and converted into useful mechanical work which in the case of the plants illustrated is immediately changed into electrical energy by the electric generator. The turbine or engine may be called the *heat utilizer*. It is evident that all the energy that has been imparted to the working substance at the source is not removed by the heat utilizer, because some is carried away by the cooling water in the condenser of the steam power plant or by the atmosphere with which the hot gases from the exhaust of the internal combustion engine mix. The condenser or atmosphere is referred to, thermodynamically, as the *refrigerator*. The four essential elements, then, of a heat power plant, are

The working substance.

The heat source.

The heat utilizer.

The refrigerator.

25. A Compressed-air System.—Figure 7 is a diagrammatic representation of the essential elements of a compressed-air plant. It consists of a compressor, which takes in air at *A*, at atmospheric conditions of pressure and temperature (14.5 psia and 70 F), and delivers it at *B* at a higher pressure, for example, 100 psia as indicated, to a

receiver near the compressor. The utilizer or engine, located usually at a considerable distance from the compressor, spends the energy of the compressed air and discharges its exhaust back into the atmosphere.

The air will be delivered from the compressor at *B* at a higher temperature than 70 F by virtue of the work done upon it. But since it pauses for a time in the receiver and must be conveyed through pipes of considerable length to the utilizer, it will be cooled, by conduction of heat to the surrounding air, to a temperature nearly if not quite as low as that of the atmosphere itself. In the figure it

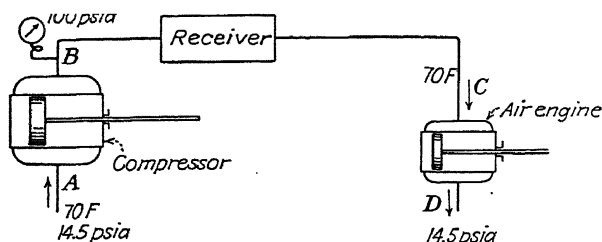


FIG. 7.

has been assumed that it has been cooled back to 70 F by the time it has arrived at *C*, the entrance to the air engine.

A compressed-air plant is not a heat engine in the same sense that the steam power plant is a heat engine. Its purpose is to *transmit* mechanical energy from one point to a more or less distant point or points, not to *derive* mechanical energy from the primary source of energy, *viz.*, heat. But the air in the compressed-air plant performs the same function as does the steam in the steam plant. It is a working substance or energy vehicle, which receives energy here and disposes of it there, and is subjected to heating effects at some points and cooling effects at others.

26. The Refrigerating Plant.—Steam may be called a vaporous substance because, within the range of temperature and pressure included in its cycle, it is at times in the gaseous and at other times in the liquid state. In the use of steam as a working medium in a heat engine, heat is

first imparted to it, then energy is extracted from it as mechanical work, and finally the residue of heat is discarded at a lower temperature to some body capable of receiving it—usually to cooling water in a condenser, by means of which the steam is liquefied.

Now it is possible to reverse all the above operations. Let the condensate pump of Fig. 5 take water *from* the hot well and deliver it into the condenser shell, partly submerging the tubes. With a vacuum such that the boiling

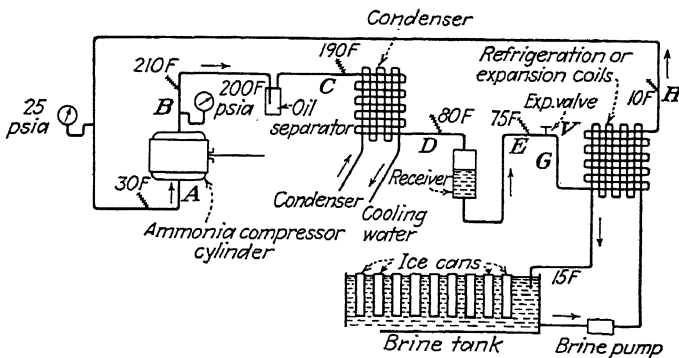


FIG. 8.

temperature of what is now the feed water is lower than the temperature of the circulating water inside the tubes, evaporation will now take place, and what was the condenser of the heat engine now becomes a boiler. If work is done upon the steam thus formed, by compressing it, the steam will be changed from a low-pressure, low-temperature state to high pressure and correspondingly high temperature. This may be accomplished by turning the utilizer of the heat-engine system into a compressor. Finally, the compressed steam could be condensed in the boiler (which now becomes the condenser in the reversed cycle) by circulating gases of somewhat lower temperature than the saturation temperature of the steam but still many degrees higher than that of the water from which the heat originally came. Thus, by the use of essentially the same elements employed in a steam heat engine and by reversing

the cycle of operations, it becomes possible to "pump" heat from a lower to a higher temperature level, and the plant becomes in effect a *mechanical refrigerating system*.

Mechanical refrigeration can be effected by the use of a "permanent gas" like air, as the working medium, but the advantages of a "vaporous substance," which like steam changes in state from liquid to gas and back again in the course of its complete cycle, are so pronounced that practically all machines are of the vapor type. There are a number of vaporous substances that have properties that make them suitable as refrigerants. Those most commonly used are ammonia, NH_3 ; Freon, F12, CCl_2F_2 ; carbon dioxide, CO_2 ; Carrene, CH_2Cl_2 ; and water, H_2O .

CHAPTER V

WORKING SUBSTANCES

27. Changes of State Accomplished by Heating a Substance.—All substances may appear in three phases. The state of a substance changes as its properties change, and the change of state is usually brought about by performing work or adding heat. Let water be chosen as a working substance, and let its properties be investigated as heat is added to it. Starting with 1 lb of ice at -40°F , and exposed to atmospheric pressure (14.696 psia), as heat is added at constant pressure the enthalpy increases and the temperature rises. The temperature continues to rise as heat is added until a temperature of 32°F is reached. This information may be shown on a diagram such as Fig. 9. The heat added between A and B could be calculated by Eq. (3)

$$h_B - h_A = \bar{W}c_i(t_B - t_A) = 1 \times c_i(32 - [-40])$$

As more heat is added to the water, the ice melts without an increase in temperature and the phase changes from solid to liquid. The heat added is called the *latent heat of fusion*, $h_{if} = h_c - h_B$. After all the ice has melted, the temperature again rises with the addition of heat and continues to rise until a temperature of 212°F is reached, point D . The heat added between C and D is

$$h_D - h_c = \bar{W} \times c_f(t_D - t_c) = 1 \times c_f(212 - 32).$$

If more heat is added, boiling takes place without a rise of temperature. Another name for the boiling point is *saturation temperature*. The heat added during the change of phase from liquid to vapor is called the *latent heat of evaporation*, $h_{fg} = h_E - h_D$. After all the liquid water has

been vaporized, the temperature will rise above the saturation temperature and, as such, will be called *superheated steam*. At some point such as F , say 800 F, an amount of enthalpy h_F will be stored in the pound of water

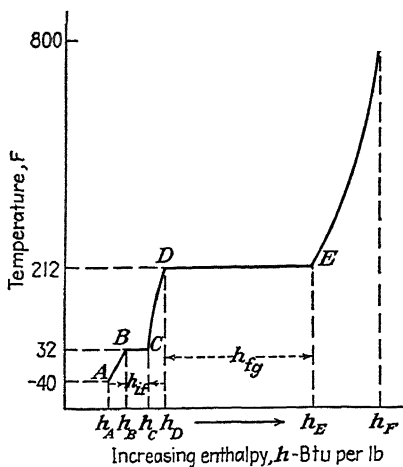


FIG. 9.

vapor. The heat added beyond the point E is called the *heat of superheat* and equals

$$h_F - h_E = \bar{W}c_g(h_F - h_E) = 1 \times c_g(800 - 212).$$

The calculation of heat added from A to F could be repeated for differences of enthalpy between any two points. It would be desirable to choose some reference point to use in the calculation of enthalpies at the various points. Point C is a convenient point for power-plant calculations, and enthalpies for steam are usually referred to the enthalpy in 1 lb of water at 32 F. For working substances used in refrigeration plants, liquid at -40 F frequently has been used as a reference point. Figure 9 may be redrawn as Fig. 10 in which point C becomes the reference point and the enthalpies given along the abscissa are the enthalpies above that in 1 lb of liquid water at 32 F.

Molecules are continually leaving the free surface of any liquid, and as they leave the surface they become vapor

WORKING SUBSTANCES

molecules above the surface. Some of these vapor molecules meet the surface and become liquid. When the rate at which molecules leave the surface equals the rate at which molecules enter the surface, equilibrium exists and there will be a density of molecules above the surface which develop a certain pressure called the *vapor pressure*. If the temperature of the liquid is raised, molecules leave the surface and enter it at a faster rate and the velocity and density of molecules above the surface is increased, providing a greater vapor pressure. For each temperature of a given substance, there is a definite vapor pressure. Now imagine a bubble to be formed under the surface. A vapor pressure exists in this bubble depending upon the temperature. If the vapor pressure is less than the pressure applied on the surface, the bubble will collapse or would not have formed. When water is heated to a temperature of 212 F, its vapor pressure becomes 14.696 psia, and thus bubbles are able to form and boiling takes place. The pressure corresponding to the saturation temperature is called the *saturation pressure*, and it is the vapor pressure equal to the pressure on the surface when ebullition takes place.

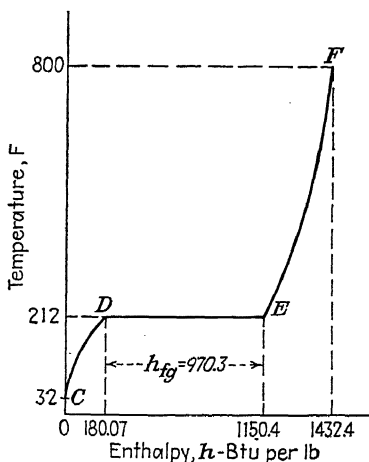


FIG. 10.

If the pressure on the surface of a liquid is increased, the saturation temperature is higher, and the liquid will have to be heated more before ebullition takes place. At higher saturation temperatures and pressures, the latent heat of evaporation becomes smaller, *i.e.*, points *E* and *D* are closer as shown in Fig. 11. At a temperature of 705.4 F, the saturation pressure is 3206.2 psia and the latent heat of evaporation is zero. Point *G* is called the *critical point*,

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and the saturation pressure and temperature at G are called the *critical pressure* and *critical temperature*. Any point on the line $CDD'D''G$ will represent the liquid phase. Any point on the line $EE'E''G$ will represent the vapor phase. Points to the right of this line will also represent the vapor phase, but the vapor will be superheated. Inside the steam dome $CDD'D''GE''E'E$, a mixture of liquid and vapor exists. The concentration of the vapor is called the *quality*. If half the latent heat has been added, half a pound of liquid is mixed with half a pound of vapor and the

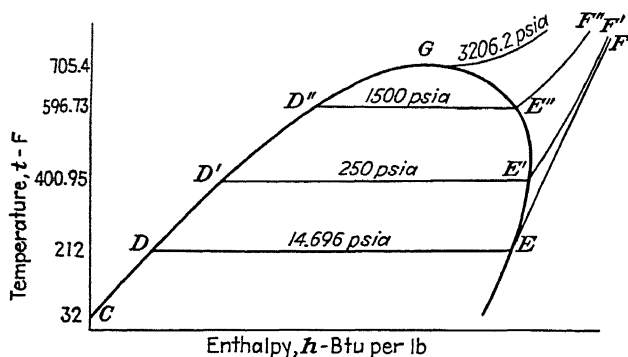
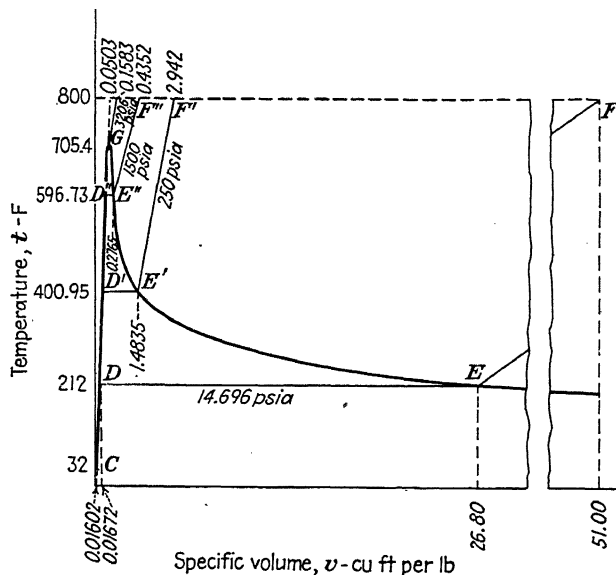


FIG. 11.

quality is 50 per cent. If nine-tenths of the latent heat has been added, nine-tenths of a pound of vapor is mixed with one-tenth of a pound of liquid and the quality is 90 per cent. When all the latent heat has been added to 1 lb of liquid but no heat of superheat has been added, we have 1 lb of dry and saturated vapor or 100 per cent quality, represented by a point such as E . The word "saturated" refers to the saturation temperature. Dry and saturated means 100 per cent quality and at the saturation temperature. Wet and saturated means that the quality is less than 100 per cent, but the steam is at the saturation temperature. Saturated liquid would refer to liquid, 0 per cent quality, at the saturation temperature. Per cent moisture is 100 minus per cent quality. Thus 90 per cent quality could be expressed as 10 per cent moisture.

If specific volume should be used as the property for the abscissa, the history of the change of state would be represented by a similar graph. Figure 12 shows temperature plotted against specific volume with points for identical states lettered the same as in Fig. 11. At a quality of 50 per cent, one-half of the increase in volume v_{fg} has taken place; at 90 per cent quality, nine-tenths of the



perature are called *vapors*, and as such the fluids do not behave in accordance with the gas laws. The distance between molecules is no longer large compared with the molecular diameters, and there are attractive forces between molecules. The higher the temperature and the smaller the pressure, the more nearly do the vapor molecules behave like gas molecules. Very highly superheated vapors approach the perfect gas states, and the gas laws

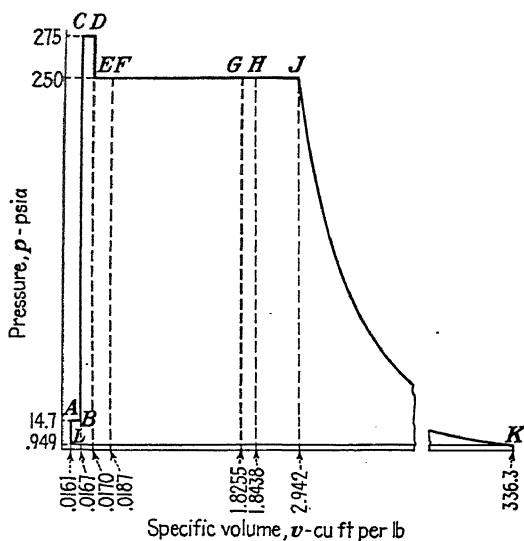


FIG. 13.

apply with reasonable accuracy. The properties of vapors must be determined by experiment and are usually tabulated in vapor tables. The vapor table for water is referred to as a steam table. The most accurate data of steam are tabulated in Keenan and Keyes "Thermodynamic Properties of Steam,"¹ and the data used in the examples in this book are taken from these Steam Tables.

29. The Pressure-volume Diagram.—In Fig. 13, pressures in pounds per square inch absolute are represented by vertical, and volumes in cubic feet by horizontal coordi-

¹ KEENAN, J. H. and F. G. KEYES, "Thermodynamic Properties of Steam," John Wiley & Sons, Inc., New York, 1936.

nates. Consider 1 lb of the steam (or water or mixture). It is proposed to picture the manner of variation of these two properties, *viz.*, pressure and volume, as the pound of substance passes through a complete cycle in the main circuit of the power plant of Fig. 5.

As a helpful preliminary step, the circuit diagram of Fig. 14 is first drawn, and the data of Fig. 5 are marked thereon. Beginning at the point where water is about to enter the

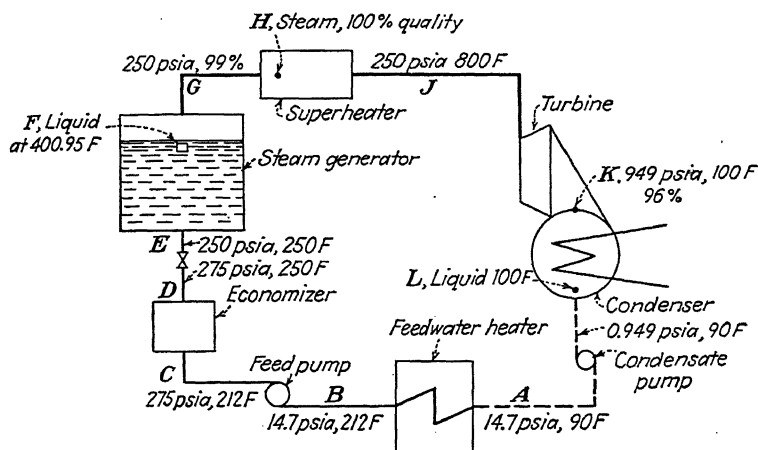


FIG. 14.

heater, its temperature is observed to be 90 F, and it is subjected to the barometric pressure, since the heater is open to the atmosphere. What volume is occupied by the pound of substance? Water is practically incompressible, so its volume is not sensibly affected by the pressure, although it is influenced by the temperature. Values for the volume occupied by 1 lb of water, in cubic feet, at various temperatures, are given in column 3, Tables 1 and 2 of the Steam Tables. At 90 F, the specific volume is 0.01610 cu ft. The pressure at the point of observation in the circuit is atmospheric, or about 14.7 psia. Let point A, Fig. 13, represent 14.7 psia pressure and 0.01610 cu ft volume.

In a similar manner, the specific volumes at various other points of the circuit can be determined and set down in

Table 1, from which the values for the pressure-volume sketch of Fig. 13 can be conveniently taken.

TABLE 1.—DATA FOR PRESSURE-VOLUME SKETCH OF FIG. 13

Point in circuit (Fig. 14)	State of working substance	Pressure, pounds per square inch absolute	Temperature, degrees Fahrenheit	Specific volume, cu ft/lb	Steam Table reference	
					Table	Column
A	Water	14.7	90	0.0161	1	4
B	Water	14.7	212	0.0167	1	4
C	Water	275	212	0.0167	1	4
D	Water	275	250	0.0170	1	4
E	Water	250	250	0.0170	1	4
F	Water	250	400.95	0.0187	2	3
G	99% Steam	250	400.95	1.8255	2	3, 4
H	100% Steam	250	400.95	1.8438	2	4
J	Superheated steam	250	800	2.942	3	
K	96% Steam	0.949	100	336.3	1	6
L	Water	0.949	100	0.0161	1	4

30. The Pressure-volume Diagram, Neglecting Water Volumes.—All the changes of volume of the water, as pic-

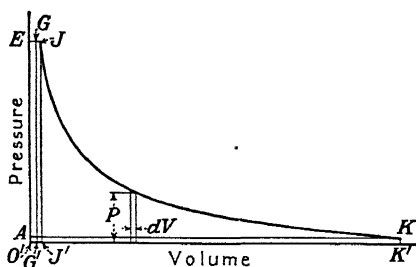


FIG. 15.

tured in Fig. 13, are trifling as compared with the maximum volumes of the diagram. In fact, the entire volume of the water itself is so minute that it is usually negligible in the consideration of the volumes involved in the complete circuit. Disregarding water volumes, the diagram is drawn as in Fig. 15.

It is to be noted that the diagram of Fig. 15 (or Fig. 13) is not an *indicator diagram*. An indicator diagram is a picture of the relation between pressures and volumes within the cylinder of an engine. The diagrams of Figs. 13 and 15 picture the relations of pressure and volume of the water throughout its cycle in the plant.

31. Significance of Area on the pV Diagram.—During a part of its cycle, the pound of steam (*i.e.*, the working substance) increases in volume, thus overcoming resisting forces and doing external work; while in other operations of the cycle, work must be done upon the steam to compress and condense it. The net result is that the steam does more work than is done upon it, which is the desired effect or ultimate objective of the whole assemblage of mechanisms. The area of the diagram of Fig. 15 is proportional to the net work done by the steam. The area under any line that represents a given portion of the process as, for example, the area $G'GJJ'$, under the line GJ (which is the line of superheating), shows the work done by the steam because of its growing volume while being heated in the superheater. The area $K'KAO'$ is the work that must be done upon the steam in pushing it into the condenser. The area $AEJK$ is the net work of the entire cycle, neglecting minor losses.

The unit of work is the foot-pound, which, in general, is the product of force in pounds times distance in feet; or pressure in pounds per square foot times change of volume in cubic feet. The actual area of the diagram of Fig. 15 even if drawn to scale does not represent work in foot-pounds, because the pressure is expressed in pounds per square inch, for convenience in conforming to the common unit of measurement of pressure.

In the case of the work area $G'GJJ'$, the work done by the steam in the operation is

$$250 \times 144 \times (2.942 - 1.8255) = 40,176 \text{ ft-lb.}$$

In the case of the area $J'JKK'$, the computation is not so

simple. The expression

$$\text{Work} = \int p \, dV \quad (6)$$

must be written and evaluated by a knowledge of the law of the curve JK . In the case of steam, a more direct method can be employed by computing or observing the net loss of enthalpy from J to K , as will be shown later.

32. Entropy.—The pV diagram may be considered as a work diagram. One coordinate, the pressure, represents what may be called an *intensity* factor; the other, the volume, represents a *distribution* factor. Their product, in general, is work, a form of energy. Both of these factors are readily perceptible to the senses, and there is no difficulty in forming a conception of them.

Heat is also a form of energy, and a compound quantity, the product of two simpler elements. One of these is an *intensity* factor, and the other is a *distribution* factor.

The intensity factor is temperature, which is readily perceptible to the senses and is capable of direct measurement with instruments that respond to its influence. But temperature alone is no more a measure of heat than pressure alone would be of work. A distribution factor must be conceived which bears a relation to heat similar to that which volume bears to work. This distribution factor is called *entropy*. Entropy, unlike pressure, volume, and temperature, is a property that is not perceptible to the senses, nor is any instrument yet devised responsive to its influence.

33. The Temperature-entropy Diagram.—In Fig. 16 the ordinates represent temperatures; and the abscissas, entropy. Absolute temperature, degrees Fahrenheit, is used, just as absolute pressure is employed for the work diagram. In the case of entropy, some arbitrary starting point must be made use of, since the true zero of entropy is at negative infinity (see Art. 106).

In Fig. 16, let AB represent the path of a change. The entropy changes from S_A to S_B . The heat supplied to the

working substance is the area under the path of change, *viz.*, A_1ABB_1 , just as area under a path of change on a pV diagram represents work.

34. Representation of Changes of State of the Steam.

In Fig. 17, it is proposed to picture the manner of variation of temperature and entropy and to show the heat changes in 1 lb of the steam of the main circuit of Fig. 5, just as the variations in pressure and volume were pictured in Fig. 13

or 15. The unit of entropy has not yet been considered, but this will be deferred for the present. The values of

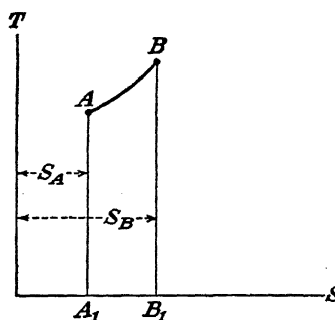


FIG. 16.

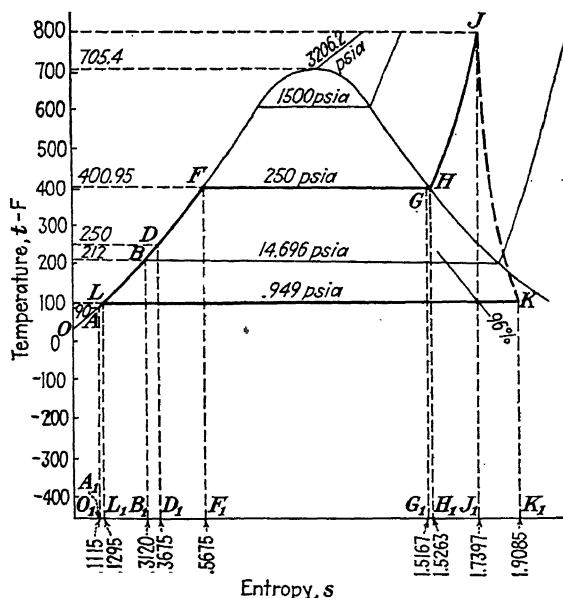


FIG. 17.

entropy will be taken from the Steam Tables, corresponding to those of the observed properties as marked on Fig. 5, or the circuit diagram of Fig. 14.

It will be convenient to tabulate the values of temperature and entropy for each point in the circuit, which is done in Table 2, below.

No thermal changes take place in the feed pump or condensate pump, and hence there is no entropy change. Point *F*, Fig. 14, represents 1 lb of water in the boiler which has been heated to the boiling temperature of 400.95 F and is just ready to be evaporated into steam. Point *G* represents the pound of water consisting of a mixture of 0.99 lb of steam and 0.01 lb of water, and the total entropy consists of the entire entropy put into the pound of water in bringing its temperature up to 400.95 F (0.5675) plus the entropy of evaporation of 0.99 lb of water into steam (0.99×0.9588). Point *H* represents the place in the superheater at which the evaporation of the remaining 0.01 lb of water is just completed. From this point on, the temperature of the steam is increased, departing farther and farther from the saturation temperature corresponding to the pressure, as heat is received in the superheater.

TABLE 2.—DATA FOR TEMPERATURE-ENTROPY DIAGRAM OF FIG. 17

Point in circuit (Fig. 14)	State of H ₂ O	Temperature, degrees Fahrenheit	Entropy	Steam Table reference		Calculations
				Table	Column	
<i>A</i>	Water	90	0.1115	1	10	0.5675 + (0.99 × .9588)
<i>B</i>	Water	212	0.3120	1	10	
<i>D</i>	Water	250	0.3675	1	10	
<i>F</i>	Water	400.95	0.5675	2	8	
<i>G</i>	99% steam	400.95	1.5167	2	8, 9	
<i>H</i>	100% steam	400.95	1.5263	2	10	0.1295 + (0.96 × 1.8531)
<i>J</i>	Superheated steam	800	1.7397	3		
<i>K</i>	96% steam	100	1.9085	1	10, 11	
<i>L</i>	Water	100	0.1295	1	10	

At point *K*, where the H_2O leaves the turbine, there is only 0.96 lb of vapor to give up entropy of evaporation.

Hence, the total entropy at *K* is made up of the entropy of evaporation of 0.96 lb of steam plus the entropy of a full pound of water which the water will contain when completely condensed at *L*, Fig. 14, at 100 F.

Table 2 now furnishes all the necessary data in columns 3 and 4 for the construction of the temperature-entropy diagram of Fig. 17. In such a diagram, areas to represent heat quantities received or given out during a given operation must extend down to the absolute zero of temperature. However, all Steam Table data are given in terms of thermometer temperatures. It is therefore convenient to lay off the temperature scale in terms of ordinary temperatures and to place the base line of the diagram at -460 F, as shown in Fig. 17. In this way, the cumbersome process of adding 460 to Steam Table temperature is avoided, and yet heat quantities are properly represented by areas on the diagram.

Enthalpies in the Steam Tables are those in excess of the enthalpy in 1 lb of liquid water at 32 F. Areas on the temperature-entropy diagram represent heat when temperature is measured in degrees Fahrenheit absolute. Hence, it becomes desirable to record the entropies in the Steam Tables as the entropy in excess of the entropy of 1 lb of liquid water at 32 F. It is to be observed that the temperature-entropy diagram of Fig. 17 is similar to the temperature-enthalpy diagram of Fig. 11 and the temperature specific volume diagram of Fig. 12. At the critical point, the entropy of liquid and vapor are equal.

35. Vapor Tables.—During the vaporization of a liquid, there is a large change in volume, and considerable heat is added during the change of phase unless the vapor state is very close to the liquid state. The latent heat of evaporation may be divided into two parts. The external latent heat of evaporation is that part of the latent heat of evaporation which is transformed into external work while

the volume is increasing. Using the data of Fig. 13, the volume is increased to 1.8438 cu ft at point *H* where it is dry and saturated steam, from a volume of 0.0187 cu ft at point *F* where it is saturated liquid. The volume increases at a constant pressure of 250 psia, and hence the work per pound is $250 \times 144(1.8438 - 0.0187) = 65,704 \text{ ft-lb} = 84.4 \text{ Btu}$. The Steam Tables show a latent heat of evaporation, $h_{fg} = 825.1 \text{ Btu per lb.}$ The other part of the latent heat is the internal latent heat of evaporation and equals $825.1 - 84.4 = 740.7 \text{ Btu per lb.}$ The internal part of the latent heat energy is used to pull the molecules apart and overcome the attractive forces. Even after the liquid has become a vapor, there exist attractive forces between molecules, although smaller than those occurring during vaporization, and a part of the heat of superheat of a vapor overcomes these forces as the volume continues to increase and the remainder of the heat of superheat raises the temperature of the vapor, *i.e.*, increases the velocity of the molecules. At high superheats, the molecules are far enough apart so that the molecular attractions are negligible, and then all the additional heat raises the temperature and the fluid behaves very nearly as a perfect gas.

The specific heats c_i , c_f , and c_g are not constant. The vapor tables consider the variation of specific heat. For example, the enthalpy of saturated liquid, h_f , at 250 psia from Table 3 of the Steam Tables is 376.00 Btu per lb. The heat added between 32 F and 400.95 F is therefore ${}_cQ_D = \bar{W} \times c_f(t_D - t_c)$, or $376 = 1 \times c_f(400.95 - 32)$, from which $c = 1.0191$ the average specific heat for the temperature range 32 – 400.95 F, and not 1.0000 as sometimes assumed. If ${}_cQ_D$ should be calculated using 1.0000 as the specific heat, an error of about 2 per cent would ensue. For larger temperature differences, the error becomes larger.

The Steam Tables are divided into three tabulations. In Table 1 the first column is saturation temperature. In Table 2 the first column is absolute pressure. Both tables

give the same information, but the duplication is provided as a matter of convenience. Table 3 is a superheat table and gives the enthalpy, specific volume, and entropy for superheat states. The internal energies for states not in the superheat region are given in Table 2.

Vapor tables for other working substances are available, although they are not so complete as the Steam Tables.

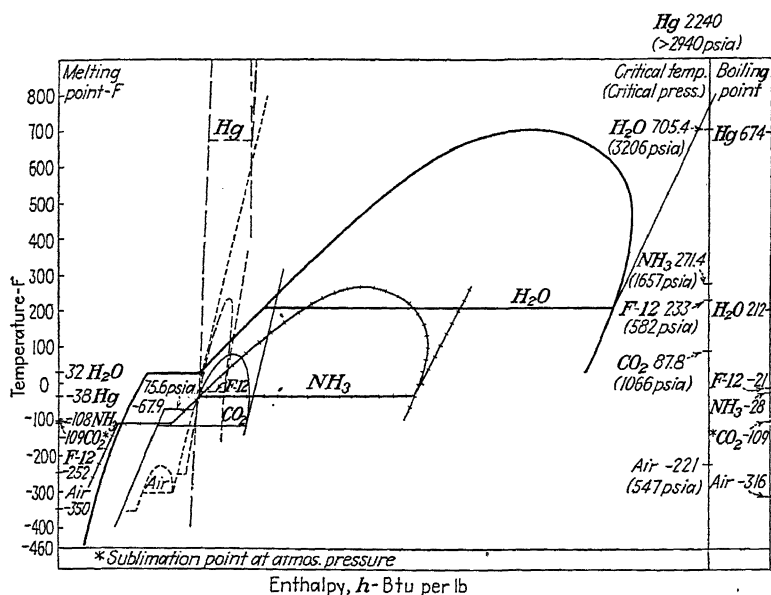


FIG. 18.

Temperature-enthalpy diagrams for six common working substances used in power plants and refrigeration plants are shown on Fig. 18. The similarity of shape and the variation of the saturation temperature at atmospheric pressure are to be noted. At 700 F, all six substances are in the gaseous state at atmospheric pressure. Air is highly superheated, but mercury is a vapor only slightly superheated.

The enthalpies of each point in the circuit of the power plant of Figs. 5 and 14 are tabulated in Table 3. With

the observational data of pressure, temperature, and quality, the enthalpies may be read from the Steam Tables.

TABLE 3.—ENTHALPIES IN POWER-PLANT CIRCUIT

Point in circuit (Fig. 14)	State of the H ₂ O	Pressure, pounds per square inch absolute	Temperature, degrees Fahrenheit	Enthalpy (water at 32 F = 0)		Remarks
				Area (Fig. 17)	Btu Steam Tables	
A	Water	14.7	90.0	O_1OAA_1	57.99	
B	Water	14.7	212.0	O_1OBB_1	180.07	
D	Water	275.0	250.0	O_1ODD_1	218.48	
F	Water	250.0	400.95	O_1OFF_1	376.0	Enthalpy of liquid for 250 psia.
G	99% steam	250.0	400.95	O_1OFGG_1	1192.85	= 376.0 + 0.99 × 825.1
H	100% steam	250.0	400.95	O_1OFHH_1	1201.1	Enthalpy saturated steam
J	Superheated steam	250.0	800.0	O_1OFHJJ_1	1422.7	Enthalpy superheated steam
K	96% steam	0.949	100.0	O_1OLKK_1	1063.68	= 67.97 + 0.960 × 1037.2
L	Water	0.949	100.0	O_1OLL_1	67.97	Enthalpy of liquid at 0.949 psia

Having at hand the enthalpies at all points of the circuit, the amount of heat contributed or withdrawn in each element of the plant can easily be found. The procedure is indicated in Table 4, which shows the quantity of heat transferred per pound of steam in each element and the total for the steam cycle. Table 4 shows that in the course of its progress around the circuit of the plant each pound of steam receives 1364.71 Btu and rejects 1005.69 Btu. The difference, 359 Btu, is the amount of heat energy that has been *transformed* into work energy by the utilizer, or, in this case, the turbine. *Thermal efficiency* of a plant (in respect to the steam cycle) is the ratio of the amount of heat transformed into work to the amount of heat supplied to the steam. In the case of our example, thermal efficiency:

$$\eta_{th} = \frac{359}{136} = 0.263, \text{ or } 26.3 \text{ per cent}$$

TABLE 4.—HEAT TRANSFERRED BY EACH ELEMENT OF THE PLANT

Element of plant (Figs. 5 and 14)	Operation (Fig. 17)	Heat quantities			
		Area (Fig. 17)	Difference in enthalpy (Table 3)	Btu per pound of steam	
				Received	Rejected
Feed-water heater.	<i>AB</i>	A_1ABB_1	180.07 — 57.99	= 122.08	
Economizer.....	<i>BD</i>	B_1BDD_1	218.48 — 180.07	= 38.41	
Boiler.....	<i>DFG</i>	D_1DFGG_1	1192.85 — 218.48	= 974.37	
Superheater.....	<i>GHJ</i>	G_1GHJJ_1	1422.7 — 1192.85	= 229.85	
Condenser.....	<i>KLA</i>	A_1ALKK_1	1063.68 — 57.99	=	1005.69
Complete circuit..	1364.71	1005.69

36. The Entropies of the Steam Tables.—In the case of a constant-temperature change, of which evaporation is an example, the relation among temperature, entropy, and heat quantity is quite simple. In Fig. 19, the line *AB* represents a change at the constant temperature of T_A . The heat supplied during the operation A_1ABB_1 is the product of the temperature times the change in entropy. Or

$$A_1ABB_1 = T_A \times (S_B - S_A)$$

For example, it is found from the Steam Tables that the latent heat of steam at 250 psia is 825.1 Btu, which may be represented by area A_1ABB_1 of Fig. 19. The temperature of evaporation

$$T_A = 400.95 + 459.69 = 860.64 \text{ F abs}$$

Hence

$$\text{Change of entropy from } A \text{ to } B = \frac{825.1}{860.64} = 0.9588$$

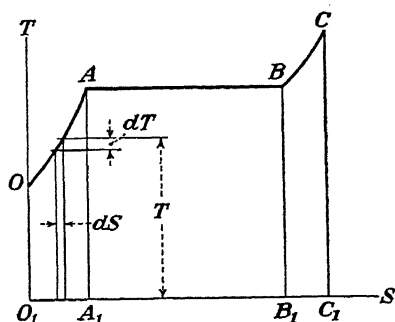


FIG. 19.

which agrees with the entropy of evaporation given in the Steam Tables for 250 psia. All entropies of evaporation given in the Steam Tables are calculated in the above manner.

When heat is applied, accompanied by a change of temperature, as in the case of heating water, or superheating steam, the simple relation above does not hold and an integration of a differential expression must be resorted to. Consider the heating of 1 lb of water along OA , Fig. 19. The area of the differential strip is

$$dQ = T dS$$

In applying the heat dQ , the temperature of the water will be raised dT . If the specific heat is c , then the amount of heat is given by another expression

$$dQ = c dT \quad (3)$$

whence

$$T dS = c dT$$

$$dS = c \frac{dT}{T}$$

The entire change of entropy from O to A is given by the integration of the last expression between T_o and T_A .

$$S_A - S_o = c \int_{T_o}^{T_A} \frac{dT}{T} = c \log_e \frac{T_A}{T_o} \quad (20)$$

To compute the change of entropy for a given change of temperature, it is only necessary to know the mean value of the specific heat and the terminal temperatures.

In Fig. 19, let point A again represent the saturation temperature at 250 psia. The area O_1OAA_1 is the amount of heat required to raise the temperature of 1 lb of water from 32 F to 400.95 F (enthalpy of liquid), or 376.00 Btu. The change of temperature is 400.95 F - 32 F = 468.95. The mean specific heat of the water is therefore 376.00/468.95 = 1.0191. Entropy change from

$$O \text{ to } A = 1.0191 \quad \frac{400.95 + 459.69}{32 + 459.69} = 0.5675,$$

which agrees with the entropy of liquid given for 250 psia.

If the steam at 250 psia is superheated to 800 F at point *C*, Fig. 19, the mean specific heat of superheat at constant pressure is equal to the heat of superheat B_1BCC_1 , divided by the temperature change from *B* to *C*.

$$\begin{aligned}\text{Enthalpy of steam at 250 psia and 800 F} \\ = 1422.7 \text{ Btu (Table 3)}\end{aligned}$$

$$\begin{aligned}\text{Enthalpy of steam at 250 psia saturated} \\ = 1201.1\end{aligned}$$

$$\begin{aligned}\text{Heat of superheat} = \text{area } B_1BCC_1 \\ = 221.6\end{aligned}$$

$$\begin{aligned}\text{Temperature change from } B \text{ to } C = 800 - 400.95 \\ = 399.05\end{aligned}$$

Let c_p = mean specific heat at constant pressure.
Then

$$c_p = \frac{221.6}{399.05} = 0.5553$$

$$\begin{aligned}\text{Entropy change from } B \text{ to } C \\ = 0.5553 \log_e \frac{800 + 459.69}{400.95} = 0.2134\end{aligned}$$

$$\text{Total entropy of point } C = (\text{entropy at } B) + 0.2134$$

Entropy at *B* is the "entropy of saturated vapor" from the Steam Tables and for 250 psia is found to be 1.5263. Hence entropy at *C* = 1.5263 + 0.2134 = 1.7397, which agrees exactly with the value in the Steam Tables for 250 psia and 800 F.

37. Structure of the Mollier Diagram.—It has already been pointed out that any two independent properties of steam may be used as coordinates to represent changes of state. For the Mollier diagram, enthalpy and entropy are chosen (see Fig. 20). The enthalpy and entropy are measured above their values for 1 lb of liquid water at 32 F, and hence point *O* is on the liquid line at 32 F. As heat is applied, the entropy increases, the state point changing along the liquid line *OA* which is similar to that

of the temperature-entropy diagram. Suppose evaporation begins at *A*. As heat is applied, the entropy increases in direct proportion, and the evaporation line *AB* is straight. During superheating, the entropy continues to increase with the heat although with a slight curvature of the line *BC*. The line *OABC* is a line of constant pressure.

Now let other constant-pressure lines, such as *ODEF* and *OGHJ*, be drawn. Through the points *B*, *E*, and *H*, located by plotting the values of enthalpy and entropy for the 100 per cent quality conditions for the several

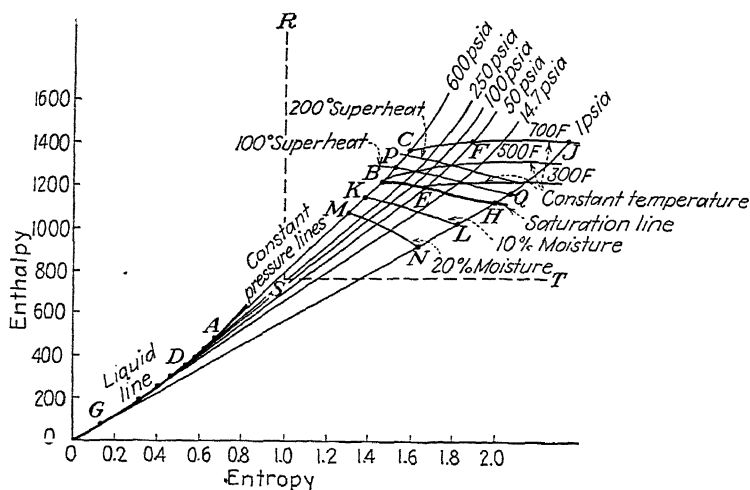


FIG. 20.

pressures, the line *BH* is drawn; it is the line of 100 per cent quality, or the saturation line. The area below *BH* is the region of wet steam; and that above is the region of superheat. *KL* and *MN* are constant-quality lines. *PQ* is a line of constant superheat. Horizontal lines represent constant enthalpy, and vertical lines constant entropy.

A working Mollier diagram, drawn to a large scale, is included with the Steam Tables and consists of the upper right-hand part (enclosed within the angle *RST*) of the complete chart represented by Fig. 20.

38. Diagrams.—The change of state has been pictured on several combinations of coordinates. Pressure-volume diagrams are of particular interest because areas on them represent work. The temperature-entropy diagram is very useful for showing heat quantities since areas on the T - S diagram represent heat. In addition to diagrams whose coordinates are properties, flow diagrams serve the purpose of showing the sequence and data involved in flow processes. Figure 14 is typical of a flow diagram. Flow diagrams showing the distribution of energy or heat-balance diagrams such as Fig. 38 are an aid to visualizing the distribution of work and heat in thermodynamic processes. The student will find the use of diagrams very helpful in setting up and in solving problems as well as an aid to the understanding of thermodynamic principles.

Problems

1. For a change of 1 psia pressure, does the temperature of saturated steam change more in the high-pressure region, or in the low? Show by plotting the curve of temperature (as ordinate) against pressure (as abscissa) through the range 1 to 2500 psia.

2. Show, by a curve, how the volume of 1 lb of dry, saturated steam (specific volume) changes with pressure for the range of 1 to 300 psia.

3. Show, by a curve for some particular pressure, for example, 100 psia, how specific volume of superheated steam varies with absolute temperature, laying off scales from zero of absolute temperature and specific volume. For a perfect gas, this curve would be part of a straight line passing through the zero of coordinates, according to Charles's law.

4. When 1 lb of water at 50 psia pressure and 281 F temperature is changed completely into steam at the same pressure and temperature, what is the increase in volume?

5. Steam containing moisture is exhausted from a turbine at 29 in. Hg vacuum. What is its temperature? If the steam contained no moisture and had no superheat, what volume would 1 lb occupy?

6. In a water-vapor refrigeration plant, a vacuum pump maintains a pressure of 0.1475 psia in a vessel containing water. Some of the water evaporating at this pressure cools the remainder to a temperature of 45 F. In a 100-ton plant, about 1200 lb of water per hour would have to be evaporated. How many cubic feet of steam per hour must be handled by the vacuum pump?

7. Calculate the volume of 4 lb of steam at 1800 psia, 90 per cent quality.

8. What is the entropy of 1 lb of steam at 575 psia, 575 F?

9. Exercises in the use of the Mollier diagram.

a. Ascertain the enthalpy for each of the following conditions:

- (1) 200 psia pressure, 500 F temperature.
- (2) 450 psia pressure, 540 F superheat.
- (3) 160 psia pressure, 2.2 per cent moisture.
- (4) 100 psia pressure, 98 per cent quality.
- (5) 15 psia pressure, 270 F temperature.
- (6) 16 psia pressure, 1.620 entropy.
- (7) 0.2 psia pressure, 30 per cent moisture.
- (8) 1.0 psia pressure, 1.717 entropy.

b. (1) What is the percentage moisture in steam at 20 psia pressure and 1120 Btu enthalpy?

(2) What is the pressure and quality of steam at 982 Btu enthalpy and 1.825 entropy?

(3) What is the percentage moisture in steam at 120 psia pressure and 1174 Btu enthalpy?

(4) At what constant pressure does it require the maximum amount of heat to make 1 lb of saturated steam from water at 32 F?

10. Fill in the blank spaces of the following table using Steam Tables and Mollier diagram. Under condition, state degrees of superheat or quality.

p , psia	v , cu ft/lb	t , F	h , Btu/lb	s , units/lb	Condition
100	5.589
....	350	1000
....	1.7566	$x = 100\%$
3000	$\Delta t = 104.6\text{ F}$
60	$x = 45\%$
60	250

11. Steam is conveyed at the rate of 30,000 lb/hr through a 6-in. pipe line 200 ft long, with a drop of 5 psi in pressure from the initial value of 100 psia. Loss of heat to the air is 300 Btu/hr — sq ft. If the initial moisture content is 1 per cent, what is the moisture content at the point of delivery?

12. Determine the mean specific heat of superheated steam for the ranges 0° to 100° and 0° to 200° of superheat for pressures of 15, 200, 600, and 1200 psia. (It will be found convenient to use the Mollier diagram in determining the necessary enthalpies.)

13. *True or instantaneous* specific heat is the number of Btu required to raise the temperature of 1 lb of a substance 1 Fahrenheit degree. Determine approximately the instantaneous specific heat of *water* at temperatures in the vicinity of 70, 212, 400, 500, 600, and 705 F.

14. Plot specific heat vs. temperature for the temperatures of Prob. 13.

15. Determine the instantaneous specific heat (at constant pressure) of *superheated steam* near the saturation temperatures for 15, 200, 600, 1200, 1800, and 3206 psia.

16. A pound of air is heated at constant pressure from 40 F to 440 F. Take specific heat as 0.243 Btu.

a. Calculate amount of heat required, and change of entropy.

b. Calculate change of entropy from 40 F to intermediate temperatures of 140, 240, and 340 F.

c. Plot curve on temperature-entropy sketch, and indicate area that represents heat required to raise temperature from 40 F to 440 F.

d. If 2 lb of air were specified instead of 1, in what way would the temperature-entropy curve be different from that of c?

17. If the specific heat of a liquid or gas (superheated steam, air, etc.) has a constant value, will the path that represents a heating or a cooling on a temperature-entropy diagram be a straight line? Explain.

18. The following data are obtained from observations of a steam power plant similar to that represented by Fig. 5:

Boiler pressure.....	400 psia
Exhaust pressure in condenser.....	0.596 psia
Temperature of water leaving hot well.....	75 F
Temperature of water leaving heater.....	210 F
Temperature of water leaving economizer.....	250 F
Temperature of steam leaving superheater.....	700 F
Temperature of condensate leaving condenser..	75 F
Quality of steam leaving boiler and entering superheater.....	99%
Moisture in steam leaving turbine and entering condenser.....	10%

a. Draw a sketch of a circuit diagram similar to Fig. 14, and indicate values thereon.

b. Draw a free-hand temperature-entropy sketch as a guide, lettered to correspond with (a).

c. Tabulate values of temperature and entropy as in Table 2.

d. Plot data of (c) to scale in a manner similar to Fig. 17.

e. On the temperature-entropy sketch of (b), indicate the areas that represent heat supplied to each pound of the working substance in the feed-water heater, the economizer, the boiler, and the superheater; also, the heat rejected in the condenser.

f. Determine the number of heat units supplied to each pound of the working substance in the feed-water heater, the economizer, the boiler, and the superheater. Also find the number of heat units rejected to the condenser and the number taken out by the turbine to be converted into mechanical work (neglecting "radiation" losses). What is the thermal efficiency of the steam cycle?

19. Construct to scale a temperature-entropy diagram, showing the constant-value lines indicated below. Make the limits of the scale 1000 F and 2.40 units of entropy. Start temperature scale at 0 F instead of at 0 F abs, in order to use a larger scale.

a. Constant-pressure lines for 1, 12, 67, 250, 680, 1500, and 3206.2 psia.

b. The liquid and saturation lines.

- c. Constant-moisture lines for 10 and 50 per cent.
- d. Constant-superheat lines for 200 and 400° superheat.
- e. Constant-enthalpy lines for 1400, 1300, 1160, and 1000 Btu.
- f. A constant-volume line for 5 cu ft.

20. Calculate the entropies of the liquid, of evaporation, of superheat, and total entropy for 100 psia pressure and 400 F temperature, and compare answers with values of Steam Tables.

21. As you proceed from lower to higher pressures, does the external-work part of the enthalpy (above 32 F) of *saturated* steam, expressed as a percentage, increase or decrease? It is suggested that the percentage be calculated for pressures of 0.20, 1.0, 10, 20, 100, 300, 500, 1000, 2000, and 3206.2 psia and the results tabulated and plotted in the form of a curve.

22. Imagine 1 lb of water to be placed in a *closed vessel*, whose volume is 8 cu ft. Let the pound of water be brought to a temperature of 32 F, and then apply heat, continuing to do so until, by means of windows in the vessel, it is observed that the last vestige of water has just turned into steam.

- a. What then is the pressure of the steam?
- b. How many Btu will have been required to make 1 lb of saturated (100 per cent quality) steam from water at 32 F?
- c. How does this compare with the enthalpy of 1 lb of saturated steam at same pressure as given in the Steam Tables?
- d. Why the difference if any?

23. Steam at 170 psia, dry and saturated, expands at constant entropy to a pressure of 14.696 psia. Calculate the quality and enthalpy, and check your results by reading the moisture and enthalpy on the Mollier diagram.

24. Calculate the per cent of total enthalpy of steam at 14.696 psia and 600 F in the enthalpy of the liquid, enthalpy of evaporation, and enthalpy of superheat. Compare with steam at 2500 psia, 840 F.

25. Calculate the internal energy and external work of the following:

- a. One pound of saturated water at 50 psia.
- b. One pound of dry and saturated steam at 50 psia.
- c. One pound of saturated water at 1500 psia.
- d. One pound of dry and saturated steam at 1500 psia.

Compare with values in Steam Tables.

26. From Fig. 18, determine the phase of each working substance at 0 F, 14.7 psia.

CHAPTER VI

SECOND LAW OF THERMODYNAMICS

39. Insufficiency of the First Law.—The *first law* states that heat and mechanical work are convertible one into the other, but it gives no hint as to whether or not the conversion can be realized completely in both directions or with equal facility. Proceeding by the guidance of the first law alone, the engineer in the design and operation of heat engines might be led to two fallacious conclusions:

1. He might reason, since heat is always convertible into work, that the efficiency of a heat engine is of little ultimate importance, since whatever heat “got by” one engine could be converted into work in another. Work ultimately goes back into heat, and this heat could be reconverted into work, and no one would ever need be anxious about our store of useful energy running out.

2. He might be led to believe that 100 per cent thermal efficiency could be attained in a heat engine. For example, if 1 lb of coal possesses a heating value of 14,000 Btu, it would seem entirely possible that a perfect heat engine might be conceived that would deliver

$$14,000 \times 778 = 10,892,000 \text{ ft-lb}$$

of work in using that pound of coal.

40. The Thought Underlying the Second Law.—The conclusions supposed in the preceding article we know to be erroneous from general experience, notwithstanding the fact that they do not violate the first law. In every heat engine that has been considered, a refrigerator has been found to be an indispensable element of the system. Of the entire amount of heat Q supplied to a working substance by the source of heat, the utilizer cannot convert

100 per cent into work. A portion of the heat supplied must be disposed of in a refrigerator as heat, in order to make possible a completion of the cycle (and continuous operation of the engine) by a restoration of the working substance to its initial state.

It is evident, therefore, that in order to make thermodynamics a science broad enough to account for all the facts a limitation must be formulated that will preclude such fallacies as have been indicated. The formulation of this limitation really constitutes a statement of the *second law of thermodynamics*.

41. The Second Law of Thermodynamics.—*It is impossible to transform the whole of an amount of heat, Q , imparted to a working substance, into mechanical work, in any heat engine.* The maximum amount that can possibly be transformed into work is given by the expression $[(T_1 - T_2)/T_1]Q$ and is called the *available* part of the heat supplied; the remainder $(T_2/T_1)Q$ is the *unavailable* part. T_1 and T_2 are the highest and lowest absolute temperatures furnished in the heat-engine system.

The second law expresses the concept of availability and unavailability of heat, thus ascribing a *qualitative* value to heat energy.

42. Derivation of the Expressions $(T_1 - T_2)/T_1$ and T_2/T_1 .—The fact of the first law can be established by direct experiment. The quantitative expressions $(T_1 - T_2)/T_1$ and T_2/T_1 of the second law must be derived analytically. Sadi Carnot, a French engineer, who lived from 1795 to 1832, was the first to make this analysis and determine the fractions that express the proportionality of available and unavailable energy of a given supply of heat.

Carnot reasoned that if a cycle could be conceived for a perfect heat engine, and if the thermal efficiency of the cycle could be determined, this efficiency fraction would represent the proportion of available heat in the total quantity of heat supplied. He therefore set about to devise a perfect heat-engine cycle. Now a perfect cycle

must be composed of several individual operations performed in sequence, each of which must also be perfect in character. In order to determine whether any particular operation was perfect or not, Carnot invented the test of *reversibility*. If an operation was reversible, it was perfect; if irreversible, it was not perfect.

An operation is reversible if, at its conclusion, the whole process can be reversed so that every element of the heat-engine system is restored to exactly its original condition—the source, the working substance, the refrigerator, and the position of the mechanisms of the engine. In the next four articles (43, 44, 45, and 46), examples of reversible and

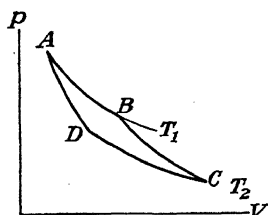


FIG. 21.

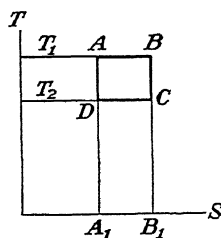


FIG. 22.

irreversible operations will be considered at some length, but, just now, it is appropriate that the *Carnot cycle* should be introduced, so that from it the temperature ratios of the second law may be derived.

Applying the test of reversibility, Carnot devised a cycle composed of two reversible isothermals and two reversible adiabatics, which is represented by Figs. 21 and 22. Beginning at *A*, the working substance receives heat along the isothermal *AB*, at the temperature of the source, T_1 ; at *B*, the heat supply is cut off, and adiabatic expansion takes place to *C*, at which point the temperature T_2 is that of the refrigerator. From *C*, the working substance is compressed isothermally to *D* and thence adiabatically to the initial state *A*.

Having devised a cycle for a perfect heat engine, Carnot's next step was to determine the efficiency of that cycle,

which is very easily done, especially with the help of the temperature-entropy diagram which is available to us, although not available to Carnot in his time. In Fig. 22, the heat supplied, Q , is represented by the area A_1ABB_1 ; and that rejected, Q_N , by A_1DCB_1 . Work output equals area $ABCD$.

$$\begin{aligned} \text{Hence, efficiency of the cycle} &= \frac{\text{work output}}{\text{heat input}} \\ &= \frac{A_1ABB_1 - A_1DCB_1}{A_1ABB_1} = \frac{Q - Q_N}{Q} = \frac{T_1 - T_2}{T_1} \quad (21) \end{aligned}$$

Then,

$$\text{Heat utilized} = \left(\frac{T_1 - T_2}{T_1} \right) Q = ABCD$$

and

$$\text{Heat rejected} = \left(\frac{T_2}{T_1} \right) Q = A_1DCB_1$$

The efficiency of the Carnot cycle is that of the perfect or ideal heat engine. By means of it the heat quantity $Q = A_1ABB_1$, Fig. 22, is divided into the two fractions:

$$\text{Available heat} = ABCD = \left(\frac{T_1 - T_2}{T_1} \right) Q \quad (22)$$

and

$$\text{Unavailable heat} = A_1DCB_1 = \left(\frac{T_2}{T_1} \right) Q \quad (23)$$

43. Reversible and Irreversible Isothermal Operations.

Let us imagine a complete heat-engine system consisting of two thermal capacities, one, the *source* of heat, at high temperature, and the other, the *refrigerator*, at low temperature; a *working substance* confined in the cylinder of a piston *utilizer*; and a storage for mechanical energy such as a weight suspended by a rope wound on a drum. The working substance may be placed in thermal communication with either the source or the refrigerator at the will

of an operator or may be cut off from both. Let this system be effectually isolated from the outside world, so that no energy in any form may be received or lost.

Suppose that the temperature of the source is 2000 F abs and that of the refrigerator 500 F abs. The thermal capacities of the two are considered to be so large that the loss or reception of a quota of heat used in any operation will not materially affect their temperatures. As a first assumption, let the temperature of the working substance be 1600 F abs, with the piston in the position of minimum volume. Now let the working substance be placed in thermal communication with the source and let the piston move forward, its rate of motion being adjusted to the rate of heat flow from source to working substance, so that the temperature of the latter remains constant at 1600 F abs. This is an *isothermal expansion*, and work is expended in lifting the weight on the drum. Now suppose we attempt to reverse the operation. In order to do this, heat must be transferred from the working substance to the source and the piston returned to its original position. But with the working medium at 1600 F abs, and the source at 2000 F abs, heat will not flow back of its own accord. The only means by which the temperature of the working substance can be raised is by some kind of compression different from the expansion, and requiring more energy than that stored in the weight on the drum. The forward operation is an *irreversible isothermal*, and it is accompanied by loss of availability of energy.

Again, let the temperature of the working substance in the cylinder be kept constant at 1999 F abs during an expansion. When reversal is attempted, it will be found impossible as before, but not nearly in the same degree as in the first example. As the temperature at which the working substance receives heat becomes more and more nearly equal to that of the source, the forward isothermal operation becomes more and more nearly reversible, until, in the limit, it becomes reversible. The same reasoning of

course applies to isothermal operations when the working substance is placed in communication with the refrigerator.

The isothermal is representative of all those operations in which the working substance is in communication with a heat body. None of them can ever practically attain reversibility because of the necessity of allowing a temperature drop or thermal head to effect a sufficiently rapid heat transfer. The limiting case represents the ideal chosen by Carnot, where all the heat received during the cycle is taken in *at the temperature of the source* and all the heat given out is rejected *at the temperature of the refrigerator*.

The basis of reasoning in deciding whether a thermal operation is reversible or not is the axiomatic fact that *heat cannot pass directly from a body of given temperature to one of higher temperature*.¹

44. Reversible and Irreversible Adiabatics.—In an adiabatic operation, the working substance of our isolated heat engine is cut off from communication with both the heat source and the refrigerator. During expansion, for example, work is done upon the piston at the expense of internal energy of the working substance. Whether this operation is reversible or not depends entirely upon how the transformation of heat into work takes place.

As one example, the temperature of the working substance at the beginning of expansion may be higher than that of the cylinder walls, but, as the operation continues, the temperature of the working substance will become lower than that of the walls because of its cooling in doing work. Heat will thus flow out of the working substance into the metal during the first part of the expansion, and into the working substance from the metal during the latter part. The external walls of the cylinder may be *perfectly insulated* against heat interchange with a surrounding atmospheric medium, yet the back-and-forth shuffling of heat goes on inside coincidentally with the transformation

¹ The second law is often stated in this way.

of heat into work. The operation of expansion, while adiabatic in the sense of there being no interchange of heat between the utilizer and an outside body, yet is not a true adiabatic with respect to the working substance itself. The external useful work represented by the elevation of the weight as a result of the kind of expansion described is not the full equivalent of the available internal energy of the working substance and as a consequence is insufficient, on a reversed operation, to restore the piston to its original position and the working substance to its initial state, and therefore the operation is an *irreversible adiabatic*.

If internal frictional effects are present during an expansion (as in the turbine type of utilizer), the work of overcoming them is transformed back into heat that is reabsorbed by the working substance. Hence, so far as the working substance is concerned, the expansion is again not truly adiabatic, although no heat exchange whatever, with outside bodies, takes place. Since a part of the entire work done by the working substance is thus consumed internally, the amount of energy appearing as useful external work is insufficient to compress the working substance to its initial state, and the expansion operation is again an *irreversible adiabatic*. All throttling processes are of the character just described and hence are irreversible and lossful operations.

If the cylinder walls of the piston engine could be made of material impervious to heat, so that heat is not transferred to or from the working substance at any time during expansion, and if all internal frictional effects could be eliminated, then the operation would be a *reversible adiabatic*. Every foot-pound of work done during expansion would be accounted for in the energy stored in the lifted weight, which would then be sufficient to effect the complete reversal of the forward operation.

Carnot chose this reversible adiabatic (the constant-entropy adiabatic) for the second operation of his ideal cycle, by means of which the temperature of the gas is brought down from its higher level to its lower, and also

as the fourth operation, by means of which the temperature is raised from the lower to the higher level.

45. Other Illustrations of Reversibility and Irreversibility.—Imagine a car on an inclined plane, held by a rope parallel to the plane, running over a pulley at the head of the incline. A weight is fastened to the other end of the rope and suspended vertically from the pulley. All mechanisms are assumed to be frictionless. From a supply of sand bags at the top of the incline, the car is loaded in just sufficient amount to lift the vertically moving weight. Then the slightest force applied by the operator will set the system in motion. When the car reaches the bottom of the incline, the potential energy of position originally possessed by the car and its load has been transferred to the weight.

The process can now be reversed if desired. With the slightest force in the other direction, the operator can cause the elevated weight to pull the car with its load back to its original position. The energy system is then in exactly the same condition that it was in the beginning. No assistance from an outside energy source has been called in. The descent of the loaded car is a reversible operation.

Now suppose there is a step or section of steeper slope near the bottom of the incline. In negotiating this, the force produced by the car on the rope is greater than that necessary to raise the vertically moving weight. The excess is expended in accelerating the masses of the system. At the end of the operation, the loaded car has done exactly the same amount of work as it did on the continuous incline. But the whole of the energy thus expended has not been transferred to the vertically lifted weight, whose position (of lesser elevation than before) now represents what available energy there is, to effect a reversal of the forward operation or to produce any other mechanical effect. That part which was expended in acceleration of masses was immediately transformed by impact into heat energy (chaotic molecular activity) which is not available either

for assisting in reversal or for producing any other mechanical effect. The original operation is irreversible. There has been a departure from the orderly state of balance between force and resistance, and at that point a loss in availability of energy resulted.

Again, imagine an isolated energy system consisting of a tank of water elevated above the surface of a pond. For the purposes of illustration it may be assumed that water can get from the elevated tank to the pond in three ways: through a short piece of pipe with a valve in it, attached to the tank; by leakage through cracks or holes in the tank; and through a water turbine of 100 per cent efficiency.

If the valve in the short piece of pipe is opened, water will spill from the tank to the pond. The process is obviously an irreversible one, because, without help from the outside, the spilled water can never be restored to the tank. The leakage of water from the tank is an irreversible process for the same reason. The latter operation differs from the former in that it is not controlled. It corresponds exactly to the thermal process of "radiation."

If the water is directed through the perfect turbine, the power developed may be stored up and recalled later to operate the turbine as a pump. The power so developed is sufficient to reverse the original process and restore the used water to the tank, leaving the energy system as it was originally.

46. An Irreversible Operation Means Loss of Available Energy.—The high-head energy of a source is eager to break forth and must be confined. If its escape is properly directed, useful results may be enjoyed from its availability. On the other hand, if the escape is improperly directed, or uncontrolled, not only is availability of energy destroyed without rendering any useful purpose, but the action may be accompanied by the most disastrous results, as, for example, the failure of a dam or the burning of a building.

From the various illustrations offered, it is evident that the operations of a cycle for developing work from heat energy may vary in degree all the way from complete reversibility, on the one hand, to complete irreversibility on the other. *A reversible operation is one in which there is maintained a perfect balance between a yielding resistance and a motive force.* There is an orderly and continuous transfer of energy. The potential energy of the loaded car on the inclined plane is gradually and smoothly transformed into potential energy of the vertically lifted weight. The water, passing from the elevated tank, through the perfect water turbine, to the pond, gradually imparts its potential energy to the wheel, sinking without splash or commotion from one level to the other.

In an irreversible operation, there is a disordered, discontinuous, chaotic transfer of energy, accompanied by secondary transformations. When the loaded car drops at the step in the incline, the motive force exceeds the resistant force, and the excess becomes manifested in a kinetic energy of the masses. When the car bumps upon the bottom of the step, kinetic energy is suddenly destroyed in impact, which results in the creation of heat. Some of the energy that should have gone into elevating the weight has found its way into unavailable heat. When water from the elevated tank is allowed to spill into the pond, either by the opening of a valve, or by leakage, there is similarly, first, an unresisted transformation of potential energy into the kinetic form, and then a further transformation into heat. The whole operation is accompanied by disturbance, commotion, or splash. Again, the potential energy of the water, that might have been directed into some useful effect, now appears as useless heat. Parenthetically, it may be noted that the step of transforming potential energy into the kinetic form does not necessarily imply irreversibility. Kinetic energy is a mechanical form of energy and as such is capable of being conserved and put to useful purpose, provided appropriate means—a suitable

engine—is at hand for the purpose. In the illustrations used above, there were no suitable means of utilizing the kinetic energy, so it was inevitable that it should waste itself in impact—which is really a frictional effect.

The purely thermal irreversible process of direct passage of heat from a hot body to a cold one is analogous to the spilling of the water into the pond. Temperature is the heat head that corresponds to the gravity head of the water system. There is what may be called, having in mind the picture of the tank of water, a “thermal splash.”

47. The Carnot Cycle Represents the Highest Possible Efficiency.—In order to satisfy himself that his test of

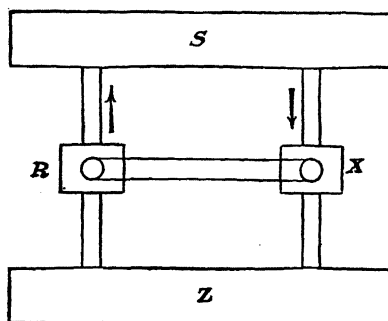


FIG. 23.

reversibility as applied to the individual operations of a cycle was sound, Carnot resorted to a very ingenious general proof to show that his cycle (or any reversible cycle) represents the highest efficiency possible. Imagine two heat engines *R* and *X*, Fig. 23, of the same horsepower operating between the same source *S* and refrigerator *Z*. Let *R* be a reversible engine, while *X* operates upon some cycle more efficient than that of *R*. *X* is arranged to drive *R*, so that *R* becomes a heat pump or refrigerating machine. The system is segregated from any outside communication and possesses a certain amount of available energy, represented by the amount of heat in *S*, and its tendency to flow toward *Z*.

Now let the system be put into operation. X , being supposed to be more efficient than R , will take less heat from the source than would R , if R were running direct and developing the same amount of power. Or, operating with R reversed, X will take less heat from the source than R will deposit in it. As a consequence, after a given time of operation, it would be found that the amount of heat in S was greater than it was in the beginning, which would mean that the available energy of this segregated system had been increased. This result is contrary to the general law of common experience that *in an isolated energy system no change can take place that will increase the availability of energy*. This statement is similar to that other general law of universal experience, *viz.*, that *heat cannot pass directly from a body of given temperature to one of higher temperature*. The two statements really amount to the same thing, since if an exception to one could be found, it would likewise be an exception to the other. It must, therefore, be concluded that the assumption of an engine of greater efficiency than a reversible engine is an absurdity. Hence, a reversible engine represents the highest possible efficiency for a heat engine, and the definite expression of Art. 41 to show the maximum possible portion of a given quantity of heat supplied, that might be converted into work, is established by the expression derived for the efficiency of a reversible engine in Art. 42.

$$\text{Heat supplied} = Q$$

$$\text{Amount available} = \left(\frac{T_1 - T_2}{T_1} \right) Q \quad (22)$$

$$\text{Amount unavailable} = \left(\frac{T_2}{T_1} \right) Q \quad (23)$$

where T_1 and T_2 are the absolute temperatures of source and refrigerator.

It is to be noted that these expressions are derived without regard to the properties of any substance. They

are therefore perfectly general and independent of any working medium.

48. Other Reversible Cycles.—There are other possible reversible cycles besides that of Carnot. The ideal cycle of the Stirling hot-air engine (Arts. 146 and 147), making use of the idea of the heat regenerator, is an example. The working substance is passed bodily through the regenerator—an element consisting of a checkerwork of some substance, with a considerable heat-storage capacity. As the charge of gas at temperature T_1 enters the first section of the checkerwork, it meets with a temperature only slightly less than its own and imparts some of its heat to the checkerwork. It then proceeds to the next zone at a slightly reduced temperature and again meets with a temperature a little less than its own, where it imparts more of its heat. This process continues until the gas finally emerges from the regenerator at the temperature T_2 , and in no case, theoretically, has there been more than an infinitesimal drop in temperature between the two bodies, gas and checkerwork. The constant-volume operations are here ideally reversible, as well as the isothermals.

Another example of a reversible cycle is that alluded to in Art. 71, where it is suggested in connection with Fig. 35 that the condensate (feed water) might be returned by way of a jacket around the barrel of the turbine in such a manner that the water at rising temperature would always be meeting surfaces in contact with steam at a trifle higher temperature, which would constitute essentially a reversible operation. The practical difficulty with this, as indeed with all processes approaching reversibility involving the transfer of heat, is the slowness of heat flow with small differences of temperature.

49. Availability of Energy Is Continually Decreasing.—While the total *quantity* of energy in an isolated system cannot be altered by any operation taking place therein, the *availability* of the energy may be altered. If the operation is a reversible one, availability is unchanged; if irre-

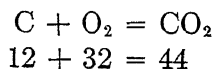
versible, availability is decreased. Since a reversible operation is wholly ideal, and therefore never quite attainable, it follows that every actual operation is in some degree irreversible and consequently contributes to a decrease in availability of energy, with a corresponding increase in unavailability.

50. Illustration of the Continual Decrease of Available Energy.—As an example to show how every change that takes place helps to render some energy unavailable, a pictorial history of the energy of a pound of fuel is presented in the succession of figures that follow (Figs. 24 to 28), wherein the energy is shown in its various forms and substances from combustion of the fuel in the furnace of a steam power plant to final dissipation into the atmosphere in the form of completely unavailable heat.

For the example, most of the conditions of the power plant of Fig. 5 will be used. The heating value of 1 lb of carbon as fuel will be considered rather than that of 1 lb of coal, to avoid the complication of giving attention to the ash.

The figures are all drawn to the same scale, so that temperatures and entropies at various stages are correctly represented by the coordinates of the figures, and heat quantities are proportional to the areas.

51. The Heat of Combustion—Zero Air Excess.—Take the heating value of 1 lb of fuel as 14,400 Btu. Assume both the carbon and the air with which it is to be burned to be at a temperature of 70 F before combustion.



Each pound of carbon will require $32 \div 12 = 2.67$ lb of oxygen. The oxygen of air constitutes 23 per cent of the weight of the air.

$2.67 \div 0.23 = 11.6$ lb of air required to burn 1 lb of carbon.

It will be assumed that there is no loss of heat from the furnace and that after combustion all the heating value of the fuel appears as enthalpy of the gases, of which there are 12.6 lb for each pound of fuel. For the purposes of this illustration, the specific heat of the gases will be assumed to be 0.318 and to remain constant throughout the temperature range encountered. This, of course, is contrary to the actual facts, Art. 6, but the lack of precision will in no wise detract from the objective sought in the example, while at the same time it will simplify the procedure and better enable the reader to follow the thought that is intended to be brought out.

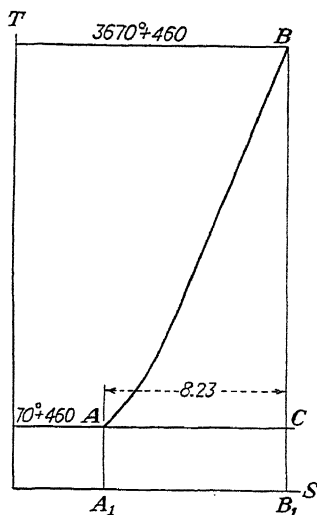


FIG. 24.

The theoretical temperature that will be attained by the 12.6 lb of gas in absorbing 14,400 Btu is

$$\frac{14,400}{12.6 \times 0.318} \quad 70^\circ \text{F} = 3670^\circ \text{F}, \text{ or } 4130^\circ \text{F abs}$$

The change of entropy that results from raising the temperature of 12.6 lb of gas from 70 to 3670 F at constant pressure is given by the expression

$$\begin{aligned} \text{Change of entropy} &= \bar{W}c_p \log_e \frac{3670 + 460}{70 + 460} \\ &= 12.6 \times 0.318 \times 2.0532 = 8.23 \end{aligned}$$

The area A_1ABB_1 , Fig. 24, represents the 14,400 Btu that has just been received as heat by 12.6 lb of furnace gas, as a result of the combustion of 1 lb of fuel. It is conceivable that the furnace gas itself might be employed as a working medium. In that case, the best way to use it would be to introduce it into a utilizer at state B and then

allow it to expand at constant entropy to the minimum temperature possible, which is just short of 70 F, at *C*. Heat could then be removed at constant temperature by transmission through the walls of the utilizer to the surrounding atmosphere, to bring the state of the charge back to *A*. Thus we build up an ideal cycle *ABC*, adapted to the prescribed manner in which the working substance must receive heat. From Fig. 24

Heat supplied, $A_1ABB_1 = 14,400$

Unavailable heat, $A_1ACB_1 = 530 \times 8.23 = 4,360$

Available heat, $ABC = 10,040$

With this cycle, 69.7 per cent of the heating value of the fuel is available heat.

52. The Heat of Combustion—50 Per Cent Air Excess.

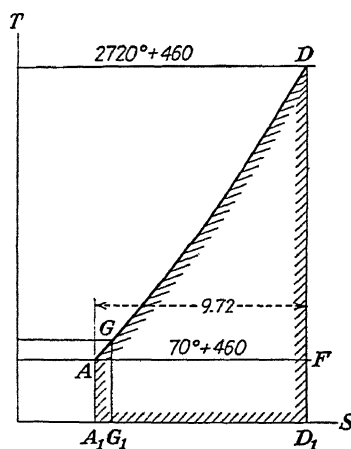


FIG. 25.

Practically, it is impossible to burn fuel, particularly solid fuel, with no air excess. Assume now that the furnace of the plant of Fig. 5 uses 50 per cent excess of air. Then the number of pounds of furnace gas per pound of fuel is equal to $(11.6 \times 1.50) + 1$, or 18.4.

Calculating the temperature of combustion and the change of entropy in the same manner as in Art. 51, with the slightly different value of 0.295 for the specific heat, it is found that

the temperature will be 2720 F, or 3180 F abs, and the entropy increase will be 9.72.

In Fig. 25, A_1ADD_1 represents the 14,400 Btu developed by the combustion of the pound of carbon. The area is of the same general shape as that of Fig. 24, but it is to be noted that it is wider and not so tall.

Assuming again that the furnace gas is the working medium, the ideal cycle (Fig. 25) is ADF .

$$\begin{aligned}\text{Heat supplied, } A_1ADD_1 &= 14,400 \\ \text{Unavailable heat, } A_1AFD_1 &= 530 \times 9.72 = 5,150 \\ \text{Available heat} &= 9,250\end{aligned}$$

The available energy is now only 64.2 per cent of the entire enthalpy of the furnace gases; 5.5 per cent more of the heating value of the fuel has been rendered unavailable because of the necessity of burning with excess air.

While the ideal cycles of Figs. 24 and 25 are thermodynamically possible, they would be totally impossible, from the practical standpoint. Being subatmospheric in pressure, heat rejection could be effected only by heat transfer with but a small temperature differential. Practically, therefore, it becomes necessary to *transfer* heat from the hot gas to a *more suitable working substance*, which in this case is H_2O .

In the power plant of Fig. 5, the furnace gases are represented as entering the stack at 300 F, which is an unusually low temperature resulting from the installation of an economizer. Taking the specific heat of the stack gases to be 0.241, the heat that will be given up to the atmospheric air, in cooling the furnace gases back to 70 F, is the amount of heat lost from the plant through the chimney per pound of fuel.

Heat lost to stack = $(300 - 70) \times 0.241 \times 18.4 = 1020$ Btu.

$14,400 - 1020 = 13,380$ Btu, the heat that will be transferred to the steam per pound of carbon.

The area A_1AGG_1 , Fig. 25, represents the stack loss.

The area G_1GDD_1 represents the heat transferred by the hot gases to the more practicable working substance water, in economizer, boiler, and superheater.

53. The Enthalpy of the Steam.—In the plant of Fig. 5, steam is delivered to the main unit at 250 psia and 399.05

degrees superheat: The heat required per pound of steam, with hot-well temperature at 90 F, is 1364.7 Btu (Art. 35).

$13,380 \div 1364.7 = 9.80$ lb of steam produced for each pound of carbon burned.

The increase of entropy from water at 90 F to steam at 250 psia pressure and 399.05 degrees superheat (Art. 36) is $1.7397 - .1115 = 1.6282$ (Fig. 17) units per lb of steam, or a total of 16.49 units corresponding to the pound of carbon. The heat received by the steam is pictured by the area H_1HJKLL_1 , Fig. 26. The reception of heat by the steam is represented by the line $HJKL$. While the steam is receiving heat, a corresponding quantity of furnace gas is losing heat. Hence, the increase of the enthalpy in the

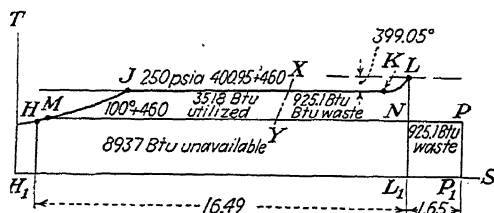


FIG. 26.

steam is to be associated with the decrease of enthalpy from the gases. The operation $HJKL$, Fig. 26, is simultaneous with the cooling operation DG , Fig. 25.

The lowest attainable temperature of the actual steam plant is 100 F, in place of the true refrigerator temperature of 70 F. The available energy, measured above the 100 F line, is 453.4 Btu per lb of steam (Art. 69) or 4443 Btu per lb of carbon. The unavailable energy is

$$13,380 - 4443 = 8937.$$

The ratio of available energy now present, to the entire heating value of the pound of carbon, is $4443 \div 14,400 = 0.308$ or 30.8 per cent. The efficiency of transferring heat from furnace gases to steam is $13,380 \div 14,400 = 0.929$, or 92.9 per cent—a very good performance. But viewed as an operation associated with a heat engine, where con-

servation of *available energy* is the real point, it is exceedingly wasteful. *Thirty-three and four-tenths per cent of the heating value of the fuel is rendered unavailable by the irreversible operation of transferring it directly from a body of high temperature to one of low.*

54. Transformation of Heat into Work.—With the acquisition of the heat by the steam, all is now ready for the climax in the series of events that are being pictured, *viz.*, the transformation of heat into mechanical work in the heat utilizer. Observation of the actual performance of the power plant of Fig. 5 showed that out of each pound of steam 359.0 Btu were converted into work at the shaft of the turbine (Art. 35). The steam had 453.4 Btu (Art. 53) available energy per pound when it entered the turbine. Hence, owing to the inefficiency of the utilizer, 94.4 Btu per lb of steam (or 925.1 Btu per lb of fuel) have been allowed to slip from availability into unavailability. The exhaust steam gains in enthalpy by the amount of the turbine's waste.

The arbitrary line *XY*, Fig. 26, is drawn to divide the area *MJKLN* into the two sections to emphasize the idea that, in effect, the turbine wastes 925.1 Btu of available energy and disposes of it as unavailable energy in the exhaust steam, where it is represented by the area *L₁NPP₁*.

Out of 14,400 Btu of energy in the fuel, 3518 Btu, or 24.4 per cent, have finally been transformed by the heat engine into mechanical energy. And out of 9250 Btu possibly available (Art. 52), only 3518 Btu are actually realized. Thirty-nine and eight-tenths per cent of the entire amount of energy has been rendered unavailable by the wasteful processes described. Yet the performance of this plant is as good as or better than that of the average steam plant.

55. The Enthalpy in the Condenser Cooling Water.—Continuing with the history of the energy of the pound of carbon, consider next the transfer of heat from the exhaust steam to the cooling water of the condenser.

Water enters the condenser of Fig. 5 at 70 F and leaves at 86 F, a rise of 16 degrees. The heat received by the water is the enthalpy of the exhaust steam, 9862 Btu, represented by the area P_1PMHH_1 , Fig. 26.

$9862 \div (1 \times 16) = 616.4$ lb of cooling water required per pound of carbon. The increase of entropy of the water is

$$616.4 \times 1 \times \log_e \frac{86 + 460}{70 + 460} = 18.25$$

The heat transferred to the cooling water is represented by the area Q_1QRR_1 , Fig. 27, where QR is the liquid line for 616.4 lb of water. The operation QR of

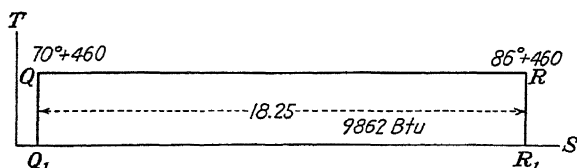


FIG. 27.

heating the water is simultaneous with the operation PMH , Fig. 26, of condensing and cooling the steam.

56. Dissipation of Heat to the Atmosphere.—The final operation to be pictured in this series is the disposal of all the heat of combustion of the pound of fuel into the surrounding atmosphere. The energy carried out of the plant by the condenser cooling water and the energy of the furnace gases delivered to the stack are deposited here; and here also is to be found ultimately the heat equivalent of those 3518 Btu, Fig. 26, that the turbine succeeded in transforming into mechanical work. No matter to what diversified utilitarian purposes this energy is applied—whether it be to drive a tool through metal or run a merry-go-round, to speed a train or propel a ship, to mine coal or weave a carpet—the opposition or resisting force, by virtue of which the work is done, is in every case really the

resistance of friction. Work is transformed back into heat that is picked up by the all-pervading atmosphere.

Of the 3518 heat units of energy transformed into mechanical work by the turbine of the plant of Fig. 5, only a part is finally applied to a useful purpose before sinking into unavailability. Through whatever path of transmission or transformation the energy is conducted, it meets on every hand frictional resistances that exact their toll from an amount that is already discouragingly small when measured in terms of the original energy.

The air of the atmosphere is of such vast heat capacity that its temperature is hardly affected by such contributions as are being considered. Its reception of heat is practically an isothermal operation. The heat of combustion of the pound of carbon is received by the air by way of three routes:

	Btu
Heat from the stack gases.....	1,020
Heat from the cooling water.....	9,862
Heat resulting from the utilization of mechanical work energy.....	3,518
Total.....	14,400

The increase of entropy of the air for each of these several divisions is found by dividing the heat quantities by the absolute temperature, *viz.*, $70 + 460 = 530$.

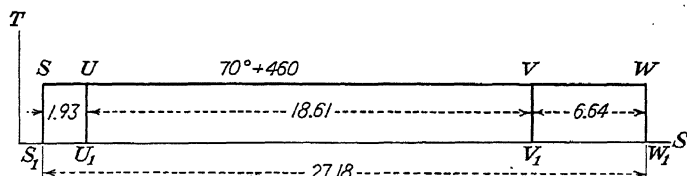


FIG. 28.

The final disposition of the energy is represented in Fig. 28.

S_1SUU_1 = heat lost in stack gases.

U_1UVV_1 = enthalpy carried away by the cooling water of the condenser.

$V_1 V W W_1$ = energy that has existed transiently as mechanical work.

$S_1 S W W_1$ = entire heating value of 1 lb of carbon, now dissipated in the atmosphere.

57. Loss of Availability in Heat Transfer.—Although the steam power plant was chosen in the example immediately preceding, to illustrate the continual transition of energy from the available to the unavailable state, it developed that the largest loss of availability occurred, not in the element of the plant in which energy is *transformed* into useful work—the utilizer—but in an element in which heat *transfer* from one medium to another took place. It is therefore evident that the effectiveness of heat energy may be consumed as readily in the purely thermal process of heat transfer as in the thermodynamic process of transformation into work; and availability must be guarded and conserved as much in the one process as in the other.

The evaporator is a good example of an apparatus in which availability is consumed entirely in effecting transfer of heat from one medium to another. The desired results are achieved by thermal action alone, and the transformation of heat into mechanical work is not at all a part of the objective.

An *evaporator* is a boiler in which the source of heat is usually low-pressure steam—frequently exhaust steam from a prime mover—at or near atmospheric pressure. The liquid to be boiled is usually a solution of some kind from which the water is to be evaporated to leave a desired residue, as in the manufacture of salt or other chemicals; or it may be simply “raw” water unfit for boiler feed or other purposes, that is to be evaporated into steam and then condensed to obtain pure (distilled) water.

Evaporators belong to that large general class of apparatus known as *heat exchangers*. Boilers and condensers are also heat exchangers in a broad sense.

58. The Single-effect Evaporator.—The diagram of Fig. 29 illustrates the principle of the evaporator, in the single-effect form. The shell of the evaporator is represented by *A*, which contains a large number of tubes, arranged in a manner similar to a surface condenser. Steam from the supply fills the tubes, which are conventionally represented by *R*.

The liquid to be evaporated is supplied at H , and the rate of flow is controlled to maintain a suitable level in A .

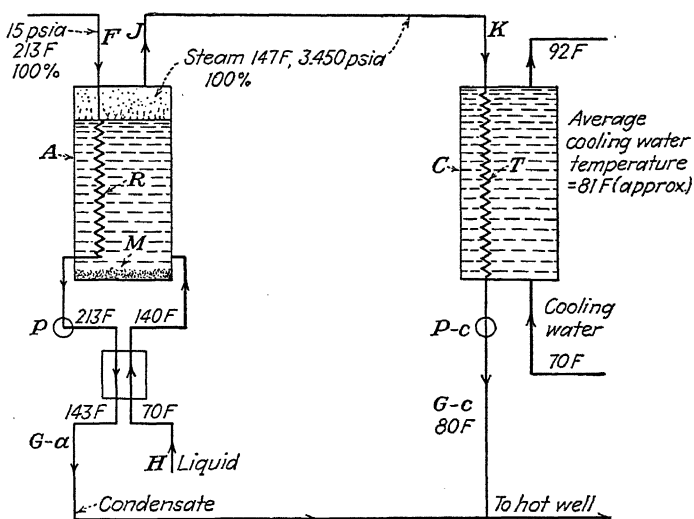


FIG. 29.

The saturation temperature of the liquid in *A* is shown as 147 F, in the sketch, and the pressure corresponding as 3.450 psia. This will be the pressure corresponding to 147 F, only in case the liquid in *A* is *water*, which is here assumed so that Steam Table data may be used. If the liquid is a solution, the relation between pressure and temperature is something different. Each kind of solution has its own set of properties, which also depend upon its concentration.

Heat is transferred from the condensing steam at 213 F to boiling liquid (water in this example) at 147 F—a drop of

66°. The hot condensate at 213 F is delivered by pump P through the *heat exchanger* E , where it is brought in contact (through tube walls) with cool liquid on its way into the evaporator. In this way, some heat that would be useless outside is returned to the evaporator.

The residue left in A by continual evaporation is represented by M , which may be the desired product, as in salt making, or merely the undesirable deposit from hard water. In either case, a suitable means of removal must be provided.

The steam generated in A is below atmospheric pressure and, hence, must be condensed so that it can be removed as water with the minimum amount of work. The shell of a surface condenser is represented by C , Fig. 29, and the tubes by T . Condensate is removed by the wet vacuum pump $P-c$.

Distilled water is delivered at *two* outlets $G-a$ and $G-c$. With respect to the product M , the system of Fig. 29 is a *single-effect evaporator*. With respect to the production of distilled water, it is a *double-effect condenser*. The elements A and C are both condensers, only in A the stream of cooling water is broken, so that continuity of flow can be maintained only by vaporization. If the boiler in which the original steam supply was made be considered, the whole plant may be thought of as a double-effect evaporator.

If the steam supply of Fig. 29 is led directly into the condenser C , thus omitting element A entirely, we would then have a *single-effect condenser*, in which the enthalpy of steam at 213 F is transferred to water at an average temperature of about 81 F.

59. The Multiple-effect Evaporator.—The diagram of Fig. 30 represents a *double-effect evaporator*, working between the same terminal conditions indicated in Fig. 29, *viz.*, heat supplied from steam at 213 F and finally rejected to cooling water at an average of about 81 F.

The double-effect evaporator of Fig. 30 differs from the single-effect only in the interposition of another element *B* between condenser *C* and element *A* of Fig. 29. Referring to the temperatures of Fig. 30, the heat from steam condensing at 213 F is transferred to water evaporating in *A* at 169 F; the heat from this new steam condensing at 169 F is transferred to water evaporating in *B* at 125 F; and finally the heat from this steam condensing at 125 F is passed on to the cooling water of the condenser.

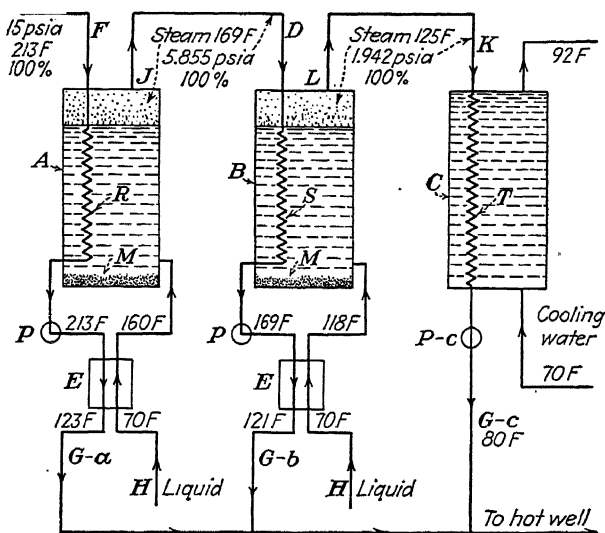


FIG. 30.

Precipitate from evaporation accumulates in both *A* and *B*. Condensate is delivered at *G-a*, *G-b*, and *G-c*. The system may therefore be considered as either a *double-effect evaporator* or a *triple-effect condenser*. Quadruple-effect evaporators are frequently used and sometimes quintuple-effect.

60. Useful Output.—Considering first the single-effect evaporator of Fig. 29, let w_1 equal rate at which steam is condensed in the tubes *R* in pounds per hour and w_2 equal rate at which water is evaporated in *A*.

Heat given up by 1 lb of steam condensing in R = enthalpy of steam at 213 F and 100 per cent quality less enthalpy of liquid at 213 F = 969.7 Btu.

Heat required to evaporate 1 lb of water in A at 140 F into steam at 147 F and 100 per cent quality = 1017.0 Btu. Assuming no loss of heat from the system (other than by way of the cooling water),

$$969.7w_1 = 1017.0w_2$$

or

$$\bar{w}_2 = 0.954w_1$$

This means that for each pound of steam supplied to the system at F , Fig. 29, 0.954 lb of water is evaporated in A , leaving an amount of precipitate at M corresponding to the strength of the solution.¹

Since the steam generated in A is condensed in the tubes T of condenser C , there will be w_2 lb of condensate delivered at $G-c$ as well as \bar{w}_1 lb at $G-a$. Or a total of 1.954 lb of distilled water will be delivered for each pound of steam supplied at F .

Considering the double-effect evaporator of Fig. 30, let w_1 = rate of condensation in R .

w_2 = rate of evaporation in A , and hence the rate of condensation in S .

w_3 = rate of evaporation in B , and rate of condensation in T .

Proceeding as before, on the principle that the heat given up by steam condensing must equal that taken up by water evaporating, and using Steam Table values, we find that if $w_1 = 1$, then $w_2 = 0.964$ and $\bar{w}_3 = 0.933$.

$\bar{w}_2 + w_3 = 0.964 + 0.933 = 1.897$ lb evaporated in A and B for each pound of steam supplied at F .

¹ In this example, the heat quantities determined, and hence the relation between \bar{w}_1 and \bar{w}_2 , are for water. The results would be different with solutions. It may also be noted here that steam from a boiling solution is superheated as it rises from the liquid.

And $\bar{w}_1 + \bar{w}_2 + \bar{w}_3 = 2.897$ lb condensate produced for each pound of steam supplied.

Thus for a *double-effect evaporator* (triple-effect condenser), the amount of precipitate from evaporation (which is proportional in amount to $w_2 + \bar{w}_3$) is nearly twice as much per pound of steam furnished as in a single-effect evaporator; and the amount of distilled water produced ($\bar{w}_1 + \bar{w}_2 + w_3$) is nearly three times as much per pound of steam as is the case with the single-effect condenser.

The useful output per pound of steam supplied to the system, whether it is salt or condensate, is seen to be roughly proportional to the number of effects. On the other hand, there is an objection to be considered.

If the supply steam of Fig. 29 was introduced directly into the condenser tubes at K , the thermal head to produce heat flow would be $213^\circ \text{F} - 81^\circ \text{F}$ (average cooling-water temperature), or 132° . In the double-effect condenser of Fig. 29, the total thermal head from primary steam to cooling water is the same as in the first case, *viz.*, 132° . But with the two effects, the total must be cut in half for each heat transfer— 66° from R to A and 66° from T to C . Hence *only one-half as much heat can be transferred per unit of time through the double-effect condenser as through the single-effect condenser; and (as can be readily seen) only one-third as much through the triple-effect condenser of Fig. 30.*

In other words, *the rate at which steam can be consumed, in pounds per hour, is about inversely proportional to the number of condenser effects.* In multiplying effects, the amount of useful product *per pound of steam* supplied is, in a general way, increased in proportion to the number of effects, but, since the number of pounds of steam per hour is reduced about in inverse proportion, the output capacity per unit of time remains about the same for any number of effects.

61. Conservation of Availability by Multiplication of Effects.—If the supply steam at 213°F (Fig. 29) is intro-

duced into the condenser at K , there will be a transfer of heat directly from primary steam to cooling water, resulting from the temperature difference $213\text{ F} - 81\text{ F} = 132^\circ$. The temperature head of 132° represents a certain amount of *availability* associated with the enthalpy of the steam. In the transfer of the heat, this availability has been exchanged for something presumably more desirable, *viz.*, a quantity of distilled water. The process is manifestly irreversible, because there is no conceivable way by which the heat now in the cooling water can be made to change the condensate back to steam at 213 F .

In the double-effect condenser of Fig. 29, the heat of the steam at 213 F has the same availability that it had before, but now the total thermal head is divided and the apparatus arranged so that the heat originally in the steam is let down 66° in one step and the remaining 66° in a second step. But *each* stage of the operation yields nearly as much of the desired product as was obtained by the single-stage transfer. Both stages of the double-effect condenser are *irreversible operations*, but neither is irreversible to the same degree as in the single-effect. It would be a good deal easier to restore the heat from 147 F to 213 F than from 81 F to 213 F .

The product that an evaporator or condenser is intended to deliver is produced merely by the *transfer of heat* from one fluid to another. No energy is consumed in the product itself. The multiplicity of effects might, theoretically, be extended indefinitely, so that 10 or 100 lb of condensed steam could be obtained from each pound of primary steam supplied. But with increase in number of effects, the temperature difference across each transfer surface decreases, and the *rate of heat transfer* decreases in the same proportion. Roughly speaking, if s lb of steam can be condensed per hour in a single-effect condenser, only one-fourth s lb can be condensed in each element of a quadruple-effect, in which each element is of the

same size (transfer surface) as the one element of the single-effect.

In the single-effect unit, the total temperature head, or availability, is consumed in one grand thermal jump; in the multiple-effect, the release of availability is more restrained and better controlled, with less spectacular local results, but with a multiplication of useful effect.

62. Entropy, a Measure of Unavailability.—A comparison of the sketches of Figs. 24 to 28 discloses the general fact that areas on temperature-entropy diagrams, representing the heat quantity (originating in the combustion of 1 lb of fuel) at various stages of its progress through the steam power plant, become lower and wider after every operation that takes place. Temperature associated with the heat quantity is always decreasing, and entropy always increasing.

The same one-way progress is sensed in the example of the multiple-effect evaporator. A substantially constant quantity of heat, originally supplied in 1 lb of steam, sweeps down through the apparatus. If the heat is portrayed by a series of temperature-entropy figures, the same transition in relation between height and width of area would be observed. Progress in the series of operations (heat transfers) is marked by continual decrease in temperature and continual increase in entropy, and both spell loss of availability and increase of unavailability.

It is, of course, the object of the designer and operator of power-plant apparatus to conserve availability of energy to the greatest degree possible, up to the very place of its final application. In pursuance of this objective, every operation that takes place ought to be as nearly reversible as possible; not that there would be any practical point in reversing a thermodynamic operation, even if it could be done. Reversal is not the desideratum; but *reversibility*, *the capability of reversal*, is an objective as a measure of perfection.

Every actual operation falls short of being reversible in more or less degree, and the amount by which it does so is a measure of its contribution to the unavailability of energy; and accompanying the increase of unavailability is the inevitable growth of entropy.

Although we have found entropy to be a tool of constant usefulness, it eludes definition as a physical concept. Originally invented as a mathematical quantity, to be associated with temperature as a means of measuring and representing *heat energy* and its disposition, entropy, like the energy laws of thermodynamics, and the concepts of reversibility and availability, has gradually come to be associated with energy in all its forms. It is as much a quality of energy as is unavailability. In fact, entropy and unavailability express the one-way tendency of all energy occurrences.

If we should lay off on coordinate paper a vertical scale representing unavailability of all the energy of a given isolated system, and a horizontal scale representing entropy, and then should plot points showing the relation between these two properties, as energy events occur in the system, the progress of a curve drawn through these points would always trend toward an increase of unavailability and an increase of entropy. There could never be a decline in the curve or a turning back to lower entropy, because such would imply operations that were more than reversible and hence in violation of the second law.

Problems

1. Which of the following are examples of a reversible process? Give reasons.
 - a. The mixing of 1 lb of water at 32 F with 1 lb of water at 72 F.
 - b. Adiabatic compression of a gas without friction.
 - c. The oscillation of a spring which is perfectly elastic.
 - d. The cooling of a gas during isothermal compression as work is performed on the gas.
 - e. Heating of a liquid by stirring with a paddle wheel.
 - f. The heating of a gas by a constant-temperature source of heat as the gas performs work at constant pressure.
 - g. The heating of a gas by regeneration (Art. 146) as the gas performs work at constant pressure.
 - h. The dissolving of ammonia gas in water.

2. Two and one-half pounds of air undergo a Carnot cycle between the temperature limits of 500 F and 1000 F. One thousand Btu are supplied to the cycle at the higher temperature.

- a. How much heat is rejected?
- b. How much work is done?
- c. What is the thermal efficiency of the cycle?
- d. What is the change of entropy during the isothermal processes?

3. Steam enters a condenser at 1 psia and 90 per cent quality. The cooling water enters at 60 F and leaves at 85 F. What is the decrease in available energy and the increase in entropy as heat is transferred in the condenser?

4. A gas expands at a constant temperature of 1540 F. A change of state takes place at constant pressure until the temperature is 540 F. Compression at constant temperature followed by expansion at constant pressure brings the state of the gas to initial conditions. Draw the cycle on pV and TS coordinates. Is the cycle reversible? What is the available energy of the cycle if 200 Btu of heat are supplied during the isothermal expansion?

5. Steam is furnished to a quadruple-effect condenser at 215 F, with no moisture. The average temperature of the cooling water in the fourth effect is 83 F. The temperature drops, areas of heat-transmitting surface, and coefficient of heat transmission are all assumed to be alike in the several effects. The temperature of the condensate leaving each effect is 10° lower than its temperature of condensation, and the temperature of raw water entering each effect is 75 F.

a. Determine the weight of condensate delivered by each effect (and also the total of the four) per pound of steam furnished at 215 F.

b. If the steam generated in the first effect was sent directly to the water-cooled condenser (thus cutting out the second and third effects), and if the other conditions above are specified, what weight of condensate will be collected per pound of steam furnished at 215 F?

c. How will the *rate* of producing condensate, per unit of time, in the quadruple effect compare with that of the double effect?

CHAPTER VII

CYCLES FOR VAPORS

63. Performance of an Actual Steam-heat Engine.—

In preceding demonstrations, the actual performance of the turbine of the steam power plant of Fig. 5 has been touched upon. In Art. 35, it was shown that the number of heat units realized in useful effect from each pound of steam was 359 Btu and that the thermal efficiency of the cycle was 26.3 per cent. But in arriving at these figures, a quality of exhaust steam of 96 per cent was assumed. There is no simple and accurate instrument by means of which the quality at this point can be directly observed. The actual number of heat units converted into work by the turbine, and therefore the thermal efficiency, can be determined only by securing more observations than those that have already been indicated.

Suppose the turbine of Fig. 5 to be tested by measuring the power output of the turbogenerator at the switchboard and by determining the rate at which it consumes steam. The most direct and accurate means of effecting the latter measurement would be to rearrange the piping so that the condensate pump would discharge the condensed steam into weighing tanks, in place of directly into the hot well. If a jet condenser happened to be employed in place of the surface condenser, a different plan would have to be resorted to, which will not be discussed here.

Let it be assumed that it is shown by the results of a test of the turbogenerator of Fig. 5 that the unit consumes 31,548 lb of steam per hour when carrying a load of 3136 kw. The steam consumption, or water rate, as it is frequently called, per kilowatt-hour is therefore 10.06 lb. One kilowatt is the equivalent of 3413 Btu per hour. Each pound

of steam therefore contributes 339.3 Btu. Assuming the generator efficiency to be 96 per cent, and the frictional loss in turbine bearings and sealing glands to be $1\frac{1}{2}$ per cent, each pound of steam contributes to the turbine shaft 359 Btu, which agrees with the results on the assumption of 96 per cent quality in the exhaust, because the latter figure was assumed in agreement with the test figures given above. The thermal efficiency of the steam cycle is therefore just as worked out in Art. 35, *viz.*, 26.3 per cent.

64. Need of a Standard or Ideal Cycle.—Up to this point, we have been considering the performance of one particular steam-heat engine, whose thermal efficiency with respect to the steam cycle is 26.3 per cent. A hundred other actual plants might be examined in a similar manner. The efficiency of some might run higher than the plant of our example and that of others lower. As each highest value was found and noted, there would be an eager curiosity on the part of the investigators to know whether this was the highest attainable, or whether it might be surpassed by some later performance. In fact, the very important question would arise, what *maximum possible thermal efficiency* could be developed by a heat engine of the class under consideration and working within the specified conditions? What would be the performance of *an ideal or perfect heat engine*?

It is evident that the engineer needs some standard to serve as a yardstick for the comparison of performances of actual heat engines and as a goal toward which improvements in heat engines may be directed. From the thermodynamic standpoint, an ideal heat engine is one that would operate upon an *ideal cycle*, which is not necessarily the most efficient that can possibly be conceived but is one appropriate to the class of heat engine to which it is to be applied.

65. Rankine Cycle.—The ideal cycle for the steam-heat engine is represented on the temperature-entropy plane in Fig. 31. *LF* represents heating of water; *FH*, evapora-

tion at constant pressure; HJ , superheating (if any) at constant pressure; JX , expansion in the utilizer; and XL , condensation of exhaust steam. The cycle described, $LFHJXL$, is called the *Rankine cycle*, and it is constantly referred to in steam-heat-engine work.

In the Rankine cycle, all the heat received by the working fluid may be said to be taken in at the constant upper pressure p_s , Fig. 31,¹ and all the unused heat is rejected at the constant lower pressure p_c .

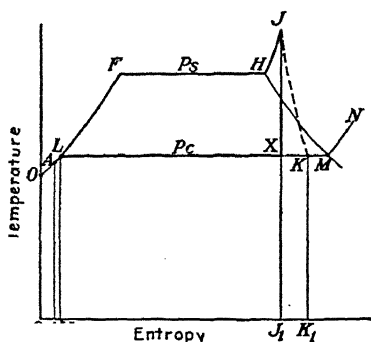


FIG. 31.

66. Comparison of Actual Cycle with Ideal.—Comparing the cycle of our actual heat engine, $AFHJKLA$ of Fig. 17 (whose lines have been reproduced on Fig. 31), with the ideal cycle $LFHJX$, it is seen that the actual

differs from the ideal in only two respects, *viz.*, in the operation LA and its reverse AL , and in the operation JK which occurs in the utilizer.

The operation LA represents a cooling of water below the temperature of condensation and the consequent requirement of an amount of heat represented by area A_1ALL_1 to bring it back to the temperature at L , which would have been unnecessary if the condensate had not been aftercooled.

The one operation of expansion in the utilizer (turbine or piston engine) is represented in Fig. 31 by a line beginning at point J and terminating somewhere on the back-pressure line p_c . In the Rankine cycle, the end point of

¹ Practically in any steam plant, heat is received by the water along the line LF at perhaps two or three different pressures (see Fig. 13). But pressure imposes so little influence upon water that the line LF would not be appreciably different for any kind of pressure variation. Hence, it is convenient and not inaccurate to say that all the heat received by the steam is taken in at constant pressure.

expansion is at X , directly beneath J ; while in the cycle of the actual engine, the end point is at K , which is to the right of X . In the Rankine engine, the amount of heat rejected in the exhaust is the area J_1XLL_1 , while that rejected in the actual engine is greater by the area J_1XKK_1 .¹ The farther the end point K lies to the right of X , the poorer is the efficiency of the cycle. It may even be possible to have the point K fall on the section MN of the constant-pressure exhaust line that lies on the outside of the saturation line HM , in which case the exhaust steam leaving the utilizer is superheated.

In the Rankine cycle, the expansion line JX , Fig. 31, is a constant-entropy adiabatic, which means that the steam neither receives nor gives out any energy *in the form of heat* during the operation, and, furthermore, that all the work that the steam does appears as *external-work energy outside the turbine or engine cylinder*. In the actual cycle, the line JK may be, and usually is, in a turbine, an adiabatic in the sense that the steam neither receives nor gives out any energy in the form of heat while expanding. The departure of JK from the ideal expansion line JX is explained by the fact that some of the work done by the steam in expanding *is consumed internally in the utilizer* in overcoming resistances. When work is consumed in this way, no matter where, the energy is transformed back into heat. In a turbine, or engine, the heat thus generated is reabsorbed by the working fluid, which as a result carries out a greater enthalpy to the exhaust than it otherwise would have done. The area J_1XKK_1 is the heat equivalent of the work consumed within the turbine because of its imperfections.

The actual path of expansion in a turbine is not exactly represented by JK , as will be seen later, and that of a piston engine may depart greatly from JK (see Art. 84). But, in either case, the terminal point K will be somewhere to the right of X .

¹ Plus, of course, the area A_1ALL_1 , if there is any aftercooling of condensate.

67. The Rankine Cycle on the Pressure-volume Plane.—

Both the Rankine and the actual steam cycles that have been discussed in connection with the temperature-entropy diagram of Fig. 31 are represented on pressure-volume coordinates in Fig. 32 by $EJXL$ and $EJKL$, respectively. Expanding from the same initial point J to the same exhaust pressure p_c , the terminal volume at K in the actual cycle is greater than that at the similar point X in the Rankine cycle, because the steam is drier. Remembering that area enclosed by a cycle on a pressure-volume plane

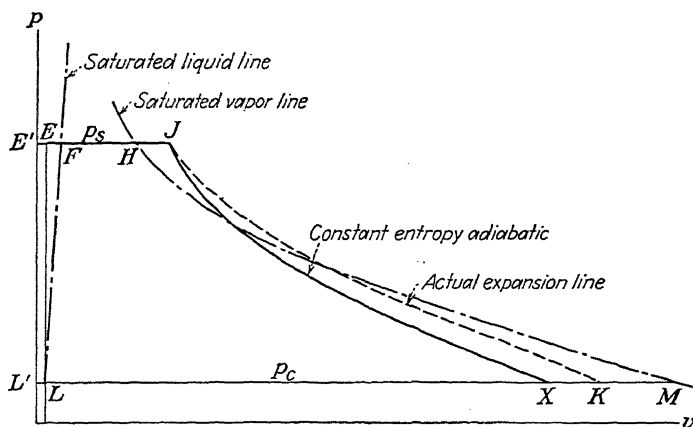


FIG. 32.

represents work, the reader will not fail to observe what appears to be a disconcerting fact, viz., that the work done in the actual cycle is greater than that in the ideal! This apparent fact is really true in respect to the *work done by the steam*, although it is not true in respect to the amount of *useful external work delivered by the utilizer*, which is the final basis upon which the merits of a turbine or engine must be judged. The area $EJXL$, Fig. 32, represents the maximum possible amount of *useful work* that can be made from the heat supplied to the steam when operating under the conditions of pressure and initial temperature specified. How the steam, operating in an actual turbine or engine, can do more gross work than in an ideal utilizer, because

of the very imperfections of the former, will be explained in Art. 81.

68. The Rankine Cycle on the Enthalpy-entropy Plane.—On the enthalpy-entropy plane, Fig. 33, $LEFHJXL$ again represents the Rankine cycle and $ALEFHJKLA$ the actual cycle. Point E lies to the left of the saturated liquid line and represents a subcooled liquid. It is at a temperature below the saturation temperature corresponding to the

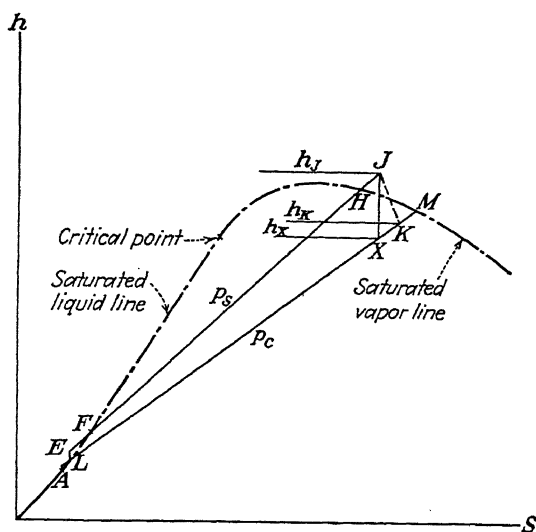


FIG. 33.

pressure p_s . The line LE represents reversible adiabatic compression of the water by the boiler feed pump. The temperature of the water during this compression is substantially unchanged so that E and L are coincident in Figs. 17 and 31. EF , Fig. 33, (of the Rankine cycle) is the heating of the liquid; FHJ is constant-pressure evaporation and superheating; JX is constant-entropy expansion in the utilizer; and XL is constant-pressure condensation or rejection of heat.

69. Efficiency of the Rankine Cycle.—Applying the general energy equation to the turbine and boiler feed

pump of the Rankine cycle we have, neglecting the kinetic energy terms, from Eq. (11)

$$\text{Work done by the turbine} = {}_J W_x = h_J - h_x \frac{\text{Btu}}{\text{lb}}$$

$$\text{Work done in the boiler feed pump} = {}_L W_E = h_E - h_L \frac{\text{Btu}}{\text{lb}}$$

Since area on a pv diagram represents work, the work done in the boiler feed pump may be represented by the area $L'LEE'$, Fig. 32. This area represents the quantity $v_{fL}(p_s - p_c)$. Hence

$${}_L W_E = h_E - h_L = \frac{v_{fL}}{J} (p_s - p_c) \frac{\text{Btu}}{\text{lb}}$$

A part of the work done by the turbine must be used to drive the boiler feed pump. Hence the net work of the cycle is

$$W = (h_J - h_x) - (h_E - h_L) \frac{\text{Btu}}{\text{lb}}$$

The heat supplied to the cycle is the enthalpy of the steam leaving the boiler minus the enthalpy of the steam entering the boiler, or

$$Q \text{ supplied} = h_J - h_E$$

Efficiency is output divided by input. The thermal efficiency of a prime mover or cycle upon which it operates is equal to the net work output divided by the heat input or heat supplied. The thermal efficiency of the Rankine cycle is then

$$\eta_r = \frac{h_J - h_x - (h_E - h_L)}{h_J - h_E} = \frac{h_J - h_x - (h_E - h_L)}{h_J - h_L - (h_E - h_L)} \quad (24)$$

The bracketed term is the work done by the boiler feed pump. Since it appears in the numerator and denominator and is small compared with $(h_J - h_x)$ and $(h_J - h_L)$, the

thermal efficiency is

$$\eta_r = \frac{h_J - h_X}{h_J - h_L} \text{ approx} \quad (25)$$

For moderate pressures, the approximate equation is sufficiently correct. However, for modern high-pressure plants, Eq. (24) should be used.

The Rankine cycle, Fig. 31, is reproduced in Fig. 34. The enthalpy at J is $h_J = 1422.7$ Btu per lb (Table 3, Art. 35). The quality at X must be determined before the enthalpy h_X is determined.

The entropies of points J and L are found directly from the Steam Tables, or, for this example, may be taken from Table 2, Art. 34. The distance

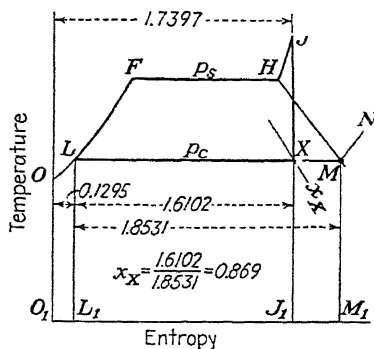


FIG. 34.

$$LX = 1.7397 - 0.1295 = 1.6102.$$

LM is the entropy of evaporation at 100 F. The quality of the steam at X is the ratio LX/LM , or

$$x_x = \frac{1.6102}{1.8531} = 0.869.$$

Enthalpy $h_x = \text{enthalpy of liquid} + x_x \times \text{latent heat}$

(all at 0.949 psia or 100 F)

$$= 67.97 + (0.869 \times 1037.2) = 969.3 \text{ Btu.}$$

Then, $h_J - h_x = 1422.7 - 969.3 = 453.4$ Btu, which is the number of heat units that would be transformed into work by an engine operating in a Rankine cycle.

The enthalpy h_L is the enthalpy of the liquid at L . From Table 3, Art. 35, $h_L = 67.97$. Neglecting the work of the boiler feed pump

$$\eta_r = \frac{1422.7 - 969.3}{1422.7 - 67.97} = 33.47 \text{ per cent}$$

The actual cycle differs from the Rankine cycle in that the state of the steam leaving the turbine is represented by point K , Fig. 31, rather than X . The work done by the turbine actually is $h_J - h_K$. $h_K = 1063.68$ Btu per lb (Table 3, Art. 35). The efficiency of the actual turbine is then

$$\eta_t = \frac{h_J - h_K}{1422.7 - 1063.7} = 26.3 \text{ per cent}$$

The efficiency of the actual turbine is therefore 78.6 per cent (26.3/33.47) of the maximum possible thermal efficiency attainable for the conditions specified. The ratio of actual thermal efficiency to Rankine cycle efficiency is called the *engine efficiency* referred to the Rankine cycle. Thus by means of the concept of an ideal cycle, we have established a definite measure of the performance of this actual heat engine and have indicated the limits of possible improvements within the utilizer itself.

Although chosen as an ideal or nearly perfect cycle for a steam heat engine, the Rankine cycle falls far short of being able to transform the entire amount of heat supplied into mechanical work. This possibility, however, is excluded for any heat engine cycle by the second law of thermodynamics.

70. Work Area on Temperature-entropy Diagram.—In the Rankine cycle, Fig. 34, the heat supplied in the cycle is represented by area J

$$= 1422.7 - 67.97 = 1354.7 \text{ Btu,}$$

and that rejected as unusable, by area L_1LXJ_1

$$= 969.3 - 67.97 = 901.3 \text{ Btu.}$$

The difference, which is the heat equivalent of useful work, is the area bounded by the cycle, *viz.*, $LFHJX$ ($= 453.4$ Btu). Thus, in a Rankine cycle, there is a definite area that represents the heat energy transformed into work, and in terms of area the efficiency of the cycle is $LFHJX/L_1$

In the actual cycle, $AFHJKLA$ of Fig. 31, the amount of heat supplied is represented by A_1AFHJJ_1 , and that rejected by A_1ALKK_1 . The difference or useful work, in this case, is *not* the area bounded by the cycle $AFHJKLA$, as might at first be supposed. The error of this supposition is readily seen when it is remembered that the displacement of the expansion line JK to the right of JX is the result of losses or imperfections in the utilizer. If the area included in the cycle represented the useful work, we should arrive at the absurd conclusion that the greater the turbine losses, the more useful work would be obtained from the steam. The temperature-entropy diagram really furnishes only a picture of the enthalpy of steam as it enters a turbine and likewise at the exhaust. For any cycle except one without internal frictional losses, such as the Rankine, there is no "natural" area that represents the heat equivalent of the energy flowing out of the utilizer in the form of work.

The area included in the cycle $AFHJKLA$, Fig. 31, is the heat equivalent of the work area $EJKL$, Fig. 32, and peculiarly enough does properly represent the whole amount of work *done by the steam*, but this is not the same thing as the *useful work delivered* by the utilizer, as will be explained in Art. 81.

71. Other Ideal Cycles.—Only a reversible cycle can give the highest possible thermal efficiency, and the Rankine cycle is not quite reversible in all respects. The reader is reminded that the familiar cycle of Carnot is a reversible cycle. An ideal must be chosen with some view toward possible conformity of operations in the actual engine to those of the ideal. The Carnot cycle is not a good pattern, especially for the steam heat engine, whereas the Rankine cycle is excellent in this respect.

In Fig. 35, $LFHWL$ represents a Rankine cycle, in which, for simplicity, initially saturated steam is indicated. In terms of areas, the efficiency is $LFHW/L_1LFHH_1$. A cycle may be conceived in which the steam, exhausting

at state W , is only partially condensed to bring its state to a point Y , directly beneath F , after which the mixture of water and steam would be compressed to F . Only the enthalpy of evaporation, F_1FHH_1 , would then be supplied from the source, and only a part (F_1YWH_1) of the latent

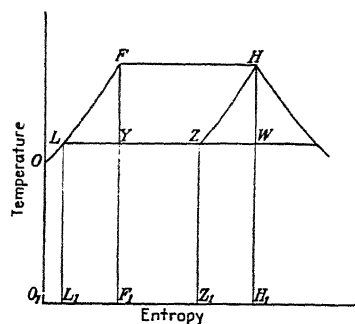


FIG. 35.

heat of the exhaust would be rejected. The efficiency of this cycle $YFHW$ is $YFHW/F_1FHH_1$, which the geometry of the figure proves to be greater than that of the Rankine cycle. The new cycle is, in form, a Carnot cycle. As a pattern for an actual engine, the cycle $YFHW$ would not be

very satisfactory, since it would be extremely difficult to condense the exhaust steam to just the right state of mixture for the beginning of compression, to say nothing of the cost and operation of the compressing apparatus.

Again referring to Fig. 35, the operation LF represents the heating of water by primary heat. Suppose, however, that the condensate of a many-stage turbine at state L is circulated through a jacket around the barrel of the turbine so that it progresses from the cool condenser end to the hot admission end, counterflow to the steam's progress inside. In this way, the water would be progressively warmed at the expense of heat taken from the working steam. Assuming frictionless expansion, the path would be HZ , Fig. 35, displaced to the left of HW , because of the loss of heat Z_1ZHH_1 to the water. If the transfer of heat from steam to water could be effected with only an infinitesimal difference in temperature, the areas Z_1ZHH_1 and L_1LFF_1 would be equal and alike in all respects. In the cycle $LFHZ$, the amount of heat supplied from an outside source is F_1FHH_1 , and that rejected is L_1LZZ_1 , which, owing to the equality and similarity of areas .

and Z_1ZHH_1 , is equal to F_1YWH_1 . The work of the cycle is $LFHZ = YFHW$, and the efficiency, $LFHZ/F_1FHH_1$, which is seen to be the same as that of $YFHW$ or the Carnot cycle. The cycle $LFHZ$, although not a Carnot cycle, is nevertheless a reversible one and has therefore the same efficiency, which is somewhat higher than that of a Rankine cycle.

The difficulties of jacketing the barrel of a turbine and transmitting heat through thick walls would seem to preclude favorable consideration of the cycle $LFHZ$ as an ideal or pattern. However, a more practical means of accomplishing the exchange of heat is embodied in the successful principle of *extraction heating*, whereby condensate is warmed serially in heaters by working steam, extracted at several points along the expansion. In a turbine employing this principle, $LFHZ$, Fig. 35, is the ideal cycle, except that the use of superheated steam modifies its form somewhat. Extraction heating is discussed in Art. 92.

72. Available and Unavailable Energy.—The amount of heat required to make 1 lb of steam at constant pressure p_s and final temperature t_s from water at a temperature t_L is represented on the temperature entropy diagram of Fig. 36, by the familiar area L_1LFHJJ_1 . It is possible, as we know, to transform some part of this heat into mechanical work in a utilizer, but we are also aware of the fact that a large part of it will not be and cannot be so transformed even in a perfect or ideal utilizer. Therefore, it is desirable to know how much is *available to do work* and how much is unavailable for this purpose.

In Fig. 37, the same heat-supply area L_1LFHJJ_1 is represented as in Fig. 36, and upon this the *Rankine cycle* $LFHJX$ is superimposed, all of whose lines coincide with lines on the original figure, except LX . As we have already seen, the area enclosed by the Rankine cycle represents the maximum amount of heat that can be changed into useful work. Hence, the double cross-hatched area of Fig. 37

is the *available* part of the heat supplied, and the area L_1LXJ_1 , the *unavailable* part.

The Rankine cycle (the ideal in this case) furnishes a dividing line LX that cuts the supply area L_1LFHJ_1 into two parts, one lying above this heat-rejection line and representing that part of the heat that is *available* to be transformed into work, and one lying below and representing *unavailable* heat. We may, therefore, think of an ideal cycle as an instrumentality for analyzing the heat supply to predetermine the amounts of available and unavailable heat.

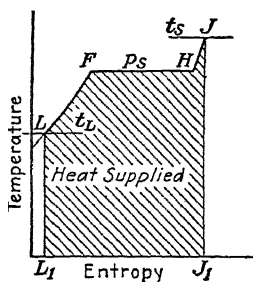


FIG. 36.

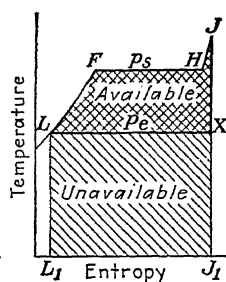


FIG. 37.

It is useful to note here that energy received or given up by a substance *as heat* is represented on the temperature-entropy diagram by a strip to the right or left of a vertical line dropped through the initial state point and extending to the base line of absolute zero; whereas, the *capability of the substance to do work* is portrayed by an area lying *above* some dividing line. In the Rankine cycle this line is horizontal. In ideal cycles for internal-combustion engines, this dividing line is not horizontal, but the same rule applies. That area which lies above the temperature or temperatures at which heat is rejected represents available heat; and that below, unavailable heat.

73. Utilization of Available Heat.—The utilizer of any heat engine has only the available part of the heat supplied to work on. The unavailable part is entirely beyond its

reach. An actual heat engine is necessarily less efficient than an engine operating in a Rankine cycle, and it will therefore never transform 100 per cent of the available heat into useful work. A turbine or piston engine has at its disposal the available heat $LFHJX$, Fig. 37, which it, in turn, will subdivide into what we may call the *utilized energy*, which flows out along the shaft as mechanical work, and the *waste energy*, which is that part of the available heat that the utilizer had to let go. The amount of waste energy is a measure of the unit's inefficiency. The utilizer has no chance at the originally unavailable heat of the steam, which is determined by external conditions; but it is responsible for that part of the available heat that it failed to transform into work and which may therefore be properly termed *waste heat*.

74. Disposition of Energy in Utilizer.—It is convenient to think of steam as a vehicle delivering to the utilizer the heat that it receives in the boiler room. The disposition of the heat within the utilizer is illustrated by the flow diagram of Fig. 38, the temperature-entropy diagram of Fig. 39, and by Table 5. For illustration, numerical values from Art. 69 are used, where it is assumed that steam is supplied at 250 psia pressure and 800 F temperature, is exhausted at 0.949 psia, and the turbine delivers 79.2 per cent of available energy of the steam as work at the shaft.

In the analysis of the performance of a turbine or piston engine, the enthalpy of the liquid corresponding to the exhaust pressure is assumed to be returned to the boiler room. In a complete steam plant this is not likely to be the case, since the water after condensation will be cooled somewhat below the steam temperature in the condenser, and it may lose further heat to the surrounding air before returning to the heating elements. However, the turbine (or engine) itself is not responsible for these heat losses, and its cycle is therefore credited with the enthalpy of liquid at the temperature of condensation.

TABLE 5.—DISPOSITION OF ENERGY IN UTILIZER
 Steam supplied at 250 psia and 800 F. Exhaust pressure 0.949 psia
 Shaft energy 79.2 per cent of available heat

Item	Heat quantity	Area (Fig. 39)	Btu
1	Enthalpy of steam as supplied.....	$O_1OLFHHJ_1$	1422.7
2	Enthalpy of liquid of condensate.....	O_1OLL_1	68.0
3	Net amount of heat supplied per pound of steam.....	L_1LFHHJ_1	1354.7
4	Available heat.....	$LFHHJX$	453.4
5	Unavailable part of heat supplied = 1354.7 - 453.4.....	L_1LXXJ_1	901.3
6	Enthalpy of unavailable heat = 901.3 + 68.0	O_1OLXXJ_1	969.3
7	Utilized heat = 79.2 per cent of 453.4.....		359.0
8	Waste heat = 453.4 - 359.0.....	J_1XKK_1	94.4
9	Enthalpy of inherently unavailable energy (Item 6).....	O_1OLXXJ_1	969.3
10	Waste heat, from available heat <i>via</i> losses in turbine.....	J_1XKK_1	94.4
11	Enthalpy of exhaust steam.....	O_1OLKK_1	1063.7
12	Enthalpy of liquid of condensed steam, returning to boiler.....	O_1OLL_1	68.0
13	Unavailable (latent) heat of exhaust to condenser water.....	L_1LKK_1	995.7

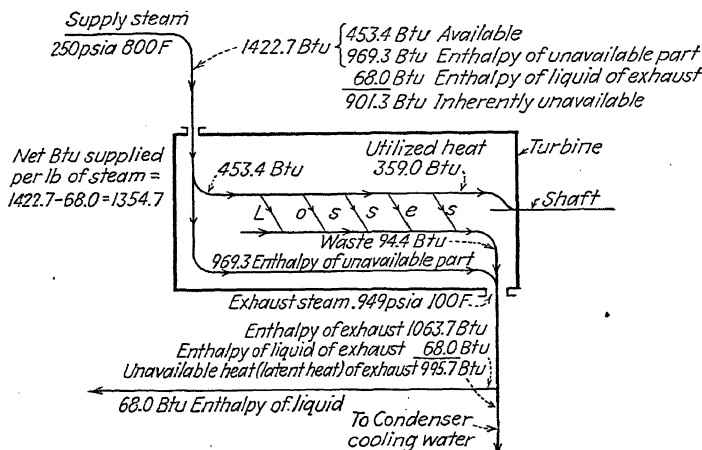


FIG. 38.

In the case of the turbine of Fig. 38, it is evident that the sum of the energy outflow at the shaft and that at the exhaust pipe must be equal to the inflow of energy in the steam supply. Hence, the less efficient the turbine, the more enthalpy there is in the exhaust. In a perfect turbine, 453.4 Btu of energy would pass out along the shaft, and the enthalpy of the exhaust would be 969.3 Btu (Item 6, Table 5). In a turbine of zero efficiency, none of the energy would pass out *via* the shaft, and the enthalpy of the exhaust would equal that of the supply, *viz.*, 1422.7 Btu. In the case specified, the turbine by its inefficiency renders unavailable 94.4 Btu of available energy and enriches the exhaust steam by just that much.

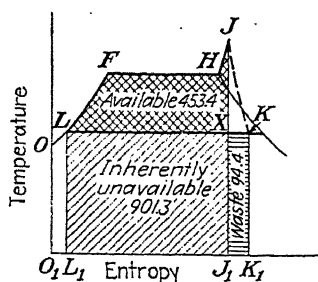


FIG. 39.

Before the steam enters the turbine, its enthalpy is represented by the area $O_1OLFHHJ_1$, Fig. 39. As it leaves the turbine, its enthalpy is O_1OLKK_1 . The available heat area $LFHJX$ has disappeared from the figure—79.2 per cent permanently in the form of work, and 20.8 per cent to reappear as the area J_1XKK_1 which is added to the originally unavailable heat area L_1LXJ_1 , making the enthalpy in the exhaust steam, O_1OLKK_1 .

75. How Availability of Heat May Be Lost.—Availability of heat may be lost in two ways:

a. By being transformed into waste energy *within the utilizer itself*, so that the *useful* work delivered at the shaft is less by that amount than the equivalent of the available heat of the steam.

b. By thermal transfers, whereby heat is removed from hot high-pressure steam, stored temporarily within the metal masses comprising that part of the utilizer that comes in contact with the steam, and then returned in

full to the steam after it has expanded to a lower pressure and temperature state.

The first loss (*a*) is characteristic of the steam turbine in which none of the second kind of loss need be encountered. The second (*b*) is characteristic of the piston engine, which operates upon a cyclic plan, with the result that the temperatures of the steam and containing metal walls are different, thus causing thermal transfers first in one direction and then in the other. It will be shown that this alternating flow of heat is a cause of loss of availability.

76. Available-heat Transformations in a Simple One-stage Turbine.—A simple one-pressure-stage impulse turbine is illustrated in Fig. 40. Steam enters the chest at

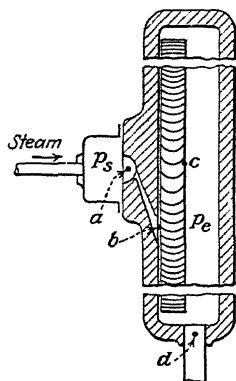


FIG. 40.

the full pressure p_s and expands in the nozzle ab to exhaust pressure p_e . With the velocity thus produced, the steam impinges upon the buckets bc and does work in turning the rotor, and finally

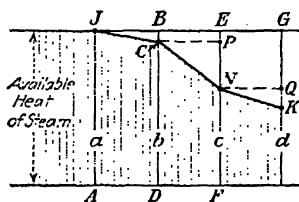


FIG. 41.

exhausts at d . In the impulse turbine of the type illustrated, all operations from the discharge end of the nozzle take place at essentially the constant pressure of the exhaust, p_e .

The progressive transformation of available heat within the turbine is illustrated by the small diagram of Fig. 41, in which vertical distances represent energy quantities; and horizontal distances, progress of the steam through the turbine. The whole width of the band between top and bottom horizontal lines represents the available heat of supply steam as it enters the nozzle at a . At b , the end of the nozzle, the available heat JA all should have been

transformed into kinetic energy BD , but, on account of the nozzle's imperfections, only the kinetic energy CD is developed. BC is the loss of available heat that occurs in the nozzle.

If the buckets performed their function perfectly, the kinetic energy CD would be transformed completely into mechanical work PF . Actually, however, only the bucket work NF is developed, PN representing the loss in the buckets.

Upon leaving the buckets, the steam momentarily occupies the casing in which the bucket wheel revolves. This atmosphere of steam offers resistance to the turning wheel, so that some of the work done upon the buckets by the nozzle jet is consumed in overcoming windage. Consequently, the work energy KL , delivered to the shaft, is less than the work developed in the buckets by the amount of the windage loss QK .

The story of the steam's progress through the turbine is told by the line $JCNK$, which shows how energy on its way to the shaft of the machine is continually reduced by internal expenditures to meet oppositional forces.

77. Return of Energy Losses to the Steam.—Changes in the state of the steam as it progresses through the turbine of Fig. 40 are illustrated by the temperature-entropy diagram of Fig. 42. If the nozzle had performed its function perfectly, the state of the steam at its discharge end would be represented by point X . But owing to friction of the jet along the sides and turbulence within the steam, there is opposition to flow. Whenever friction and turbulence appear, heat is generated, which is ultimately absorbed by the enviroing atmosphere, which in this case is the steam itself. Hence, the enthalpy of the steam at the nozzle discharge is greater than it would have been had there been no losses, and point C represents the actual state at the nozzle discharge, instead of point X . The nozzle loss, or that part of the originally available heat wasted by the nozzle, is represented by the area .

In the turbine nozzle of Fig. 40, the pressure drops from p_s to p_e , and all the while the expansion goes on, frictional effects are operative, so that the path of change is represented by the continuous line JC , Fig. 42, the exact form of which depends upon the characteristics of the nozzle.

The steam is hurled through the buckets of the turbine by the momentum acquired in the nozzle, but it no longer

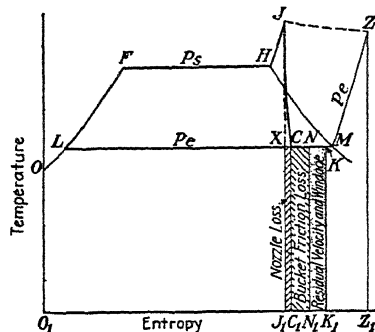


FIG. 42.

has any available heat or potential energy, because it has already expanded to the lowest pressure level provided. Some of its kinetic energy is consumed in overcoming bucket friction with resultant heat generation and absorption of this energy by the flowing steam itself, so that the state, upon leaving the buckets at c , Fig. 40, is at point N , Fig. 42. The energy wasted in this manner and returned to the steam is represented by area C_1CNN_1 .

The steam will have some left-over velocity or residual kinetic energy as it leaves the buckets, which cannot now be usefully employed and, hence, is dissipated into heat. In addition, the revolving wheel does work upon the steam occupying the casing at the moment, resulting in further generation of heat that is also reabsorbed by the steam, which finally leaves the turbine at state K . The area N_1NKK_1 represents the energy returned to the steam in the transformation of residual kinetic energy and windage work.

The complete path of the state point in the turbine of Fig. 40 is therefore represented in Fig. 42 by the line $JCNK$. The total amount of heat returned to the steam because of losses occurring within the turbine is represented by the area J_1XCNNK_1 . These several losses are called *reheating losses*. Reheating is not of itself a loss, or the cause of losses, but is merely a term that describes the

disposition of the heat resulting from frictional losses that take place within the steam. Friction losses in bearings outside the turbine casing are not reheating losses, because the heat generated is not reabsorbed by the steam.

The progress of the steam through the turbine is also represented on the Mollier diagram of Fig. 43, which, being lettered like Fig. 42, will be clear without further discussion.

78. Limits of Exhaust State and Relation to Efficiency.—If the turbine of Fig. 40 were *perfect* in every respect, the state of the steam issuing from the nozzles would be represented by point *X*, Fig. 42. There would be no fric-

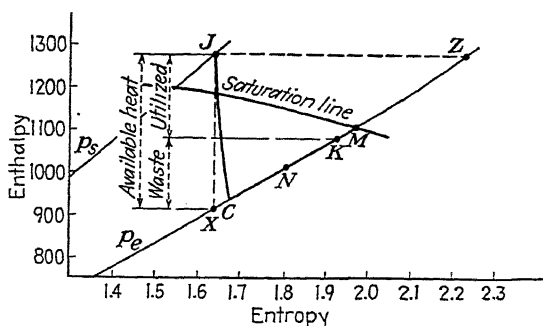


FIG. 43.

tion in the buckets, no left-over kinetic energy from them, and no windage. Consequently, the thermal state of the steam would remain unchanged from the nozzle discharge end to the exhaust, and points *N* and *K* as well as point *C*, of the typical case discussed in Art. 77, would all coincide with point *X*. The state of the exhaust steam would be at *X*, and *JX* would represent the whole change of state within the perfect turbine.

The other limit of possibility is a turbine of zero efficiency, in which case the energy outflow along the shaft is zero (neglecting bearing loss), and hence the enthalpy of the exhaust steam will be equal to that of the supply steam. On the enthalpy-entropy diagram of Fig. 43 it is evident that the point that represents the state of exhaust steam is at *Z* at the intersection of the constant-pressure line

and the horizontal line (constant enthalpy) through J . On the temperature-entropy diagram of Fig. 42, the position of point Z on p_e is such that the area O_1OLMZZ_1 is equal to the area $O_1OLFHHJ_1$.

The exhaust state for a turbine utilizing 100 per cent of the available heat of the steam is at X , Figs. 42 and 43; and that for a turbine of zero efficiency is at Z . The line ZMX , therefore, includes the exhaust-state points of all possible turbines of efficiencies between 0 and 100 per cent. In Fig. 42, the waste-heat area J_1XMZZ_1 is, of course, equal to the available-heat area $LFHHJX$. For any turbine, in general, with exhaust state K , area J_1XKK_1 represents waste heat, while K_1KMZZ_1 is an area that, rather artificially, represents the heat equivalent of the useful work delivered to the shaft.

79. The Turbine of Zero Efficiency.—Any turbine “running light,” as with shaft coupling joining it to a driven machine disconnected, is an example of a utilizer of zero efficiency. Disconnecting the coupling shuts off energy flow along the shaft as the closing of a valve stops fluid flow in a pipe. Starting with this idea, several cases of a turbine of zero efficiency are listed below in a kind of devolutionary scale.

- a. Turbine with properly designed nozzle and buckets with shaft coupling disconnected.
- b. Turbine of (a) with rotor locked to prevent rotation.
- c. Turbine of (a) with rotor removed from casing.
- d. Nozzle of (c) replaced by orifice in a thin plate.
- e. Orifice of (d) replaced by porous plug.

The changes of state of the steam flowing through the several devices listed above are illustrated in Figs. 44 and 45. In Case (a), JC is the nozzle-expansion line, while CMZ is reheating due to bucket loss and windage. The bucket loss *may* be no more, per pound of steam, than when the turbine is operating normally, but the windage loss may be fully as great in actual consumption of power, which, spread over the fewer pounds of steam consumed at no

load, is sufficient to reheat the steam to point Z . In Case (b), the nozzle-expansion line is again JC , but the kinetic energy developed is completely dissipated in the buckets and casing and returned as heat to bring the exhaust to the same state Z . In Case (c), the nozzle line is JC as before, but here the jet dissipates its velocity in the empty chamber, reheating the steam to Z before it leaves the casing.

In Case (d), the orifice is but an inefficient nozzle whose expansion line is JY , which results in the larger wastage J_1XMY_1 , Fig. 44. The kinetic energy generated is dissipated in the empty casing, and, although less in amount than in the previous cases, it is sufficient to reheat the steam

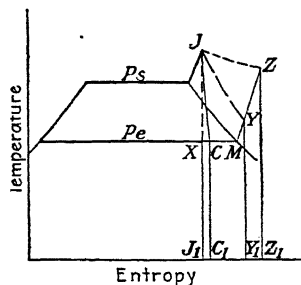


FIG. 44.

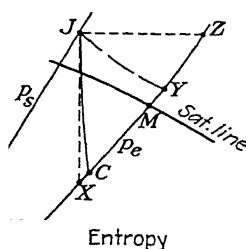


FIG. 45.

through the lesser distance YZ , to bring the final state of exhaust again to Z .

The porous plug may be thought of as a tube filled with cotton or metal cuttings, so that steam flowing through it emerges with practically no velocity. It is a nozzle of zero efficiency. The path of expansion through it is JZ , which is a constant enthalpy line. Here the loss and consequent reheating have occurred entirely within the nozzle, leaving none to be done in the empty turbine casing.

The discussion of these cases shows that so long as no energy is withdrawn from steam while passing through any kind of a device, the enthalpy of the exhaust must be equal to that of the supply, no matter what internal energy transformations take place. Whatever mechanical work is developed inside is converted into heat in full, regardless

of the simplicity or multiplicity of the internal processes. These facts are true for any kind of working fluid as well as for steam.

80. The Throttling Calorimeter.—The reader will recognize in Case (d), Art. 79, the equivalent of the *throttling calorimeter*, made use of in testing to determine the moisture content of steam. The instrument is represented in Fig. 46. The body is the equivalent of a turbine casing with rotor removed, and the simple orifice replaces the nozzle. The pressure p_e in the calorimeter is usually atmospheric or slightly above, as indicated by the manometer.

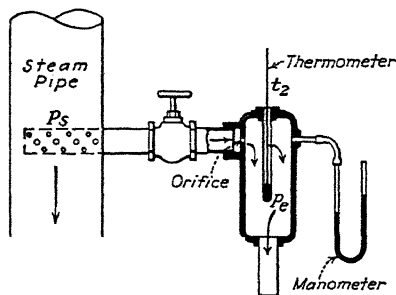


FIG. 46.

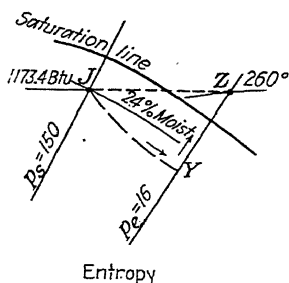


FIG. 47.

The enthalpy after expansion, at Z , Fig. 47, is equal to that at J , before expansion, if the instrument is well insulated. The actual path of the steam's state point is something like JYZ , but this is immaterial since the position of the end point Z is not influenced by the particular route from J to Z . Hence, the kind of hole or orifice used makes no difference in the indication of the instrument. A rough-punched hole is just as satisfactory as a smoothly drilled one with all burrs removed.

As an example in the use of the throttling calorimeter, suppose $p_s = 150$ psia, $t_2 = 260$ F, and $p_e = 16$ psia. What is the moisture content of the steam in the pipe?

The Mollier diagram is most convenient in the solution. At the intersection of $t_2 = 260$ F and $p_e = 16$ psia, as illustrated by Fig. 47, the enthalpy of steam in the calorimeter is found to be 1173.4 Btu. Then, the position of point J

on the diagram is located at the intersection of enthalpy line 1173.4 Btu with constant-pressure line $p_s = 150$ psia, at which point the moisture content is read as 2.4 per cent.

81. Throttling.—From the energy standpoint, it is now evident that throttling consists in transformation of energy which might have been transformed into work and ultimately appears again as enthalpy in the fluid.

The *losses in a turbine* are almost wholly of a throttling character. A turbine's inefficiency is not to be attributed to inability to transform available heat into work but in its self-consumption of a share of the energy transformed from the enthalpy of the fluid.

Throttling occurs wherever there is fluid flow, although the operation is often disguised under a different name. In flowing through a pipe, work is required to keep a column of fluid moving against the opposition of surface friction and turbulence. The loss in pressure or head of fluid is a measure of the available energy demanded for the purpose, and the throttling operation is called *pipe friction*. When a fluid is squeezed through restricted ports forming a part of a machine in which the fluid is to be used or handled, the operation is called *wire drawing*. Sometimes a *reducing pressure valve* is used in a fluid line for the purpose of reducing the pressure from a higher to a lower value better adapted for use in some particular apparatus, as in the case of steam reduced from line pressure to near atmospheric for use in a heating system. Here again available energy is consumed internally in the fluid and is thereby lost for useful external application. In throttle-governed engines and turbines, the rate of fluid flow is adjusted to the energy demand by means of a *governor throttle valve*, and available energy is again wasted in the process.

On a temperature-entropy diagram, Fig. 48, the expansion of the steam in a many-stage type of turbine is represented by the line JK , while the line for the Rankine cycle is JX . On the pressure-volume diagram of Fig. 49, the line JK necessarily lies to the right of JX , since the

steam at the terminal pressure p_e is drier at K than at X , and therefore occupies a greater volume. The work area of the actual cycle $EJKL$ is therefore unquestionably greater than that of the ideal cycle $EJXL$. The heat equivalent of the work area $EJKL$ of the pressure-volume diagram is the area $LFHJK$ on the temperature-entropy diagram, and $LFHJX$, Fig. 48, is the equivalent of $EJXL$, Fig. 49. Hence, it must be admitted that more energy is transformed in the cycle $LFHJK$ than is accounted for by the available heat $LFHJX$!

In the process of expanding along JK , the enthalpy in the steam may be thought of as being transformed into

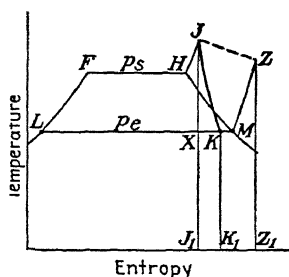


FIG. 48.

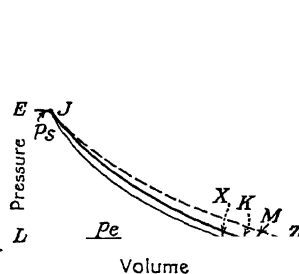


FIG. 49.

two kinds of energy, *viz.*: useful work that flows out along the shaft, and energy transformed by internal losses in the turbine. This latter, as in all cases where work is done against friction, is turned back into heat, which is reabsorbed by the steam. The area J_1JKK_1 , Fig. 48, represents this returned heat. The steam responds to this internal supply of heat, in respect to its properties, in exactly the same manner that it would if this same amount of heat were introduced at the same rate from an external source such as a hot jacket. By virtue of this heat replenishment, more energy is transformed from the steam by the amount JXK . Thus, because of the losses themselves, more energy is transformed from the steam than the equivalent of the available heat.

The situation may be illustrated in the following manner: Suppose a turbogenerator of 100 per cent efficiency, so that

the electrical energy delivered is the full equivalent of the available heat of the steam. Let the turbine casing be wound with resistance wires, and let a part of the electrical output, say 40 per cent, be diverted for use in this jacket heater. The steam-expansion line in the turbine would thus be changed from JX , Fig. 48, to some such line as JK . The heat quantity J_1JKK_1 is received by the steam from the jacket during expansion, and as a result the steam is enabled to do more work in this turbine of the (originally) Rankine cycle by the amount JKX , and the electrical output is increased correspondingly, which may be considered as being added to the 60 per cent that was delivered by the unit for useful external purposes. Unfortunately, the gain resulting (represented by JKX) is not nearly so great as the electrical energy (represented by J_1JKK_1) returned to the steam. Of this latter, the area J_1XKK_1 becomes unavailable and is carried away as enthalpy by the exhaust steam. Energy in the mechanical or electrical form is fully available for performing some useful effect and is entirely independent of temperature. But when transformed into heat again, the second law of thermodynamics prevails, and only a part of the total energy is thus available.

The process in an actual turbine is like the imaginary case discussed above, the only difference being that some of the energy transformed in the turbine produces heat then and there, whereby the enthalpy of the steam is increased resulting in the development of some further work but by no means an amount equivalent to the energy transformed by losses.

82. Losses in a Steam-engine Cylinder.—Steam-friction loss is a necessary accompaniment of velocity or flow. The steam turbine is a velocity machine; hence, its peculiar susceptibility to this loss.

The steam reciprocating engine, on the other hand, is a direct-pressure machine. Its operating principle does not involve the necessity for the generation of velocity

except for the incidental purpose of getting the steam into and out of the working chamber. Frictional effects accompany the creation of these incidental velocities, and the kinetic energy represented by them is quickly returned to the steam in the form of enthalpy. This loss is called *wire drawing*, which, as has been seen, is in reality throttling.

That part of the mechanical losses (friction horsepower) of an engine represented by valve and piston friction is also of a reheating character, the energy returning to the steam as enthalpy. But the piston engine is also vulnerable to a very serious loss peculiarly its own, which, in the steam engine, is called *initial condensation and reevaporation*.

83. Initial Condensation and Reevaporation.—Imagine a horizontal steam engine, with the piston at the crank end dead center. The steam in the head end of the cylinder, having just been released, is at its lowest temperature and is cooling the cylinder walls, piston, ports, and valve. When the piston reaches the head end, the inlet valve opens, flooding the cool surfaces with hot steam, which is partially condensed thereby. At cutoff, the space behind the piston is filled with steam (and water) whose volume is much less than it was a moment before in the chest. Hence the piston will not be pushed so far by a given weight of steam as it would if there had been no condensation. Some of the steam's ability to do work has been lost. Later operations in the cycle tend to make good this damage, but an irreparable loss of availability has occurred that can never be made good in full.

On the pV sketch of Fig. 50, the charge of steam while yet in the chest, just prior to admission, may be thought of as occupying the volume EJ . Because of initial condensation, its volume in the cylinder at cutoff is only EG . As expansion takes place, the pressure of the steam drops and with it the temperature. At first, the temperature of the steam is higher than the average temperature of the walls, and condensation, due to transfer of heat from

the steam, continues, but at a lessening rate. But after a time, the temperature of the steam falls below that of the confining walls, the flow of heat reverses, and the steam gains heat from the metal during the remainder of expansion to K , which results in the reevaporation of some of the moisture. This cycle of heat interchange between steam and metal is referred to as *initial condensation and reevaporation*.

84. Why Initial Condensation and Reevaporation Result in a Loss of Availability of Energy.—In one respect, the phenomenon just described, occurring in an engine cylinder, is like the frictional or throttling loss of a turbine. In both cases, a certain amount of energy is diverted at or near the beginning of the process, and later this energy is returned in full, or nearly so, to the working medium as enthalpy. There is no net loss of energy quantity in either case.

Although there is no loss of total-energy quantity in either case, there is loss of available energy in both. How initial condensation and reevaporation result in a loss of available energy can be well pictured on the temperature-entropy diagram.

In Fig. 51, J represents the state of the steam in the chest, and LK the exhaust pressure. NN is a level at which, in the course of the expansion, the temperatures of steam and metal are equal. The change in state of the steam from one instant, when it was in the chest to the next when it is in the cylinder at cutoff, is represented by the path JG —a line trending *to the left*, because heat is removed from the steam during the operation; and *downward*, because of *some loss of pressure* by wire drawing.

After cutoff, the pressure falls as a result of expansion and with it the temperature. However, the steam continues to lose heat until its temperature, going down, meets the metal temperature, coming up, at level NN . This second phase of the steam's heat-losing operation is represented by GM . The total amount of heat transferred from

total available energy is much less than for the single-cylinder expansion which is represented by the dotted line JMK . Since there is a less amount cut out from the available-energy area, there will be a less amount of heat rejected in the exhaust steam, so that the point Q representing the state of the steam will fall to the left of point K which represents the state of the exhaust for the single-cylinder engine, and the waste energy J_1XQQ_1 for the compound engine is less than J_1XKK_1 for the single-cylinder engine.

The steam turbine is practically free from initial condensation losses, because at any given point the steam is always at one temperature for a given load, and the metal in contact with it assumes that constant temperature. There is no periodic interchange of heat. On the other hand, this advantage on the part of the turbine is offset by the fact that it must contend with large steam-frictional losses, because it is a velocity machine.

86. The Uniflow Engine.—The uniflow reciprocating engine is an attempt to approach the favorable condition of the turbine with respect to condensation and reevaporation loss, by keeping the steam moving in one direction as far as may be possible with a reciprocating mechanism. The cool exhaust steam is not pushed out through the same passages or even the same end of the cylinder as that at which the hot steam enters but passes out of the cylinder, after its work is done, through ports uncovered by the piston itself. The idea of the uniflow engine is the most important thermodynamic achievement in the development of the steam engine since the introduction of compounding.

It is to be noted that the loss, described by the term *initial condensation and reevaporation* in connection with a steam reciprocating engine, is not peculiar to this prime mover alone. It exists in the gas engine and the oil engine and, in fact, in any utilizer in which the containing walls and other metal parts are subjected to contact with the working substance as its temperature varies during the working

operations of the cycle. It is, however, particularly aggravated in the steam reciprocating engine because of the formation of a water film on the cylinder walls, which accelerates the transmission of heat and so greatly increases the magnitude of the loss.

Jacketing the cylinders of ordinary (counterflow) engines with high-pressure steam is practically ineffective in reducing initial condensation and reevaporation. While such an arrangement may help to maintain a higher mean temperature of cylinder walls and thus reduce initial condensation, on the other hand, the greater part of the heat transmitted from the jacket is received by the working steam *at its lowest temperature, i.e.,* while the exhaust valves are open. Heat received at this time is at once rendered unavailable in the exhaust steam.

The heads of uniflow engines are always steam-jacketed, and sometimes also the ends of the cylinder barrel. But here the jackets tend to maintain the steam in the cylinder near the steam inlet ends always at the maximum temperature, while the steam in the middle part of the cylinder in the vicinity of the exhaust ports receives no heat from the jacket.

87. Means of Increasing the Efficiency of the Ideal Cycle.—The utilizer of a steam-heat-engine system may be regarded, thermodynamically, as the primary element of the plant, and its thermal efficiency depends upon how much available energy the utilizer has at its disposal and how well it performs its function of converting that available energy into useful work. *How well* the utilizer performs its function of converting available heat into work is determined by operations *internal* to itself, and this point has already been discussed in connection with the two principal losses, *viz.:* throttling, and initial condensation and reevaporation. *How much available heat* the utilizer has at its disposal is determined by the condition of pressures and temperatures at which heat is supplied to and rejected from the steam and is dependent upon the boiler

and condenser—elements *external* to the utilizer. Engineers, in recent years, have turned their attention more and more toward consideration of the various possible means of increasing the amount of heat available to the utilizer, and in the embodiment of these ideas in actual designs, they have been guided by the results of thermodynamic calculations. These various means may be enumerated as follows:

- a. *Low exhaust temperature.*
- b. *High pressure without superheat.*
- c. *High temperature with moderate pressure.*
- d. *Resuperheating.*
- e. *Extraction heating of feed water.*
- f. *The binary vapor system using mercury and steam in different ranges of the complete cycle.*

These various plans will be treated in turn to show the theoretical advantages offered by each.

88. Low Exhaust Temperature.—As a basis of comparison, consider steam furnished at 250 psia and 100 per cent quality and exhausted at 0.949 psia, which corresponds to 100 F temperature. The Rankine cycle

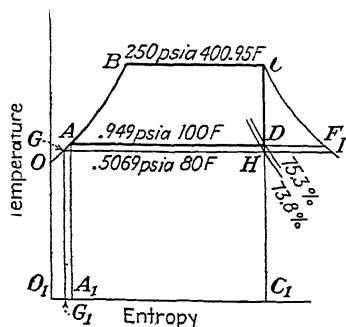


FIG. 53.

for these conditions is represented on Fig. 53 by $ABCD$. Let the back pressure be reduced to 0.5069 psia pressure, which corresponds to 80 F temperature. The Rankine cycle for the new condition is $GBCH$. The available-heat area has been increased by the strip $ADHG$, although the amount of heat supplied is also larger by the area of the strip

By calculation or reference to the Mollier diagram, it is found that $ABCD = 353.1$ Btu, and

$$GBCH = 379.1 \text{ Btu,}$$

thus showing a gain of 26 Btu, as a result of the lower

back pressure or higher vacuum. The thermal efficiency for the case of the 100 F exhaust temperature is found to be 31.2 per cent, while that for the 80 F temperature is 32.8 per cent, a gain of 1.6 per cent in efficiency.

There is, however, a very definite limit to the temperature to which the exhaust can be reduced, and that is the temperature of the cooling water, which in turn reflects that of the general environment in which a plant may be situated. In present-day high-grade plants, the excellence of condenser equipment has brought the exhaust-steam temperatures to within a few degrees of that of the leaving cooling water, for average temperature of environment, so that but little further improvement is possible in this direction.

Accompanying the theoretical gain in the use of lower exhaust pressures, there are two practical difficulties to be met. As the steam pressure becomes lower, the volume of the steam increases enormously, thus imposing a severe problem in the design of passages in the lower stages of the turbine and the exhaust connections in order to provide sufficient area for the steam to flow without undue restriction. In the example employed above, the specific volume of the steam at *D*, Fig. 53, is 264 cu ft, while that at point *H* is 467—an increase of 77 per cent for a reduction of temperature of only 20 degrees. Again, with the lower exhaust temperature, the end point of the expansion line penetrates deeper into the moisture region. Thus, at *D*, the quality of the steam is 75.3 per cent, while at *H* it is 73.8 per cent. Moisture in steam is not only detrimental to the efficiency of a turbine but by its erosive action is injurious to the nozzles and blades.

89. High Pressure without Superheat.—Let it be assumed that the steam pressure is to be 600 instead of 250 psia. The pressure increase is accompanied by a temperature increase of about 85 degrees. In Fig. 54, *ABCD* again represents the Rankine cycle for 250 psia steam pressure and 0.949 psia exhaust pressure. With 600 psia and

saturated steam, the cycle is $AJKL$. The efficiency with the higher pressure is decidedly greater, because a greater proportion of the heat supplied (A_1AJKK_1) is disposed above the exhaust line AF than is the case with the lower

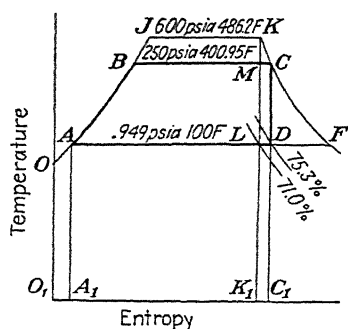


FIG. 54.

pressure. The available heat for the 600 psia case is 400.2 Btu against 353.1 Btu for 250 psia; and the efficiency of the higher pressure cycle is 35.2 per cent as against 31.2 per cent for the lower pressure—a gain of 4 per cent.

This large theoretical gain is, however, accompanied by an increase of several per cent

in the moisture content of the steam in the low-pressure stages where it is so destructive to buckets and conducive to supersaturation (a loss discussed in Art. 204) that it becomes impracticable to employ high-pressure steam without accompanying superheating and even resuperheating (see Art. 91).

In contrast to the situation met with in Art. 88, where there is a more or less absolute limit toward which the exhaust temperature can be lowered, there is no such absolute limit at the upper part of the cycle. Practically, of course, there are limits to the pressure for which the elements of a steam plant may be designed and operated, although there are plants operating at 3500 psia, which is above the critical pressure of steam.

90. High Temperature with Moderate Pressure.—High steam temperatures may be obtained by superheating, without accompanying high pressures. There is no absolute limit to which the temperature may be brought by superheating, but the usual practical limit at present is 975 F, taking into account the materials of construction.

In Fig. 55, the Rankine cycle for 250 psia pressure and 800 F temperature is $ABCNP$. It is apparent from an

inspection of the diagram that the theoretical gain from high superheat is rather small. For the maximum benefit, *all* the heat should be supplied at the maximum temperature. If higher temperatures than *BC* are to be employed, the ideal way to do is to supply the heat as heat of vaporization, so that the additional area will lie in the region above *BC*. That, however, there is some theoretical gain from superheating is evident from the figure, since there is a somewhat larger disposal of area above the exhaust-temperature line *AF* than there is without superheat. The available heat for this superheat cycle is 454.4 Btu, and the efficiency is 33.6 per cent. The theoretical gain from superheating at 250 psia to 800 F is six-tenths that with 600 psia saturated steam, notwithstanding the fact that the maximum temperature in the latter case is 314 deg less than that in the former, which illustrates and emphasizes the point that it is *not so much the maximum*

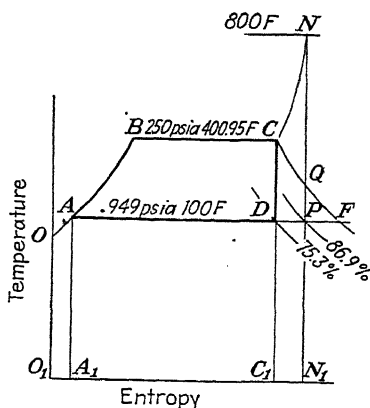


FIG. 55.

temperature of a Rankine cycle that counts as it is *the temperature at which most of the heat is received* by the working substance.

The great practical advantage of superheated steam is in the reduction of the moisture content of the steam in the turbine. In the ideal cycle, *ABCNP*, Fig. 55, the steam expands to *Q* before its state point enters the moisture region, and in the actual cycle the point of entry is still lower because of the displacement of the expansion line *NP* to the right, as a result of internal losses that must necessarily exist in any actual turbine.

91. Resuperheating.—It will be recalled that the large theoretical gain resulting from the use of the high pressure

of 600 psia, with no superheat, is considerably offset by the practical disadvantage of high moisture content of the steam in the turbine. This was shown in connection with Fig. 54, where the quality at the adiabatic end point L is 71.0 per cent. It will naturally occur to the reader that the remedy for the high-moisture condition with high initial pressure is to superheat the steam to some high temperature before it enters the turbine. In Fig. 56, R represents the state of the steam at 600 psia pressure and 800 F temperature. If expansion takes place from

this state, the adiabatic end point falls at S , where the quality is found to be 81.2 per cent, which is not much higher than that resulting from expansion from 250 psia without superheat. On account of the leftward trend of the saturation line $FTCK$, initial superheating to a fixed temperature becomes of less and less value for the higher pressures, in reducing moisture content in the turbine.

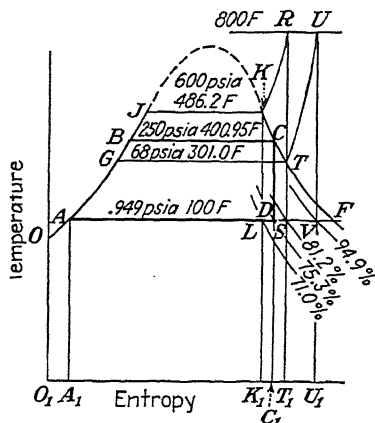


FIG. 56.

In order to avoid excessive wetness of the steam, *resuperheating* is employed in many cases.

In resuperheating, the steam is expanded from R until the superheat becomes nearly zero, *i.e.*, until the adiabatic line approaches the saturation line. In Fig. 56, the expansion is shown as continuing to T on the saturation line, where the pressure is found to be 68 psia and the temperature 301 F. At this state, the steam is withdrawn from the turbine and piped to a reheating boiler, or a high-pressure steam heater, where it is again superheated. In Fig. 56, the reheat temperature is assumed to be 800 F, represented by point U . After reheating, the steam is returned to the turbine wherein the expansion is continued

from 68 psia and 800 F to the exhaust pressure of 0.949 psia, point *V* indicating the final state at which the quality is 94.9 per cent. In plants employing resuperheating, the turbine is usually built in two separate sections: the first expanding the steam from the initial state down to the reheating pressure, and the second completing the expansion to the exhaust pressure.

The enthalpy supplied per pound of steam is represented by the area $A_1AJKRTUU_1$, of which A_1AJKRT_1 is furnished in the water preheaters, the boiler, and the primary superheater; and T_1TUU_1 is furnished in the

$$\begin{aligned} O_1OAJKRT_1 &= 1407.7 \text{ Btu Entropy at } R = 1.6343 \text{ (Steam Tables)} \\ O_1OAST_1 &= \underline{910.2 \text{ Btu Moisture content, 18.8 per cent (Mollier diagram)}} \end{aligned}$$

$$AJKRS = 497.5 \text{ Btu Available heat of steam as supplied}$$

$$O_1OAGTUU_1 = 1430.2 \text{ Btu Entropy at } U = 1.8876 \text{ (Steam Tables)}$$

$$O_1OAGTT_1 = \underline{1180.0 \text{ Btu}}$$

$$T_1TUU_1 = \underline{250.2 \text{ Btu Supplied in resuperheating (Steam Tables)}}$$

$$O_1OAGTUU_1 = 1430.2 \text{ Btu}$$

$$O_1OAVU_1 = \underline{1052.0 \text{ Btu Moisture content at } V = 5.1 \text{ per cent}}$$

$$AGTUV = \underline{378.2 \text{ Btu}}$$

$$O_1OAGTT_1 = 1180.0 \text{ Btu Entropy} = 1.6343 \text{ (Steam Tables)}$$

$$O_1OAST_1 = \underline{910.2 \text{ Btu (Mollier diagram)}}$$

$$AGTS = \underline{269.8 \text{ Btu}}$$

$$AGTUV = 378.2 \text{ Btu}$$

$$AGTS = \underline{269.8 \text{ Btu}}$$

$$STUV = 108.4 \text{ Btu Available part of heat supplied in resuperheating}$$

$$AJKRS = 497.5 \text{ Btu Available heat of steam as supplied}$$

$$STUV = \underline{108.4 \text{ Btu Available part of resuperheat}}$$

$$AJKRTUV = 605.9 \text{ Btu Total available heat of the cycle}$$

$$O_1OAJKRT_1 = 1407.7 \text{ Btu Enthalpy of steam as supplied}$$

$$T_1TUU_1 = \underline{250.2 \text{ Btu Resuperheat}}$$

$$O_1OAJKRTUU_1 = 1657.9 \text{ Btu Total enthalpy of cycle}$$

$$O_1OAJKRTUU_1 = 1657.9 \text{ Btu Total enthalpy of cycle}$$

$$O_1OAA_1 = \underline{68.0 \text{ Btu Enthalpy of liquid at exhaust pressure}}$$

$$A_1AJKRTUU_1 = 1589.9 \text{ Btu Heat supplied for the cycle}$$

$$\text{Thermal efficiency of cycle} = \frac{AJKRTUV}{A_1AJKRTUU_1}$$

$$\frac{905.9}{589.9} = 0.381 \text{ or } 38.1 \text{ per cent.}$$

$ABCM$, is now less than before, but at the same time a much less quantity of heat, B_1BCC_1 , must be supplied from the boiler than would be the case if the feed water is heated in the boiler from A to B . The efficiency of the supposed cycle is the ratio of the area $ABCM$ to the area B_1BCC_1 , which, it is readily seen, is that of the *Carnot cycle*. With steam supplied at 250 psia, 400.95 F and exhausted at 100 F, the efficiency of cycle $ABCM$ is 35.0 per cent.

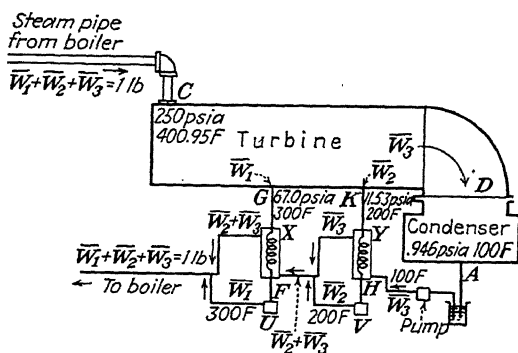


FIG. 58.

While this plan is not practical for an actual turbine, nevertheless it represents an ideal that is partially realized by the more feasible method of heating feed water in steps by steam extracted from the turbine at certain points for that specific purpose.

A diagrammatic scheme illustrating extraction heating in two stages is shown in Fig. 58. X and Y are the two closed heaters, steam for which is extracted at appropriate points from the turbine casing. For each pound of steam that enters the turbine at C , the fraction \bar{W}_1 is withdrawn at G for the heater X , and the fraction \bar{W}_2 at K for the heater Y . The remainder \bar{W}_3 expands completely in the turbine and goes on to the condenser, where it is liquefied. The steam extracted at K serves to heat the water \bar{W}_3 , from 100 F to 200 F, and that extracted at G heats the water $\bar{W}_2 + \bar{W}_3$, from 200 F to 300 F. The water leaving the heaters is here shown as having attained the temperature

of the heating steam, which is, of course, not quite probable; and the condensate from the heaters, after having been delivered by the traps V and U , is shown as joining the main stream of feed water at the same temperature, although the trap discharge is not usually disposed of in this manner. The temperatures and pressures for the extracted steam are so chosen as to divide the total temperature range into nearly equal parts, which is approximately the most favorable division.

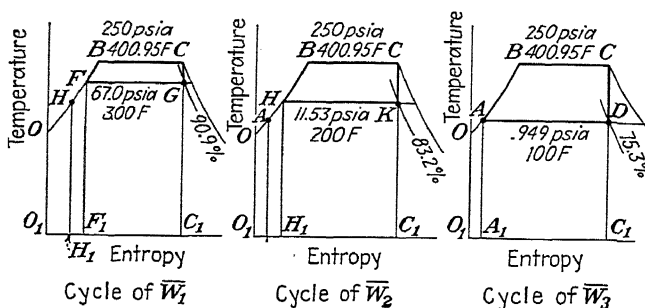


FIG. 59.

It will now be seen that a part of the steam, \bar{W}_1 , expands in the turbine from 250 to 67.0 psia and is then withdrawn to give up its latent heat to the feed water; another part \bar{W}_2 expands from 250 to 11.53 psia and is then withdrawn to give up its latent heat; while the remainder \bar{W}_3 is completely expanded in the turbine and passes on to the condenser. The cycles of \bar{W}_1 , \bar{W}_2 , and \bar{W}_3 are represented on the temperature-entropy diagrams of Fig. 59, where the lettering corresponds to that of Fig. 58. The cycle of \bar{W}_1 is $FBCG$; that of \bar{W}_2 , $HBCK$; and that of \bar{W}_3 , $ABCD$.

The weights \bar{W}_1 , \bar{W}_2 , and \bar{W}_3 can be determined by equating appropriate heat quantities, as found in the Steam Tables.

$$\bar{W}_2 \times H_1HKC_1 = \bar{W}_3 \times A_1AHH_1.$$

$$\bar{W}_2 \times H_1HKC_1 = 977.9 \text{ (latent heat at 200 F)} \times 0.832 \text{ (quality)} \times \bar{W}_2 = 813.6 \bar{W}_2.$$

$\bar{W}_3 \times A_1 A H H_1 = \bar{W}_3$ [168.0 (enthalpy of liquid at 200 F) - 68 (enthalpy of liquid at 100 F)] = $100\bar{W}_3$.

Whence

$$813.6\bar{W}_2 = 100\bar{W}_3$$

and

$$\bar{W}_2 = 0.1229\bar{W}_3$$

Similarly,

$$\bar{W}_1 \times F_1 F G C_1 = (\bar{W}_2 + \bar{W}_3) \times H_1 H F F_1.$$

$\bar{W}_1 \times F_1 F G C_1 = 910.1$ (latent heat at 300 F) $\times 0.909$ (quality) $\times \bar{W}_1 = 827.3\bar{W}_1$.

$(\bar{W}_2 + \bar{W}_3) \times H_1 H F F_1 = (\bar{W}_2 + \bar{W}_3)$ [269.6 (enthalpy of liquid at 300 F) - 168 (enthalpy of liquid at 200 F)] = $101.6(\bar{W}_2 + \bar{W}_3) = 101.6(0.1229\bar{W}_3 + \bar{W}_3) = 114.1\bar{W}_3$.

Whence

$$827.3\bar{W}_1 = 114.1\bar{W}_3$$

and

$$\bar{W}_1 = 0.138\bar{W}_3$$

From the above values, it is easily determined that for each pound of steam entering the turbine $\bar{W}_1 = 0.1095$, $\bar{W}_2 = 0.0975$, and $\bar{W}_3 = 0.7931$.

The quantity of steam, \bar{W}_1 , contributes an amount of available heat per pound represented by the area $FBCG$, whose value, determined by the usual means, is 104.2 Btu. In like manner, the available heat per pound of \bar{W}_2 is 219.5 Btu ($HBCK$), and that for \bar{W}_3 , 353.1 Btu ($ABCD$). The total available heat for every pound of steam entering the turbine is

$$\begin{array}{rcl} 0.1095 \times 104.2 & = & 11.4 \text{ Btu} \\ 0.0975 \times 219.5 & = & 21.4 \text{ Btu} \\ 0.7931 \times 353.1 & = & 280.0 \text{ Btu} \\ \hline \text{Total} & = & 312.8 \text{ Btu} \end{array}$$

The total amount of heat to be supplied in each pound of steam entering the turbine is that necessary to heat 1 lb of water from 300 to 400.95 F and vaporize it at that temperature. This heat is represented by the area $F_1 F B C C_1$, Fig. 59, and is equal to 931.5 Btu. The efficiency is

$312.8/931.5 = 0.336 = 33.6$ per cent, which, when compared with the value 31.2 per cent for the straight Rankine cycle $ABCD$, reveals a gain of 2.4 per cent in efficiency or a relative gain of 7.7 per cent for the conditions specified and using two heaters. Three, four, or more heaters might be considered, and it would be found that each additional heater would contribute a slight amount to the improvement in efficiency over that of the Rankine cycle. The maximum possible efficiency that could be attained with an infinite number of heaters would be that of the cycle $ABCM$, Fig. 57, which, for the conditions of the example, has already been found to be 35.0 per cent. It is apparent that each additional heater contributes a less gain than the one preceding, so that, practically, the number is usually limited to four or less. A single heater results in a large gain, and it is interesting to observe that the common open feed-water heater of the ordinary steam plant is, in reality, a single heater of an extraction feed-heating cycle. Each pound of steam made in the boiler is divided into two parts: the larger part passes through the utilizer, expanding through the entire pressure range, while the smaller part expands only to atmospheric pressure in noncondensing auxiliary engines and is then used to heat the condensate from the condenser by allowing it to give up its latent heat in the open feed-water heater. Viewed in the light of the advantage resulting from extraction feed heating, it is evident that even in a plant that has no steam-driven auxiliary engines it would pay to extract steam from an appropriate point in the utilizer to heat the water before introducing it into the boiler.

It may, at first, seem somewhat peculiar that the available heat of each pound of steam entering the turbine is considerably less for the extraction heating cycle than it is for the straight Rankine cycle. A little thought, however, will recall the fact that a part of the steam in the extraction cycle is diverted from its regular course, and applied to the purpose of heating feed water, while yet possessing a

good deal of available heat, with the net result, as has been seen, that the efficiency is materially increased. *The steam consumption*, in terms of pounds of steam per unit of power output, *would be increased about 13 per cent*, but, on account of the high temperature at which the water comes to the boiler, *the heat to be supplied to each pound is reduced by about 18 per cent*.

In extraction feed heating, it is to be noted that the gain over the straight Rankine cycle is accomplished, not by extending the limits of the cycle, but by a redistribution of the heat within the original limits. For this reason, the plan should have primary consideration, as well as for the fact that it is perhaps the most easily embodied in an actual steam power plant.

In the example employed in demonstrating the principle of extraction heating of feed water, steam is assumed to enter the turbine without superheat, which is the simplest case. The principle, however is applicable as well to turbines using superheated steam, and all turbines of such size as to warrant consideration of extraction heating would use superheated steam for reasons heretofore discussed.

93. The Binary Vapor System—Mercury and Steam. For moderate temperatures, steam is an ideal working agent. It is fortunate indeed that water, one of the most universal of substances, should at the same time possess properties that make it so suitable for use in a heat engine. But when it becomes necessary to advance the temperature frontiers in order to exploit available-heat territory that lies so temptingly in view, the use of steam presents difficulties that have been revealed in the paragraphs immediately preceding. Into the low-temperature region, steam carries with it its troublesome high specific volume; and into the high-temperature region, its unreasonably high pressure. No effort has been successful in finding any substance suitable for the entire temperature range demanded, but it has been found that mercury possesses

properties that render it peculiarly suited as a working substance to operate within that high-temperature territory that lies between the maximum set by the safe strength of materials, 1000 F, and that accompanying what may be considered a moderate and easily handled pressure for steam (about 400 F). The use of mercury and steam in a dual or binary system has been worked out by Dr. W. Le R. Emmet, of the General Electric Company, and the properties of mercury, in that connection, by L. A. Sheldon of the same company.¹

Briefly, the embodiment of the mercury-steam system consists of a mercury boiler in which the liquid is vaporized by the heat of the fuel; a turbine for the utilization of the heat of the mercury vapor which operates in exactly the same manner as a steam turbine; and an element which is, at one and the same time, a mercury condenser and a steam boiler, the heat given up by the mercury upon condensation being used to vaporize the water. The steam formed in this manner may be superheated by the heat of the gases of combustion and used in an ordinary steam turbine. Resuperheating may be employed, as may also extraction feed heating. In fact, the combination with the mercury cycle imposes upon the steam cycle no departure beyond ordinary practice, the only requirement being that the condensing temperature of the mercury and the vaporizing temperature of the steam shall be properly related to each other.

Having indicated the objective sought in the use of the mercury-steam system and suggested the means whereby the objective may be realized, the attention of the reader is now directed to Figs. 60 and 61, where the relations between the mercury and steam are represented on temperature-entropy diagrams. In Fig. 60, *OMN* is the liquid line for 1 lb of mercury; *NP* is the evaporation line at 45 psia pressure and 800 F temperature; *PQ* is the adiabatic expansion line; and *QM* is the condensation

¹ *Trans. Amer. Soc. Mech. Eng.*, 1924.

line at the exhaust pressure of 0.75 psia and 440 F temperature. $MNPQ$ is, therefore, the Rankine cycle for 1 lb of the mercury, and $ABCD$ is the familiar Rankine cycle for 1 lb of the steam.

It will be assumed that the water enters the steam boiler at a temperature of 300 F which corresponds to the example of Art. 92. Point F , Fig. 60, represents this state. The heat required to make 1 lb of steam is represented by the area F_1FBCC_1 , and the heat given up by each pound of mercury in condensing by M_1MQP_1 . Since the figure is drawn to scale, it is evident that 1 lb of mercury is inadequate to furnish enough heat to make 1 lb of steam, so that it will be necessary to circulate several pounds of mercury for each pound of steam. With the help of the Steam Tables, it is found that the area F_1FBCC_1 represents 931.5 Btu, and, with the help of the Mercury Tables, area M_1MQP_1 is found to be 99.1 Btu. The ratio of the two values shows that it will be necessary to circulate 9.4 lb of mercury to each pound of steam.

Figure 61 is like Fig. 60, except that the areas of the mercury cycle have been increased 9.4 times by making the entropy intercepts 9.4 times as great as those of Fig. 60. The cycle $MNPQ$, Fig. 61, is for 9.4 lb of mercury. To illustrate the idea better, the point Q of the mercury cycle is brought into line with the point C of the steam cycle. The heat-reception line FBC for the steam and the heat-rejection line QM for the mercury are simultaneous, and the area F_1FBCC_1 is equal to the area M_1MQC_1 .

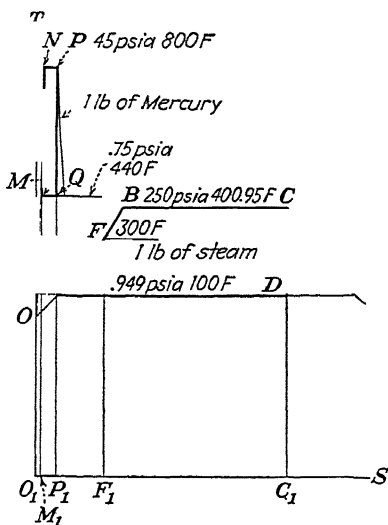


FIG. 60.

The total available heat of the system is the sum of the available heat of the steam and that of the mercury. In

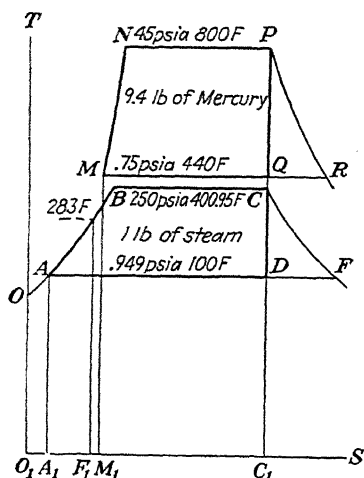


FIG. 61.

Art. 92, it was found that the available heat per pound of steam, working on the extraction heating cycle, is 312.8 Btu. The available heat of the mercury is represented by the area $MNPQ$, Fig. 61, and from the properties of mercury is found to be 352.5 Btu for 9.4 lb. The heat required for the system from an outside source is that which is necessary in the mercury boiler to heat the 9.4 lb of liquid along the line MN and vaporize along NP .

This heat quantity is 1284.0 Btu and is represented by the area $M_1MNP C_1$. The efficiency of the combined system is the ratio of the total available heat,

$$312.8 + 352.5 = 665.3,$$

to the heat supplied, 1284.0, or 51.8 per cent.

The startlingly high efficiency calculated for the mercury-steam system serves to show the wonderful possibilities in the binary vapor plan of utilizing heat. The example illustrates how peculiarly well the properties of mercury adapt it for a cycle to be superimposed above the steam cycle. Here the objective has been attained with a maximum pressure in the system of 250 psia for the steam. The saturation pressure for the mercury at 800 F is only 45 psia. With such a moderate pressure accompaniment, even higher temperatures may be successfully attempted.

94. Summary.—Inasmuch as the examples used in illustration of the several methods of increasing the effi-

ciency of the Rankine cycle are all related to each other, it will be of interest to tabulate the results for comparison.

Case	Steam pressure,	Steam temperature	Exhaust temperature, F	Efficiency of cycle, per cent	Relative efficiency, per cent
Moderate steam and exhaust temperature.....	250	Saturated	100	31.2	100.0
Low exhaust temperature.....	250	Saturated	80	32.8	102.2
High steam pressure.....	600	Saturated	100	35.2	110.0
High superheat.....	250	800 F	100	33.6	104.8
High pressure and temperature...	600	800 F	100	37.1	115.8
High pressure and resuperheating	600	800 F	100	38.1	119.0
Extraction feed-water heating...	250	Saturated	100	33.6	104.8
Mercury-steam system. Mercury, 45 to 0.75 psia					
Extraction heating in steam cycle.....	250	Saturated	100	51.8	161.5

Problems

1. Steam is generated in a boiler of a steam plant and furnished to a utilizer at 250 psia pressure and 550 F temperature. In the utilizer the steam is expanded to a condenser pressure of 1.6 psia and a quality of 97 per cent.

a. How many heat units have been contributed by each pound of steam to useful work?

b. Of the total number of heat units going to useful work, what number is to be credited to the external-work part of the enthalpy, and what number to internal energy? Neglect water volumes in solution.

2. What theoretical work, expressed in Btu, will be required to pump 1 lb of water into a boiler, under the following conditions of pressure range and temperature?

Case	Pressure-gage		Temperature, F
	Suction	Discharge	
a	0	180	190
b	80	600	276
c	200	1200	330

3. Does it require more work to pump 1 lb of cold water or 1 lb of hot water, into a boiler, through an equal pressure range? Answer by calculating the work per pound for the two cases below, and comparing the answers.

- a. Water at 70 F from 100 to 1200 psia pressure.
- b. Water at 300 F from 100 to 1200 psia pressure.

Is the water pumped against the same *head* in both cases? Explain.

4. What becomes of the energy imparted to the water by a boiler feed pump? Does it in any way increase the ability of the steam to do work in the utilizer? Explain.

5. In a given cycle, steam is generated and furnished to a turbine at 400 psia pressure and 700 F temperature and exhausted at 1.2 psia with 7.7 per cent moisture content. Condensate is returned to the boiler at 90 F.

a. Determine the number of Btu per pound of steam transformed into useful work at the turbine shaft, using the Mollier diagram.

b. Calculate the thermal efficiency of the cycle.

c. How many pounds of steam per brake horsepower-hour will be required?

6. For an ideal (Rankine-cycle) engine working between the same limits of initial pressure and temperature, and back pressure, as in Prob. 5:

a. Determine the number of Btu that would be transformed into work per pound of steam, using the Mollier diagram.

b. Calculate the thermal efficiency of the cycle.

7. For the conditions of 180 psia initial pressure, 640 F temperature, and 1 psia back pressure, calculate the thermal efficiency of a Rankine cycle, using a temperature-entropy sketch and the Steam Tables, as in Fig. 34.

8. Steam is supplied to a turbine at 250 psia pressure and 700 F temperature and exhausted at 16 psia. The turbine transforms into work 70 per cent as much heat as would be transformed by an engine operating in a Rankine cycle.

a. What is the state of steam exhausting from the turbine?

b. Indicate areas on a temperature-entropy sketch that represent: Enthalpy of steam as supplied.

Enthalpy of steam in exhaust from engine of Rankine cycle.

Enthalpy of steam in exhaust from actual turbine.

Energy that the turbine failed to transform into work because of its imperfections.

9. Calculate and compare the efficiencies of two Rankine cycles *A* and *B*, both working between the pressures of 180 and 0.8 psia; for *A*, the initial state is 4 per cent moisture; while for *B*, it is 400° superheat.

10. A test of a turbogenerator furnishes the following data: steam pressure, 120 psia with 1 per cent moisture; exhaust pressure, 20 psia; load, 2040 kw.; rate of steam consumption, 80,420 lb/hr; generator efficiency, 94 per cent; turbine-bearing loss, 20 kw. Neglect loss of heat from steam to atmosphere.

a. Determine state of exhaust steam (moisture or superheat).

b. What is the thermal efficiency of the unit?

c. What is the ratio of thermal efficiency of this actual unit to that of an engine operating in a Rankine cycle, working within the same limits?

(Illustrate operations by *TS* and *HS* sketches.)

11. Steam is supplied to a turbine at 180 psia with 200° superheat, and exhausted at 16 psia. Temperature of exhaust steam is observed to be 241 F. Load is 50 bhp.

a. Determine thermal efficiency of this unit and also that of a Rankine cycle.

b. Turbine-bearing loss is estimated at 3 hp and loss of heat to atmosphere (from turbine casing) at 10,000 Btu/hr. Determine Btu contributed to brake horsepower by each pound of steam.

c. How many pounds of steam per hour are being furnished to the turbine?

12. Determine the thermal efficiency of Rankine cycles for the following conditions:

a. Steam supply, 180 psia, 100 per cent quality; exhaust pressure, 1 psia.

b. Steam supply, 600 psia, 760 F temperature; exhaust pressure, 1 psia.

13. With steam supplied at 140 psia 2 per cent moisture and exhausting at 16 psia, determine thermal efficiency of .

a. An engine operating in a Rankine cycle.

b. A direct-acting steam pump (no expansion), and calculate theoretical loss of available heat (Btu per pound) by nonexpansive use of steam under the conditions specified.

14. A steam plant operates on a pressure of 2400 psia and 950 F, exhausts at 1 psia. Calculate Rankine cycle efficiency (a) considering pump work, (b) neglecting pump work. Calculate per cent error when pump work is neglected.

15. A turbine receives steam at 250 psia and 700 F temperature, and exhausts at 0.8 psia pressure. Neglecting loss of heat to atmosphere:

a. If it is assumed that the turbine transforms 65 per cent of the available heat of the steam into useful work, what is the moisture content in the exhaust steam?

b. If the thermal efficiency of the turbine is 21 per cent, what is the moisture content of the exhaust steam?

16. Steam is furnished to a utilizer at 300 psia and 600 F temperature. Exhaust pressure is 1 psia. Temperature of exhaust is observed to be 150 F. Neglecting loss of heat to atmosphere:

a. How many Btu per pound of steam are transformed into useful work, and how many wasted by the inefficiency of the utilizer?

b. What is the thermal efficiency of the utilizer, and what is its efficiency in the utilization of available heat?

c. How many pounds of steam are required per horsepower-hour of useful work developed?

d. Indicate areas on a temperature-entropy sketch that represent the various heat quantities involved in the solution of parts (a) and (b).

17. A 5000-kw. turbogenerator unit is furnished with steam at 214 psia pressure and 640 F temperature, and the back pressure is 1 psia. At full load, 5000 kw., the unit consumes steam at the rate of 69,000 lb/hr; and at zero load, 12,000 lb/hr. When plotted on coordinate paper, the relation between total steam per hour (vertical scale) and kilowatt load (horizontal scale) is a straight line (the Willans line).

a. Determine the thermal efficiency of this unit at full load, three-fourth, one-half, one-fourth, and zero load.

b. Determine efficiency with which available heat of steam is turned into electrical energy at the loads specified in (a).

Plot values of (a) and (b) against kilowatt load on the same sheet with Willans line, and draw curves.

18. A test of a small turbine gives the following data: steam pressure and temperature, 121 psia and 437 F; exhaust pressure and temperature, 15 psia and 247 F; load, 20 bhp.; steam consumed, 710 lb/hr. Determine loss of heat from steam in turbine to atmosphere in Btu per hour.

19. From the data given in the table below, determine the remaining quantities that should occupy the blank spaces. It is preferable to use the Mollier diagram in the solutions.

Case	Initial conditions					Back pressure, p_2 psia	Enthalpy of unavailable heat, h_2	Available heat	Unavailable heat
	Pressure, p_1 psia	Temperature, F t_1	Moisture, per cent	Enthalpy, h_1	Total entropy, s_1				
a	180		1.5			15			
b	180		1.5			1			
c	15		3.0			1			
d	180	600				1			
e	180		1.5			2			
f	425	700				1			
g	425	700				0.5			
h	425	700						20	
i	180		1.5					80	
j	5		5.0			1			

20. A simple one-stage impulse turbine, similar to Fig. 40, is supplied with 6000 lb of steam per hour at 200 psia and 1.5 per cent moisture, and exhausts at 15 psia. The nozzle is 95 per cent efficient in turning available heat into kinetic energy. The buckets transform 75 per cent of the kinetic energy of the jet into work at the rim of the wheel. The windage loss of the wheel revolving in the casing is 75 hp.

a. On a heat-entropy sketch, indicate the distances that represent the following energy quantities *per pound of steam*: available heat, kinetic energy of the jet leaving nozzle, nozzle loss, bucket loss, windage, useful work.

b. Determine the items of (a) in Btu.

c. Determine the state of the steam (percentage moisture) at the turbine exhaust.

d. What horsepower will be developed at the shaft of the turbine?

21. A turbine (see Fig. 40) is furnished with steam in the supply pipe at 160 psia and 1 per cent moisture, and exhausts at 16 psia. The turbine is

throttle governed, and at half load the pressure in the steam chest (at entrance to nozzles) is found to be 100 psia. By the use of the Mollier diagram, determine:

- a. The available heat of the steam before throttling.
- b. The state of the steam (moisture or superheat) in the steam chest after throttling.
- c. Available heat of the steam in the chest.
- d. Loss of availability in percentage, caused by throttling.
- e. On a heat-entropy sketch, indicate a distance that represents the loss of available heat by throttling.

22. What *maximum* percentage of moisture in a steam line can be determined by a throttling calorimeter discharging into atmosphere, when the line pressure is 100 psia? 200 psia?

23. When steam is throttled from a higher to a lower pressure, does its temperature increase, decrease, or remain unchanged? If there is a change, is it a matter of a few degrees or many?

Answer this question by determining the temperature after throttling, and *change* of temperature resulting, for the following cases, using Mollier diagram where possible. Throttling is from 160 psia to 16 psia in every case.

Case	a	b	c	d	e	f	g	h
Initial temperature, F.....	1000	800	600	400	363.5
Initial state.....	Dry saturated	2 per cent moisture	50 per cent moisture	Water

On one temperature-entropy sketch, with temperatures approximately to scale, show initial and final states connected by a line for each case.

24. A 1000 kw turbogenerator unit is running at full speed, but the generator is disconnected from the electric circuit and is therefore delivering no power. Under this condition, the turbine is using 3000 lb of steam per hour which is supplied at 180 psia and 500 F temperature, and exhausted at 1.40 psia. The generator loss is 70 kw and turbine-bearing loss, 8 kw. Neglecting loss of heat from steam to atmosphere, determine the state of exhaust steam in percentage of moisture or degrees superheat. Illustrate by temperature-entropy sketch.

25. A test of a turbine, supplied with dry-saturated steam at 130 psia and exhausting at 2 psia, shows the following water rates:

Load, brake horsepower.....	100	75	50	25
Water rate, pounds per brake horsepower-hour.....	17.00	18.33	21.00	29.00

Disregarding bearing friction and loss of heat to atmosphere, determine the load in brake horsepower at which the state of the exhaust will be just 100 per cent quality.

26. The turbine of Prob. 25 is throttle-governed, and the test referred to shows the pressure in the steam chest, after throttling, to be as follows:

Load, brake horsepower.....	25	50	75	100
Pressure in steam chest, psia.....	51	74	97	120

Steam supply pressure, 130 psia, quality 100 per cent, exhaust pressure 2 psia, constant at all loads.

Determine loss of available heat at each load in per cent, plot values against load, and draw a curve to show manner of variation. Furnish enthalpy-entropy sketch to illustrate principle of procedure.

27. Steam is furnished to a turbine at 180 psia, and 100 per cent quality. Determine the *available* heat per pound of steam, and *thermal efficiency* of Rankine cycle for exhaust pressures of 4, 3, 2, and 1 in. Hg abs pressure.

28. Given two turbines *A* and *B*. *A* is supplied with steam at 150 psia, 100 per cent quality; *B*, with steam at 150 psia, 500 F temperature. Both exhaust at 3 in. Hg abs. If the back pressure on both is reduced to 1 in., which unit will benefit more? Answer by calculating thermal efficiencies of Rankine cycles.

29. Given two turbines *C* and *D*. *C* exhausts at 3 in. Hg, *D* at 1 in. Both are supplied with steam at 180 psia, 100 per cent quality. If both units could be furnished with steam at 180 psia and 500 F, which would benefit more? Answer by calculating thermal efficiencies of Rankine cycles.

30. For a constant exhaust pressure of 2 in. Hg abs, determine *available heat* per pound of steam, and *thermal efficiency* of Rankine cycle for initial pressures of 140, 420, 600, 900, 1200, 1800, 2500, and 3206.2 psia, dry-saturated steam in all cases. Tabulate results, and then plot data to show how available heat and efficiency of Rankine cycle vary with initial pressure.

31. What is the thermal efficiency of a Rankine cycle for which initial pressure and temperature are 450 psia and 1000 F (not absolute), and exhaust pressure is 0.8 psia? What is the moisture content at adiabatic end point?

32. Given two Rankine-cycle turbines *E* and *F*. *E* is furnished with steam at 400 psia, 1000 F; *F* with steam at 800 psia, 600 F; both exhaust at 1 in. Hg abs; which has the higher thermal efficiency?

33. A high-pressure turbine is to be supplied with steam at 1200 psia, 800 F. It is assumed that reheating losses will be 22 per cent. Determine back pressure at which moisture content of the exhaust will be 10 per cent.

34. A turbine receives steam at 650 psia, 760 F, and exhausts at 0.8 psia. After expansion to the saturated state in a high-pressure section, the steam is resuperheated to 560 F and then expanded in a low-pressure section to final exhaust pressure. Assume constant-entropy expansion in both sections of the turbine.

a. At what pressure will the high-pressure section exhaust the steam, and how much heat is available to it per pound of steam?

b. What is the moisture content at exhaust from the low-pressure section, and what is the available heat?

c. What percentage of the total power of the turbine will be developed in the high- and low-pressure sections, respectively?

35. Determine final state of steam when throttled:

- a. From critical state (3206.2 psia, 705.4 F) to 1500 psia.
- b. From 3206.2 psia and 1000 F to 1500 psia.
- c. From 1500 psia, and 100 per cent quality to 600 psia.
- d. From 200 psia and 1 per cent moisture to 15 psia.

36. With 190 psia pressure and saturated steam, and 100 F exhaust temperature, calculate the efficiency of a cycle extracting steam for a single heater at 212 F.

37. A turbine receives steam at 450 psia and 900 F and exhausts at 0.8 psia. Steam is extracted for feed-water heating at two places (say *X* and *Y*), where the pressures are 100 and 12 psia, respectively.

a. Draw *TS* sketches (to scale) like those of Fig. 59, and determine Btu available for work in the turbine per pound delivered to *X*, to *Y*, and to main condenser.

b. Assuming that condensate pipes are connected as in Fig. 58, and that temperature of condensate passed on from *Y* to *X* is 195 F, and that from *X* to boiler, 320 F; determine the fractions \bar{W}_1 , \bar{W}_2 , and \bar{W}_3 , whose total make up 1 lb of steam entering the turbine.

c. Determine the thermal efficiency of the extraction cycle above and compare with that of "straight" Rankine cycle.

38. At 100 psia pressure, the temperature of saturated mercury vapor is 906.9 F and the entropy of vaporization is 0.0912. At 29 in. vacuum, the temperature is 408 F and the entropy of vaporization is 0.14638. The specific heat of liquid mercury is 0.0316. If mercury operating between these pressures is used in combination with a steam cycle whose limits are 200 psia pressure, saturated steam, and 80 F exhaust temperature without extraction heating:

a. Picture the cycle of 1-lb weight of each of the mercury and the steam on a temperature-entropy diagram, similar to Fig. 60, to scale.

b. Determine the number of pounds of mercury that it will be necessary to circulate for each pound of steam in order that the heat given up by the mercury in condensing may be just sufficient to make the steam.

c. Picture the cycle for the number of pounds of mercury as determined in (b), and 1 lb of steam, similar to Fig. 61, to scale.

d. Calculate the number of Btu available in the mercury cycle, the number available in the steam cycle, and the number to be supplied from an outside source, and thence the efficiency of this binary vapor system.

39. Steam at 400 psia and 740 F enters a turbine and exhausts at 2 psia. Steam is extracted at 58 psia for heating feed water to a temperature 5 degrees less than the saturation temperature of the extracted steam. The extracted steam leaves the feed-water heater as saturated liquid. Assume isentropic expansion in the turbine. What is the efficiency of the cycle?

40. Calculate the Rankine cycle efficiency for the following conditions:

a. Steam supply 250 psia, dry and saturated. Exhaust 15, 10, 5, 1, 0.5 psia. Plot efficiency vs. decreasing exhaust pressure.

b. Steam supply 400, 700, 1200, 2000, 2500, 3206 psia, dry and saturated. Exhaust 15 psia. Plot efficiency vs. increasing supply pressure.

c. Steam supply 250 psia, 500, 600, 800, 1000 F. Exhaust 15 psia. Plot efficiency vs. increasing temperature.

CHAPTER VIII

PERFECT GASES

95. Relation among the Properties of Gases—Boyle's and Charles's Laws.—All knowledge of the properties of gases must rest fundamentally upon experimental evidence. For steam and other vapors, the relations among the several properties, deduced from experiment, are quite complex, and for that reason tables of their properties are constructed, thus avoiding the use of cumbersome equations in the solution of problems. On the other hand, for gases the laws of relationship are quite simple, and it is easier to employ the equations expressing them than to resort to tables of properties. The laws of Boyle and Charles express the two fundamental relations for gases.

The first was discovered by Boyle in 1662. It states that: *at constant temperature, the volume of a given weight of gas is inversely proportional to the absolute pressure.* This law was also discovered independently a few years later by Marriotte, whose name is sometimes given to it.

The other law was discovered by Charles in 1787 but was not made public by him at the time. About 1802, Gay-Lussac published it, giving Charles the credit for its discovery however. It is sometimes spoken of as Gay-Lussac's law. It states that: *with pressure constant, the volume of a given weight of gas varies directly as the absolute temperature.*

The statement of Charles's law is equivalent to saying that the coefficient of expansion of a gas is constant; *i.e.*, its change of volume per degree of temperature change is constant. If, for example, the temperature of a gas is changed from 32 F to 33 F or from 32 F to 31 F, its volume will increase or decrease $1/491.69$ of the original, no matter what the pressure is, so long as the pressure is held constant.

If its temperature is changed from 32 F to 52 F, or from 32 F to 12 F, its volume will increase or decrease $20/491.69$ of the original.

96. Graphical Representation of Charles's Law—Absolute Zero of Temperature.—In Fig. 62, temperature is represented on the vertical scale, and volume on the horizontal. The section AB of the line ABC may be said to be within the range of ordinary temperatures, and it is a straight line, according to Charles's law. If the line AB is continued straight, it would cut the vertical axis at C , a distance of about 460 degrees below the zero of the Fahrenheit scale. This gives the first conception of the absolute zero of temperature. It is the temperature at which volume of a gas would apparently disappear. The line AB would represent the result of an experiment at some pressure p_1 held constant while temperature and volume were varied. The line DE would correspond to another and higher pressure p_2 on the same weight of gas.

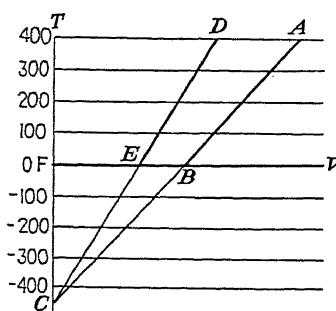


FIG. 62.

Charles's law also expresses the relationship between temperature and pressure when the volume is kept constant, as follows: *With volume constant, the absolute pressure varies directly, as the absolute temperature.*

97. Characteristic Equation of a Gas.—From Boyle's law,

$$p_1V_1 = p_2V_2 = p_3V_3 = \dots pV = \text{constant}$$

From Charles's law

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{V_1}{V} = \frac{T_1}{T}$$

Also

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \quad \text{or} \quad \frac{p_1}{p} = \frac{T_1}{T}$$

From the two, or from Charles's law alone, may be derived the very useful *characteristic equation*, viz.:

$$pV = \bar{W}RT \quad (26)$$

In Fig. 63, the initial state of the gas is p_1 , V_1 , and T_1 at state (1), and it is to be changed to state (2), with values of pressure, volume, and temperature, all different from those at state (1).

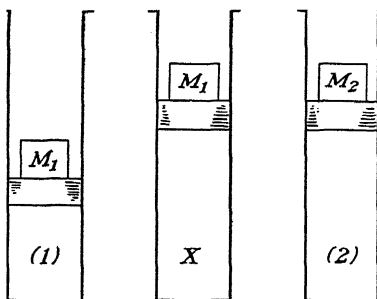


FIG. 63.

First, suppose the gas to change to an intermediate state (X) at constant pressure, and from state (X) to state (2) at constant volume. Then

$$p_x = p_1 \quad \text{and} \quad V_x = V_2$$

From Charles's law, for the first step:

$$\frac{T_x}{T_1} = \frac{V_x}{V_1} = \frac{V_2}{V_1}; \quad T_x = T_1 \frac{V_2}{V_1}$$

Also from Charles's law, for the second step:

$$\frac{T_x}{T_2} = \frac{p_x}{p_2} = \frac{p_1}{p_2}; \quad T_x = T_2 \frac{p_1}{p_2}$$

As a result

$$T_1 \frac{V_2}{V_1} = T_2 \frac{p_1}{p_2}$$

or

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \dots = \frac{pV}{T} = \text{a constant} \quad (27)$$

which is designated by $\bar{W}R$.

Whence

$$pV = \bar{W}RT \quad (26)$$

where p = pressure, psfa

V = volume, cu ft

\bar{W} = weight of gas, lb

R = gas constant, ft-lb per lb-F

T = temperature, F abs

$v = V/\bar{W}$ = specific volume, cu ft per lb

$$pv = RT \quad (28)$$

The same result will be reached if the gas is supposed to change from (1) to (X) at constant temperature, and from (X) to (2) at constant volume; or from (1) to (X) at constant volume, and from (X) to (2) at constant pressure.

98. The Value of R .—If the specific volume of a gas is known for any one condition of pressure and temperature, the values of p , v , and T can be inserted, and R can be found. For air, the specific volume at 14.7 psia and 32 F is known, from experiment, to be 12.39 cu ft per lb.

$$R = \frac{(14.7 \times 144) \times 12.39}{491.69^*} = 53.34 \frac{\text{ft-lb}}{\text{lb-F}}$$

Knowing R , the specific volume of air can be computed for *any* pressure and temperature condition. The gas constant R has a definite value for each gas. However, if in the equation

$$\frac{pV}{T} = \bar{W}R \quad (26)$$

\bar{W} is made equal to the molecular weight, M , of the gas, the

* In establishing the value of R , the more accurate value of the absolute temperature is used instead of the approximate 492.

product MR will be a constant independent of the kind of gas. The value of this universal gas constant is 1544. Hence

$$R = \frac{1544}{M} \quad (29)$$

99. Perfect Gas.—Actual gases are found to deviate slightly from strict adherence to the laws just discussed and the characteristic equation derived from them. In Fig. 62, the lines ABC or DEC would not be quite straight for any actual gas. The term *perfect gas* is employed to express *the conception of a gas that would conform exactly to the laws of Boyle and Charles*.

The absolute zero of temperature is not, therefore, exactly established by the method indicated in Fig. 62 but is located by applying as closely as possible corrections due to the imperfections of air, or any other gas, as measured by the standard of a perfect gas.

100. Specific Heat at Constant Volume and at Constant Pressure.—The specific heat of a solid or a liquid is very nearly the same, for any pressure; *i.e.*, it requires the same number of heat units to change the temperature of 1 lb of water from 60 to 100 F, no matter if there is a constant pressure of 100 psia, or 1000 psia, or a variable pressure. But the case is entirely different for a gas. The application of heat to a gas induces such an increase in the activity of its molecules as to make the gas eager to push out and occupy much more space. If its confining walls yield gradually to the increased activity of the gas, then the gas does work. In such a case all the heat that has been supplied does not appear in temperature effect. A part of it has passed on through the gas and appears as mechanical energy.

It is evident that the quantity of heat necessary to change the temperature of a given weight of gas 1° may have *an infinite number of values*, depending upon how much work the gas is allowed to do during the operation.

The two cases of special interest are:

- a. Application of heat at constant volume.
- b. Application of heat at constant pressure.

Let c_v represent the amount of heat required to raise the temperature of 1 lb of gas 1° when the volume is held constant; and c_p , the amount when the gas can expand at

TABLE 6.—SOME PROPERTIES OF COMMON GASES*

Name of gas	Chemical symbol	Molecular weight M		Specific heat		$k = \frac{c_p}{c_v}$	Gas constant R	MR	Weight per cubic foot in pounds at 32 F and 14.7 psia
		Approximate	Exact $O_2 = 32$	c_p	c_v				
Helium.....	He	4	4.0	1.250	0.75	1.66	386.0	1544.0	0.0112
Argon.....	Ar	40	39.9	0.124	0.075	1.66	38.70	1544	0.1112
Air.....		29	28.95	0.241	0.172	1.40	53.34	1544	0.0807
Oxygen.....	O ₂	32	32	0.217	0.155	1.40	48.25	1544	0.0892
Nitrogen.....	N ₂	28	28.02	0.247	0.176	1.40	54.99	1540.8	0.0783
Hydrogen.....	H ₂	2	2.016	3.42	2.44	1.40	765.86	1544	0.00562
Nitric oxide.....	NO	30	30.01	0.231	0.165	1.40	51.40	1542	0.0838
Carbon monoxide...	CO	28	28.00	0.243	0.172	1.41	55.14	1543	0.0780
Hydrochloric acid...	HCl	36.5	36.47	0.191	0.136	1.40	42.35	1544	0.1017
Carbon dioxide.....	CO ₂	44	44.00	0.205	0.160	1.28	35.09	1544	0.1227
Nitrous oxide.....	N ₂ O	44	44.02	0.221	0.171	1.26	35.03	1542.0	0.1229
Sulphur dioxide....	SO ₂	64	64.06	0.154	0.123	1.25	24.10	1543	0.1786
Ammonia.....	NH ₃	17	17.03	0.523	0.399	1.31	90.50	1541	0.0476
Acetylene.....	C ₂ H ₂	26	26.02	0.350	0.270	1.28	59.34	1544.0	0.0725
Methyl chloride....	CH ₃ Cl	50.5	50.48	0.24	0.20	1.20	30.59	1544	0.1407
Methane.....	CH ₄	16.0	16.03	0.593	0.450	1.32	96.31	1543	0.0447
Ethylene.....	C ₂ H ₄	28.0	28.03	0.40	0.33	1.20	55.08	1543	0.0780
Steam†.....	H ₂ O	18.0	18.016	0.46	0.36	1.28	85.70	1544.0	

* MARKS, "Mechanical Engineers' Handbook," McGraw-Hill Book Company, Inc., New York, 1941.

† Steam under pressures less than 1 psia (states confronted in the thermodynamics of air conditioning) behaves very nearly as a perfect gas.

constant pressure—i.e., c_v and c_p are the specific heats of a gas at constant volume and constant pressure, respectively. In the constant-volume case, the effect of the heat put into the gas is only to raise its temperature. In the constant-pressure case, the heat put in not only raises the temperature but causes the gas to do some work in addition. Consequently, the specific heat of a gas at

constant pressure (c_p) must be greater than that at constant volume (c_v).

The relation c_p/c_v is one of great importance. For convenience, this ratio is represented by the symbol k .

$$c_p \div c_v = k \quad (30)$$

The values of c_p , c_v , k , R , M , and MR for some important gases are given in Table 6.

The values of specific heat given in Table 6 are correct for gases at atmospheric pressure and 60 F. For higher temperatures and pressures such as are met in furnaces and internal combustion engines, values of specific heat at constant pressure are more accurately determined from equations such as (4a-f) or from curves.¹ Specific heat at constant volume may be determined by Eq. (34), Art. 105. Since the difference between the specific heats at constant pressure and constant volume remains constant, it is evident that their ratio, k , must necessarily change, decreasing in magnitude as temperature increases.

$$c_p - c_v = \frac{R}{J} = \frac{1544}{MJ} \quad (34)$$

$$Mc_p - Mc_v = \frac{1544}{778} = 1.985$$

The quantities Mc_p and Mc_v are known as *molar* specific heats at constant pressure and constant volume and represent the specific heat of 1 mole, *i.e.*, one molecular weight, M lb.

101. Constant-volume and Constant-pressure Lines on the Temperature-entropy Plane.—Point A, Fig. 64, represents the initial state of 1 lb of a gas that is to be heated at constant volume, from T_1 to T_2 . Since heat must be supplied, the line representing the change of state will pass in

¹ Variation of specific heat with pressure and temperature may be determined from curves published in ELLENWOOD, F. O., N. KULIK, and N. R. GAY, The Specific Heat of Certain Gases over Wide Ranges of Pressures and Temperatures, *Cornell Univ. Eng. Exp. Sta. Bull.* 30, 1942.

a direction to the right of A in order to cover an area that will represent heat supplied. The general direction of the path will be AB . The heat supplied is A_1ABB_1 .

The area of the differential strip = $c_v dt$ and also $T dS$. Hence,

$$T dS = c_v dT$$

$$dS = c_v \frac{dT}{T}$$

and

$$S_B - S_A = c_v \int_{T_1}^{T_2} \frac{dT}{T} = c_v \log_e \frac{T_2}{T_1}$$

which is the equation¹ of the line AB for 1 lb wt.

When heat is supplied at constant pressure, starting from the same initial state A , the general direction of the path of change AC is the same as AB . But it will take more heat to raise the gas to the temperature T_2 . The area A_1ACC_1 is greater than A_1ABB_1 . The equation of the line AC is

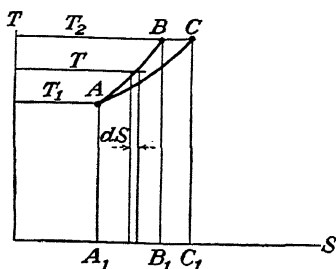


FIG. 64.

$$S_C - S_A = c_p \log_e \frac{T_2}{T_1} \quad (20)$$

c_p is larger than c_v ; consequently, C is located farther to the right than B , and the general conclusion may be stated that, on the temperature-entropy plane, constant-volume lines are steeper than constant-pressure lines.

102. Joule's Law.—The following experiment was performed by Joule (the same man who first determined experimentally the value of the mechanical equivalent of heat).

¹ This same equation was derived in Art. 36 in connection with entropies of the Steam Tables.

Two vessels *A* and *B*, Fig. 65, connected by a pipe in which there was a valve *V*, were immersed in water contained in a tank *M*. The vessel *A* contained air compressed to about 22 atm, while *B* was practically exhausted of air. The whole apparatus was allowed to stand until equilibrium of temperature among the several elements and the atmospheric air was certain to have been attained.

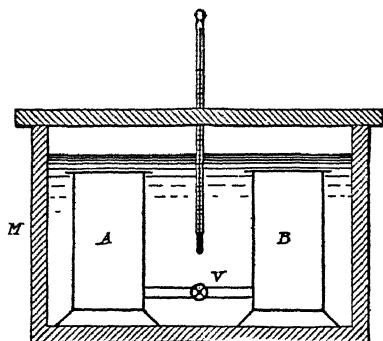


Fig. 65.

Then the valve *V* was opened, and the air rushed from *A* into *B*, filling both vessels at approximately 11 atm pressure. The thermometer indicated that the temperature of the water after the experiment was exactly the same as before. An analysis of the observations of this experiment leads to some important conclusions:

a. Since the temperature of the water was the same after the operation as before, no heat passed from the confined air to the water or from the water to the air. Hence, no net amount of heat was supplied to, or rejected from, the confined substance.

b. No external work could have been done by the air since there was no means of conveying it to or from the apparatus.

c. The general energy equation for a nonflow process is

$$-U_1 \quad {}^1W_2 \text{ Btu} \quad (19)$$

Since no heat transfer took place, ${}_1Q_2 = 0$; and since no work was performed, ${}_1W_2 = 0$. Therefore, $U_2 - U_1$ must equal zero. Although a large change in the pressure and volume took place, no change in the temperature or internal energy took place and we might surmise that the internal energy of a perfect gas is a function of tempera-

ture only. This statement conforms with the conclusions of Art. 4 based on the kinetic theory of gases.

The result of the experiment is thus stated in what is called Joule's law: *When a perfect gas expands without doing external work, and without taking in or giving out any heat, its temperature remains unchanged and there is no change of internal energy.*

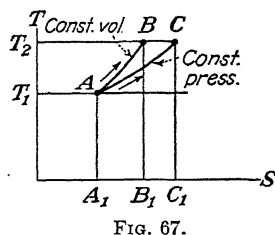
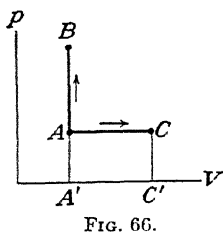
103. Deviations from Joule's Law.—In Joule's experiment, the final temperature was nearly the same as the initial temperature, but not quite, as more accurate investigations by Thomson revealed later. The deviation is very slight for the permanent gases at ordinary temperatures, and it is assumed that a perfect gas would conform exactly to Joule's law. The deviation for actual gases grows larger, the lower the temperature at which they are investigated, the gases becoming more and more influenced by the increasing proximity of their points of liquefaction. At ordinary temperature ranges, actual permanent gases conform quite closely to Joule's law and may be treated without much error as perfect gases.

104. Internal Energy of a Perfect Gas.—Starting from initial state A , Figs. 66 and 67, lines AB represent a constant volume heating of \bar{W} lb of gas from $T_1 F$ to $T_2 F$. ${}_A Q_B$, the heat supplied = $\bar{W} c_v (T_2 - T_1)$ area $A_1 ABB_1$, Fig. 67. No work is done either by or on the gas; hence ${}_A W_B = 0$, and no area is covered by AB , Fig. 66. Substituting in the general energy equation (19),

$$\bar{W} c_v (T_2 - T_1) = U_2 - U_1 + 0 \quad (31)$$

In general, when Q Btu are supplied to \bar{W} lb of gas, raising its temperature from T_1 to T_2 F, a part of Q is absorbed as internal energy, while the remainder is changed into external work. Since internal energy of a gas is a function of temperature, it follows that *the amount of heat absorbed as internal energy is the same for all kinds of changes between the same temperature limits, T_1 and T_2 .*

105. Enthalpy of a Perfect Gas.—The lines AC , Figs. 66 and 67, represent a constant-pressure heating.



The heat supplied

$${}_A Q_C = \bar{W} c_p (T_2 - T_1) = \text{area } A_1 A C C_1, \text{ Fig. 67}$$

c_p = specific heat at constant pressure.

The work done is

$${}_A W_C = p(V_2 - V_1) = \text{area } A' A C C', \text{ Fig. 66}$$

Substituting in the general energy equation (19)

$$\begin{aligned} \bar{W} c_p (T_2 - T_1) &= \dot{U}_2 - U_1 + \frac{p(V_2 - V_1)}{J} \\ &= U_2 - U_1 + \frac{p_2 V_2}{J} - \frac{p_1 V_1}{J} \quad (32) \end{aligned}$$

Rearranging,

$$\begin{aligned} \bar{W} c_p (T_2 - T_1) &= \left(U_2 + \frac{p_2 V_2}{J} \right) - \left(U_1 + \frac{p_1 V_1}{J} \right) \\ &= H_2 - H_1 \quad (33) \end{aligned}$$

Hence, the important relation for a gas, the enthalpy change, is equal to the heat transferred at constant pressure.

Enthalpy is a property, and therefore the change of enthalpy between any two states is independent of the path of the change between states. Although the constant-pressure process was used for deriving Eq. (33), the relation holds for any process.

Equation (32) may be written

$$\begin{aligned}\bar{W}c_p(T_2 - T_1) &= \bar{W}c_v(T_2 - T_1) + \frac{\bar{W}R}{J}(T_2 - T_1) \\ c_p &= c_v + \frac{R}{J} \\ c_p - c_v &= \frac{R}{J}\end{aligned}\tag{34}$$

106. The Zero of Enthalpy and Entropy for a Gas.—

A constant-pressure line for a gas is represented by XA , Fig. 68. The absolute zero of entropy of such a line is at an infinite distance to the left, because the entropy at A is equal to $c_p \log_e (T_A/0)$. The enthalpy of 1 lb of the gas, referred to the absolute zero of temperature, is $c_p(T_A - 0)$. If X is the point (in infinity) of zero absolute temperature and entropy, the area XAA_1 , which is finite, represents the enthalpy at state A .

The enthalpy of a gas is independent of the pressure. If another constant-pressure line XB , for a different value of pressure, is drawn on Fig. 68, the enthalpy at the temperature $T_B = T_A$ is $c_p T_A$, and the area that represents it is XBB_1 , which is equal to XAA_1 . This means that if a weighted piston could be placed upon 1 lb of (perfect) gas in a cylinder at zero absolute temperature, it would require the same amount of heat to bring the temperature of the gas up to 200 F, regardless of the magnitude of the weight on the piston, whether 100 psia, or 25 psia, or any other value. In this respect, there is nothing to distinguish one pressure line from another on the temperature-entropy plane. A single line might represent all pressures, or a 25 psia line might be above or below a 100 psia line. The enthalpy is indifferent to the relative position of constant-pressure lines.

However, the *availability of the heat* requires a definite relationship among these lines. If the gas expands from 100 psia pressure and 200 F to 25 psia pressure at constant

entropy, we know that its temperature decreases, and the path must be vertically downward along AC , Fig. 68. The end point of the adiabatic AC determines a point C through which the 25 psia pressure line must pass in order to be in proper relation with the 100 psia pressure line. In the same manner, all constant-pressure lines may be properly related to one another.

It might seem reasonable to suppose that an arbitrary zero of temperature and entropy could be chosen for gases

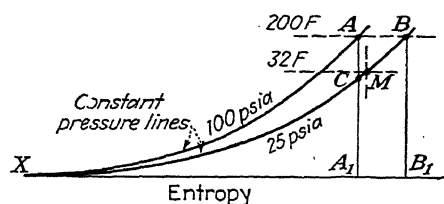


FIG. 68.

as was done for steam. For example, suppose that a state of 32 F and 25 psia is chosen, represented by M , Fig. 68. Then, all constant enthalpy lines (constant-pressure lines) would have to pass through M , which is seen to be impossible. In the case of steam and other vaporous substances, all constant-pressure lines merge into the common liquid line and any point on this line may be chosen as a zero.

107. Entropy Change of a Gas.—Although the absolute entropy of a gas at any state is infinite and therefore indefinite, and it is impossible to establish an arbitrary zero of reference, the *change of entropy* from one state to another in the course of an operation or series of operations can be calculated. In Fig. 69, if a gas expands adiabatically (constant entropy) from 100 psia at A to C at 25 psia, the entropy remains unchanged, and the temperature at C can be calculated. If heat is added while expansion is going on, the path will be something like AD , and, if the rate of supplying heat is adjusted properly to the rate at which the gas does work, AD will be a polytropic some particular value of n , Art. 109. The specific

heat c_n can be determined by the relation

$$c_n =$$

Art. 109, where k is the value of n for the reversible adiabatic case. For the path AD , the value of c_n will be negative because temperature decreases despite the fact that heat is added. The change of entropy from A to D is $c_n \log_e (T_D/T_A)$, Art. 36, or for convenience in handling logarithms, $-c_n \log_e (T_A/T_D)$ per pound of gas.

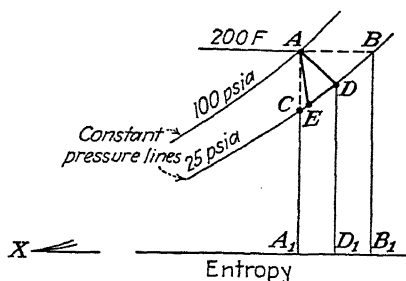


FIG. 69.

If the path of change from A to D is AED , as would be the case in a simple one-stage turbine (see Fig. 40), the change of entropy from A to D can be determined by calculating first the temperature at C and then the entropy increase along the constant-pressure line CD by the expression $c_p \log_e (T_D/T_C)$. If the gas is throttled from A on the 100 psia line to 25 psia, the temperature at the terminal point B will be the same as that at A . The increase of entropy can be determined by calculating the temperature at C and, then, the increase of entropy from C to B .

108. Available Heat of a Gas.—In a cycle in which heat is received by a gas at *constant pressure outside the utilizer* and unavailable heat rejected in the same manner, as in a compressed-air system, it is convenient to express the available heat of the cycle in terms of enthalpy, as was done for steam. Referring to Fig. 68, the enthalpy at A

is $c_p T_A$, which is represented by the area XAA_1 . After adiabatic expansion to 25 psia, the enthalpy is $c_p T_c$; and the area, XCA_1 . The available heat per pound of gas is $c_p T_A - c_p T_c$, which is represented by the area XAC .

In the case of the expansion AD , Fig. 69, if all the available energy is converted into useful work and none of the available energy reappears as increased enthalpy of the exhaust because of reheating losses, the heat A_1ADD_1 comes entirely from an outside source, and the heat equivalent of the work of the utilizer is $c_p T_A + A_1ADD_1 - c_p T_D$ and is represented by the area XAD . If reheating losses occur in the utilizer, the expansion is an irreversible adiabatic expansion and the line AD may still properly represent the path of change. In that case, A_1ADD_1 is the amount of heat returned to the gas because of imperfections of the utilizer. The available heat is XAC , and the waste caused by the inefficiency of the utilizer is A_1CDD_1 , Figs. 68 and 69. The useful work delivered by the utilizer is $XAC - A_1CDD_1$. In the case of throttling from A to B , the waste is A_1CBB_1 , which is equal to the available heat XAC .

109. Polytropic Processes.—A polytropic process is any process that may be defined by the relation

$$pV^n = \text{constant}$$

Nearly all thermodynamic processes follow such a relation. In the general energy equation

$${}_1Q_2 = U_2 - U_1 + \frac{{}_1W_2}{J} \text{ Btu} \quad (19)$$

${}_1Q_2$, the heat supplied or rejected, equals $\bar{W}c_n(T_2 - T_1)$ where c_n is the specific heat and its value depends upon the kind of process.

$$U_2 - U_1 = \bar{W}C_v(T_2 - T_1) \quad (31)$$

$${}_1W_2 = \int_1^2 p \, dV \quad (6)$$

but

$$pV^n = p_1V_1^n = p_2V_2^n = \text{constant} \quad (35)$$

Hence

$$p = \frac{p_1V_1^n}{V^n}$$

and

$$\begin{aligned} {}_1W_2 &= p_1V_1^n \int_1^2 \frac{dV}{V^n} \\ &= p_1V_1^n \frac{(V_2^{1-n} - V_1^{1-n})}{1-n} \end{aligned}$$

since $p_1V_1^n = p_2V_2^n$

$${}_1W_2 = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{\bar{W}R(T_2 - T_1)}{1-n} \text{ ft-lb} \quad (36)$$

Then the general energy equation for a nonflow process becomes

$${}_1Q_2 = \bar{W}c_n(T_2 - T_1) = \bar{W}c_v(T_2 - T_1) + \frac{\bar{W}R(T_2 - T_1)}{J(1-n)} \quad \text{Btu} \quad (37)$$

or

$$= \frac{c_v}{R} (p_2V_2 - p_1V_1) + \frac{p_2V_2 - p_1V_1}{J(1-n)}$$

and since

$$\frac{R}{J} = c_p - c_v \quad (34)$$

$$\bar{W}c_n(T_2 - T_1) = \frac{c_v(p_2V_2 - p_1V_1)}{(c_p - c_v)J} + \frac{p_2V_2 - p_1V_1}{J(1-n)}$$

substituting $\frac{c_p}{c_v} = k$

$${}_1Q_2 = \bar{W}c_n(T_2 - T_1) = \frac{p_2V_2 - p_1V_1}{J(k-1)} + \frac{p_2V_2 - p_1V_1}{J(1-n)} \quad \text{Btu} \quad (38)$$

From Eq. (37)

$$c_n = c_v + \frac{R}{J} \left(\frac{1}{1-n} \right)$$

Substituting the equivalent of R/J ,

$$c_n = c_v + \frac{c_p - c_v}{1 - n}$$

Eliminating c_p , by substituting its value

$$\begin{aligned} c_p &= kc_v \\ c_n &= c_v + \frac{kc_v - c_v}{1 - n} \\ c_n &= c_v \left(\frac{k - n}{1 - n} \right) \end{aligned} \quad (39)$$

The specific heat of a gas is a peculiar property, in that it represents the ratio of the entire amount of heat supplied, during any change, to the change of temperature. It wholly ignores the other effect of the heat, *viz.*, the external work done by, or upon, the gas.

The specific heat of a gas receiving or rejecting heat may have values ranging all the way from zero to infinity and may even have negative values.

The entropy change

$$S_2 - S_1 = c_n \log_e \frac{T_2}{T_1} \quad (20)$$

The relation among the properties p , V , and T may be obtained from the two important equations

$$p_1 V_1^n = p_2 V_2^n \quad (35)$$

and

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad (27)$$

$$\left(\frac{V_1}{V_2} \right)^n = \frac{p_2}{p_1}, \quad \text{or} \quad \frac{V_1}{V_2} = \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \quad (40)$$

also

$$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} = \left(\frac{V_1}{V_2} \right)^n \frac{V_2}{V_1} = \left(\frac{V_1}{V_2} \right)^{n-1} \quad (41)$$

and

$$\frac{T_2}{T_1} = \frac{p_2}{p_1} \left(\frac{p_2}{p_1} \right)^{-\frac{1}{n}} = \left(\frac{p_2}{p_1} \right)^{1 - \frac{1}{n}} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \quad (42)$$

110. Determination of the Value of n from an Actual Compression Line.—In Fig. 70 let MN represent a polytropic line. Let the values of the ordinates be measured at the two points (1) and (2) and introduced in the equation

$$p_1 V_1^n = p_2 V_2^n \quad (35)$$

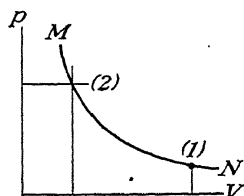


FIG. 70.

n is the only unknown, and its value is easily determined. Or if T_2 and T_1 are known, n may be determined by the relation

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \quad (42)$$

111. Derivation of the Equation of the Reversible Adiabatic, $pV^k = \text{Constant}$.

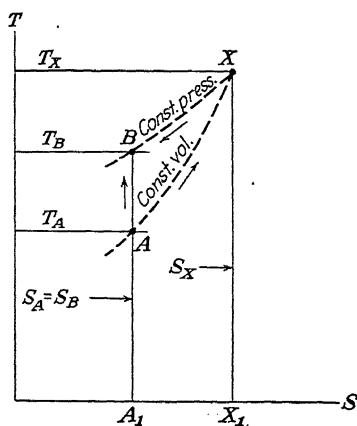


FIG. 71.

It is proposed to assume a change from A to B by first passing from A to some point X , by way of a constant-volume line AX ; and from X to B by a constant-pressure line XB . Inasmuch as the aim now is to establish a relation of conditions or properties between A and B , and not an energy relation, it is permissible to get from A to B by any route whatever. In Fig. 71,

$$S_X - S_A = c_v \log_e \frac{T_X}{T_A} \quad (20a)$$

also

$$S_X - S_B = S_X - S_A = c_p \log_e \frac{T_X}{T_B} \quad (20b)$$

Whence,

$$c_p \log_e \frac{T_X}{T_B} = c_v \log_e \frac{T_X}{T_A}$$

But

$$\frac{T_x}{T_B} = \frac{V_x}{V_B} = \frac{V_A}{V_B}$$

and

$$\frac{T_x}{T_A} = \frac{p_x}{p_A} = \frac{p_B}{p_A}$$

Whence

$$\begin{aligned} c_p \log_e \frac{V_A}{V_B} &= c_v \log_e \frac{p_B}{p_A} \\ \frac{c_p}{c_v} \log_e \frac{V_A}{V_B} &= \log_e \frac{p_B}{p_A} \\ \frac{c_p}{c_v} &= k \end{aligned} \quad (30)$$

and

$$\left(\frac{V_A}{V_B} \right)^k = \frac{p_B}{p_A}$$

or

$$p_A V_A^k = p_B V_B^k = p V^k = \text{constant} \quad (43)$$

which is the equation of a *constant-entropy* adiabatic line.

112. Adiabatic Processes.—The relations among the properties pressure, volume, and temperature during a reversible adiabatic process¹ may be obtained by Eqs. (40), (41), and (42), letting $n = k$.

The work done during a reversible nonflow adiabatic process will be expressed by Eq. (36) in which $n = k$.

$$\begin{aligned} {}_1W_2 &= \frac{p_2 V_2 - p_1 V_1}{1 - k} = \frac{\bar{W}R(T_2 - T_1)}{1 - k} \text{ ft-lb} \\ &= \text{area } F'FAA', \text{ Fig. 72} \end{aligned} \quad (44a)$$

By definition of an adiabatic, there is no heat added or rejected during the process. There is no area below the line AF , Fig. 73.

Hence

$${}_1Q_2 = U_2 - U_1 + \frac{{}_1W_2}{J} = 0 \text{ Btu} \quad (19)$$

¹ A reversible adiabatic process is called an *isentropic process*.

Therefore

$$\frac{1}{J} \bar{W}_2 = -(U_2 - U_1)$$

i.e., the work done during a nonflow adiabatic process is done at the expense of the internal energy. As the gas expands, the pressure drops. If work is done by the gas, the internal energy decreases and the temperature drops, *i.e.*, the velocity of the gas molecules decreases. If the gas is compressed, work must be done on the gas and the pressure and temperature increase. Letting $n = k$ in Eq. (39),

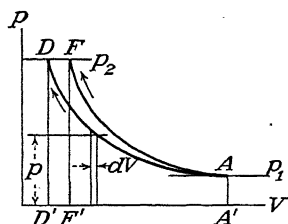


FIG. 72.

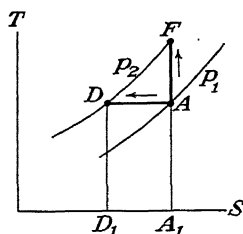


FIG. 73.

the specific heat $c = 0$ for a reversible adiabatic. Also in the equation ${}_1Q_2 = \bar{W}c(T_2 - T_1)$, \bar{W} is finite, there is a temperature change during a reversible adiabatic process, and therefore $c = 0$.

Equation (43) was derived on the basis that the change of entropy equals zero, *i.e.*, a reversible adiabatic. Therefore, $S_2 - S_1 = 0$ for a reversible adiabatic but $-S_2 - S_1$ does not equal zero for an irreversible adiabatic. The change of entropy (an increase) will then depend on the degree of reversibility. The exponent n will not equal k . If the end states are known and n is constant, the exponent n and the property changes may be calculated by the equations of Art. 109. The entropy which must increase for compressions and expansions may be evaluated by

$$S_2 - S_1 = \bar{W}c_v \frac{(k - n)}{(1 - n)} \log_e \frac{T_2}{T_1} \quad (20c)$$

Although an irreversible adiabatic curve may be represented by $pV^n = \text{constant}$ wherein n does not equal k , at the same time $c_n = c_v(k - n)/(1 - n)$ does equal zero since

$${}_1Q_2 = \bar{W}c_n(T_2 - T_1) = 0.*$$

The work done in an irreversible adiabatic process may be determined by applying the general energy equation. For example, if we have an irreversible steady-flow adiabatic process

* Entropy, the measure of unavailability, is defined as the ratio of the unavailable heat, Q_N , to the absolute temperature, T_2 , at which the unavailable heat is rejected.

$$dS = \frac{Q_N}{T_2}$$

Referring to Eq. (21), Art. 42,

$$\frac{Q - Q_N}{Q} = \frac{T_1 - T_2}{T_1}$$

or

$$\frac{Q_N}{T_2} = \frac{Q}{T_1}$$

but this relation holds only for reversible processes such as those of the Carnot cycle. Then $Q = T_1 dS$ only for reversible processes, and the area on a temperature-entropy diagram has meaning only for a reversible process. The area under a line such as JK , Fig. 48, representing an irreversible expansion, does not represent heat transferred from an external source, although the same area under the same line when representing a reversible polytropic process represents the heat transferred in the polytropic process. Although an irreversible process may be drawn on a TS diagram and a value of n calculated for the exponent of the hyperbolic path, the area under the line does not represent transferred heat, and if the process is an irreversible adiabatic, the calculation of c_n by Eq. (39) has no physical significance. Since entropy is a property, the change of entropy between two states is the same regardless of path, reversible or irreversible. Therefore, to calculate the change of entropy for an irreversible process, we may calculate the entropy changes for any combination of reversible processes which will enable the working substance to arrive at the final state from the initial state. When the hypothetical value of c_n for an irreversible adiabatic is used in Eq. (20c), the resulting entropy change is identical with that calculated by using a combination of reversible processes to effect the same change of state (see Art. 107).

$$\begin{aligned}
 \frac{{}_1W_2}{J} &= H_2 - H_1 + \frac{\bar{W}(\bar{V}_2^2 - \bar{V}_1^2)}{2gJ} \text{ Btu} \\
 &= \bar{W}c_p(T_2 - T_1) + \frac{\bar{W}(\bar{V}_2^2 - \bar{V}_1^2)}{2gJ} \\
 &= \frac{k\bar{W}R}{(k-1)J}(T_2 - T_1) + \frac{\bar{W}(\bar{V}_2^2 - \bar{V}_1^2)}{2gJ} \\
 &= \frac{k}{(k-1)J}(p_2V_2 - p_1V_1) + \frac{\bar{W}(\bar{V}_2^2 - \bar{V}_1^2)}{2gJ} \\
 {}_1W_2 &= \frac{k}{(k-1)} p_1V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{\bar{W}(\bar{V}_2^2 - \bar{V}_1^2)}{2g} \text{ ft-lb} \\
 &\quad (44b)
 \end{aligned}$$

113. Isothermal Processes.—The isothermal process is defined by $T = \text{constant}$. In the equation

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = \text{constant}$$

$T_1 = T_2$ and $p_1V_1 = p_2V_2 = \text{constant}$ (Boyle's law)
Comparing with

$$\begin{aligned}
 p_1V_1^n &= p_2V_2^n = \text{constant} \\
 n &= 1
 \end{aligned} \quad (35)$$

Since the temperature does not change, the internal energy change $U_2 - U_1 = 0$ and the general energy equation

$${}_1Q_2 = U_2 - U_1 + \frac{{}_1W_2}{J} \quad (19)$$

becomes

$$\begin{aligned}
 {}_1Q_2 &= 0 + \frac{{}_1W_2}{J} \text{ Btu} \\
 &= \text{area } D_1DAA_1, \text{ Fig. 73}
 \end{aligned} \quad (45)$$

Heat must be added at the same rate that work is being performed by the gas.

Substituting $p_1V_1 = p_2V_2$ and $n = 1$ for an isothermal process in Eq. (36) results in an indeterminate expression for the work. However, if $p = p_1V_1/V$ is substituted in the general expression for work

$${}_1W_2 = \int_1^2 p \, dV = p_1 V_1 \int_1^2 \frac{dV}{V} \quad (6)$$

$$\begin{aligned} {}_1W_2 &= p_1 V_1 \log_e \frac{V_2}{V_1} = \bar{W} R T_1 \log_e \frac{V_2}{V_1} \text{ ft-lb} \\ &= \text{area } D'DAA', \text{ Fig. 72} \end{aligned} \quad (46)$$

and

$${}_1Q_2 = \frac{p_1 V_1}{J} \log_e \frac{V_2}{V_1} = \frac{\bar{W} R T_1}{J} \log_e \frac{V_2}{V_1} \text{ Btu} \quad (47)$$

Letting $n = 1$ in Eq. (39) results in the specific heat $c = \infty$. Letting $T_1 = T_2$ and $c_n = \infty$ in Eq. (20), Art. 36, result in an indeterminate expression for the change of entropy, but since $dS = dQ/T$ (Art. 36) for a reversible process

$$S_2 - S_1 = \frac{1}{T} \int_1^2 dQ = \frac{{}_1Q_2}{T} = \frac{\bar{W} R}{J} \log_e \frac{V_2}{V_1} \quad (48)$$

114. Constant-pressure Processes.—When $p_1 = p_2 =$ constant in

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad (27)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \text{ (Charles's law)} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{V_2}{V_1}$$

Comparing with Eq. (41), Art. 109

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{V_2}{V_1} \right)^{1-n} \\ n &= 0 \end{aligned} \quad (41)$$

Letting $n = 0$ in Eq. (36), Art. 109

$$\begin{aligned} {}_1W_2 &= p(V_2 - V_1) = \bar{W} R(T_2 - T_1) \text{ ft-lb} \\ &= \text{area } A'ACC', \text{ Fig. 74} \end{aligned} \quad (49)$$

${}_1dQ$ is an imperfect differential. ${}_1Q_2$ represents the integral, $\int_1^2 dQ$ and its value depends on the path of change between states 1 and 2 [isothermal in Eq. (48)]. ${}_1Q_2$ represents the heat transferred while the state changes from 1 to 2.

$${}_1Q_2 = \bar{W}c_p(T_2 - T_1) \quad (3a)$$

$$= \text{area } A_1ACC_1, \text{ Fig. 75}$$

$$S_2 - S_1 = \bar{W}c_p \log_e \frac{T_2}{T_1} \quad (20d)$$

$$c_p - c_v = \frac{R}{J} \quad (34)$$

$$\frac{c_p}{c_v} = k \quad \frac{kc_p}{k} - \frac{c_p}{k} = \frac{R}{J}$$

$$c_p = \frac{Rk}{J(k-1)} \quad (50)$$

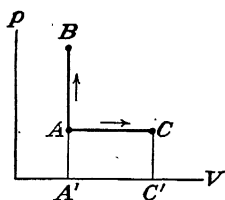


FIG. 74.

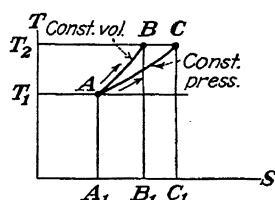


FIG. 75.

115. Constant-volume Processes.—When

$$V_1 = V_2 = \text{constant in}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad (27)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \text{ (Charles's law)} \quad \text{or} \quad \frac{T_2}{T_1} = \frac{p_2}{p_1}$$

Comparing with Eq. (42), Art. 109

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \quad (42)$$

$$n = \infty$$

Letting $n = \infty$ in Eq. (36), Art. 109

$${}_1W_2 = 0$$

If there is no change in volume, no work is performed and there is no area under the line AB, Fig. 74.

$${}_1Q_2 = \bar{W}c_v(T_2 - T_1) \quad (3b)$$

$$= \text{area } A_1ABB_1, \text{ Fig. 75}$$

$$S_2 - S_1 = \bar{W}c_v \log_e \frac{T_2}{T_1} \quad (20e)$$

From Eq. (50)

$$c_v = \frac{c_p}{k} = \frac{R}{J(k - 1)} \quad (51)$$

116. Throttling of Gases.—When air, or any gas considered as a perfect gas, is reduced in pressure without doing any useful external work as, for example, when it is throttled in a valve, passes through an orifice, is wire-drawn through restricted ports, or suffers a frictional effect while flowing in a pipe, *the operation is accompanied by no change of temperature.* This is a necessary conclusion from Joule's experiment (Art. 102). The general energy equation for a nonflow process is

$${}_1Q_2 = U_2 - U_1 + \frac{{}_1W_2}{J} \quad (19)$$

In a throttling operation, both ${}_1Q_2$ and ${}_1W_2/J$ are zero in value. Hence, $(U_2 - U_1)$ must be zero. Inasmuch as $(U_2 - U_1)$ is expressed in value by the term $\bar{W}c_v(T_2 - T_1)$, it necessarily follows that the operation of throttling is one of constant temperature, or rather one of equality between the final and initial temperature.

As an illustration, if *A*, Fig. 76, is a tank that is continually supplied with compressed air through the pipe *C*, and *D* is a throttle valve through which it escapes, the temperature of the escaped air at *F* will be the same as that in the pipe *E* leading to the throttle valve, and this temperature continues unchanged unless that of the supply air at *C* is altered.

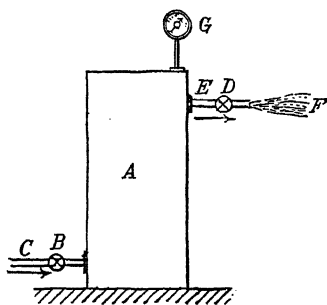


FIG. 76.

117. Throttling from an Unsupplied Reservoir.—If in Fig. 76, alluded to in the preceding article, the valve B in the supply pipe is closed, while the compressed air of the tank continues to escape at D , then the temperatures in A and at E and F all begin to go down. The depression of the temperature is, however, a result of *the expansion of the air confined in the tank* up to any particular instant and *not to throttling* as such. The tank A might be imagined to be filled with a number of small balloons, each containing air, and squeezed compactly into the tank. As the pressure is reduced by the escape of some of the air, the content of any particular balloon expands, and work is done by it in so doing, just as if the air had expanded in a cylinder behind a piston. The work done is applied in forcing some of the air of the tank out through the restricted throttling passage, thereby imparting velocity or kinetic energy to it.

If p_1 and T_1 represent the initial pressure and temperature in the tank A , Fig. 76, and p_t the pressure observed immediately after a very rapid release through D , so that the expanding air in the tank has not sufficient time to be influenced by the temperature of the containing walls, and expansion takes place adiabatically, then the final temperature T_t , at the instant when p_t is observed, is given by Eq. (42), *viz.*,

$$T_t = T_1 \left(\frac{p_t}{p_1} \right)^{\frac{k-1}{k}}$$

At the place of throttling, since no work is usefully done to detract from the store of internal energy, and since the air (considered as a perfect gas) has no capacity whereby it may store or contain a different amount of energy in a latent form at the two pressures, it follows, as was pointed out in Art. 116, that the temperature of the air after throttling must be the same as that with which it approached the throttling valve from the tank. Although

the temperature of the discharged air is continually decreasing, it is *not the result of throttling* but is caused by the air of the tank doing work by expansion without a replenishment of its internal energy.

118. Experimental Means of Determining k .—The general principles that have been discussed in Art. 117 furnish the basis of a very ingenious scheme for experimentally determining the value of k without reference to the values of the individual specific heats of which k is the ratio. The apparatus is sufficiently illustrated in Fig. 77.

In the experiment, the tank A is charged with compressed air to any convenient pressure. With the supply valve B closed, the apparatus then stands undisturbed until every part, including the charge of air, has attained the temperature of the room in which it is placed. The pressure indicated by the gage G is then noted, after which the valve D is opened quickly and some of the compressed air allowed

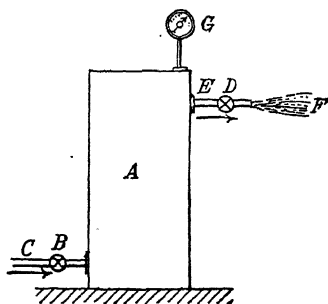


FIG. 77.

to escape. Valve D is then closed, and the indication of gage G is instantly noted. The apparatus again is permitted to stand undisturbed until every part including the residuum of the charge of air has attained the temperature of the room, which must be exactly what it was initially; after which, the pressure at gage G is again observed. Thus, the observations consist of only three pressure readings, and from these the value of k may be deduced.

Let p_1 = initial absolute pressure of the air
 p_x = absolute pressure at the moment of closure of the valve D , after a part of the air has been allowed to escape
 p_2 = absolute pressure after final restoration of equilibrium of temperature between room and residuum of air charge

Let p_1 = initial absolute pressure of the air

p_x = absolute pressure at the moment of closure of the valve D , after a part of the air has been allowed to escape

p_2 = absolute pressure after final restoration of equilibrium of temperature between room and residuum of air charge

T = temperature of room, and therefore of the charge of air in tank A at both initial and final conditions, F abs

T_x = temperature of the air at the instant of closure of valve D , F abs

The operation of allowing some of the air to escape must be done so quickly that the air remaining in the tank, a continually diminishing quantity, expands adiabatically, the walls of the container not having sufficient time to impart heat to the air as it expands. Hence,

$$\frac{T_x}{T} = \left(\frac{p_x}{p_1} \right)^{\frac{k-1}{k}}$$

representing an adiabatic change. Also, as a relation between the initial and final conditions for the residuum of air left after some has been allowed to escape

$$\frac{T_x}{T} = \frac{p_x}{p_2}$$

representing a constant-volume change.

Whence

$$\frac{p_x}{p_2} = \left(\frac{p_x}{p_1} \right)^{\frac{k-1}{k}}$$

and

$$\log \frac{p_x}{p_2} = \frac{k-1}{k} \log_e \frac{p_x}{p_1}$$

an equation in which k is the only unknown.

119. Summary.—In Figs. 78 and 79, let A indicate the initial state of a gas. Through A may be drawn a family of an infinite number of lines, all having the same general equation $pV^n = C$ and differing from each other only in the value of n .

Consider only the four special cases in which one property of a gas remains constant during a change, *viz.*, entropy, temperature, volume, or pressure.

For a constant-entropy compression from A , the general direction of the line is AB , as has already been discussed.

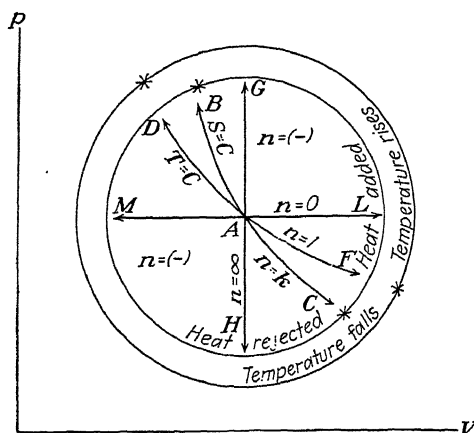


FIG. 78.

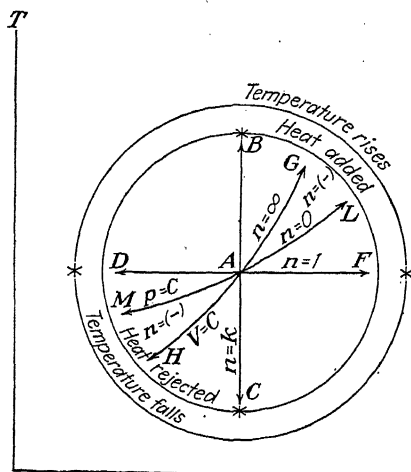


FIG. 79.

The value of n is k . For adiabatic expansion, the direction is AC . To decide whether the path should proceed upward or downward on the TS plane, it is necessary only to appreciate the fact that the temperature rises during compression by virtue of the work done upon the air.

For isothermal compression, the general direction of the compression line is AD . Whether the compression line should fall below or above the adiabetic on the pV plane may be reasoned out thus: If the gas is compressed from the same initial state to the same final pressure, the volume after the isothermal operation will be less than after the adiabetic on account of the cooling. Hence, the isothermal compression line falls below the adiabetic. By similar reasoning, or by a continuation of the isothermal through A , it is seen that the constant-temperature expansion line AF lies above the adiabetic on the pV plane. On the temperature-entropy plane the isothermal is necessarily horizontal and the direction for compression or expansion is determined by answering the question as to whether heat is withdrawn or supplied during the operation. The value of n is unity.

The directions taken by constant-volume and constant-pressure lines on the temperature-entropy plane, for the case

TABLE 7.—WORKING EQUATIONS FOR PERFECT GASES

Process	Polytropic, $pv^n = C$	Isentropic, $S = C$	Isothermal, $T = C$	Constant pressure, $p = C$	Constant volume, $V = C$
n	n	k	1	0	∞
c	$c_n = c_v \left(\frac{k-n}{1-n} \right)$	0	∞	$c_p = \frac{kR}{(k-1)J}$	$c_v = \frac{R}{(k-1)J}$
${}_1Q_2$	$\bar{W}c_n(T_2 - T_1)$	0	$\frac{p_1 V_1}{J} \log_e \frac{V_2}{V_1}$	$\bar{W}c_p(T_2 - T_1)$	$\bar{W}c_v(T_2 - T_1)$
${}_1W_2$	$\frac{p_2 V_2 - p_1 V_1}{1-n}$	$\frac{p_2 V_2 - p_1 V_1}{1-k}$	$p_1 V_1 \log_e \frac{V_2}{V_1}$	$p(V_2 - V_1)$	0
$H_2 - H_1$	$\bar{W}c_p(T_2 - T_1)$	$\bar{W}c_p(T_2 - T_1)$	0	$\bar{W}c_p(T_2 - T_1)$	$\bar{W}c_p(T_2 - T_1)$
$U_2 - U_1$	$\bar{W}c_v(T_2 - T_1)$	$\bar{W}c_v(T_2 - T_1)$	0	$\bar{W}c_v(T_2 - T_1)$	$\bar{W}c_v(T_2 - T_1)$
$S_2 - S_1$	$\bar{W}c_n \log_e \frac{T_2}{T_1}$	0	$\frac{\bar{W}R}{J} \log_e \frac{V_2}{V_1}$	$\bar{W}c_p \log_e \frac{T_2}{T_1}$	$\bar{W}c_v \log_e \frac{T_2}{T_1}$
p, V, T	$p_1 V_1^n = p_2 V_2^n$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{n-1}$	$p_1 V_1^k = p_2 V_2^k$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k}}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1}$	$p_1 V_1 = p_2 V_2$	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	$\frac{p_1}{T_1} = \frac{p_2}{T_2}$

of heat supplied, are AG and AL , respectively. For the case of cooling, or withdrawal of heat, the directions are AH and AM . On the pV plane a constant-volume change is represented by a vertical line. Whether the direction is up or down is decided by the answer to the question: Is heat supplied or withdrawn, and does the pressure increase or diminish with the operation? The direction of the horizontal constant-pressure line on the pV plane is settled in the same manner.

For lines that fall within the angles LAH and GAM , the values of n are positive. For lines lying within the angles GAL and MAH , the values of n are negative.

Table 7 is a summary of the working equations for perfect gases.

Problems

1. The air in a room 22 by 40 by 12 ft is at a temperature of 60 F, and the barometric pressure is 29.22 in. Hg. What weight of air is contained in the room?

2. What is the density of air (weight per cubic feet) at an altitude where the barometric pressure is 20.00 in. Hg and the temperature 20 F?

3. What is the size of a cube that would contain 1 lb of atmospheric air at 80 F and 14.7 psia pressure?

4. A cylindrical tank 6 in. in diameter and 40 in. long contains oxygen at 2800 psia pressure and 75 F temperature. What is the weight of the gas contained?

5. What weight of hydrogen would be contained in the cylinder of Prob. 4 under the same conditions?

6. A weight of 2 lb of air is expanded from an initial state of 100 psia and 120 F on a polytropic path for which $n = 1.20$. Determine the volumes at 100, 60, 30, and 15 psia, and plot pV curve to scale.

7. One pound of air at the initial state of 20 psia and 60 F temperature is subjected to a polytropic change for which $n = -0.80$. Determine the pressures when the volumes are two, three, and four times the initial volume, and plot the curve to scale.

8. By scaling the compression line of an indicator diagram from an air compressor at two points, it is found that the pressure and volume represented are: 25 psia and 1.25 cu ft; 90 psia and 0.45 cu ft. Determine the value of n for a polytropic that will pass through the two points.

9. A tank contains compressed air at 120 psia and 70 F temperature. A valve is opened allowing the pressure to drop to 50 psia. Assuming that the expansion occurs so quickly that the operation is adiabatic, what is the temperature of the air in the tank at the moment when the pressure is 50 psia?

10. Referring to Fig. 83, suppose the pressure, volume, and temperature at the beginning of compression (point *A*) to be 15 psia, 80 cu ft, and 65 F, respectively. The final pressure is to be 120 psia. Determine the volumes at pressures of 30, 60, and 120 psia for each of the three polytropics, *viz.*, the isothermal, the adiabatic, and one for which the value of n is 1.25. Plot the values and construct the three curves.

11. Helium is twice the weight of hydrogen, its value of R being 386. How does the "net lifting capacity" of the two gases compare when used in balloons of the same gas volume? By net lifting capacity is meant the lifting force available after deducting the weight of the gas itself, but not deducting weight of the bag.

12. A volume of 20,000 cu ft of hydrogen at atmospheric pressure 14.7 psia and 70 F temperature is run into a 25,000-cu ft balloon bag, at ground level. As the balloon rises, atmospheric pressure decreases allowing the hydrogen to expand. Let it be assumed that cooling of the hydrogen by expansion just keeps pace with decrease of atmospheric temperature with elevation.

a. What lifting force will be exerted by the hydrogen at ground level?

b. At what atmospheric pressure will the volume of the gas just fill the bag; what will then be the temperature and what the lifting capacity of the hydrogen?

13. One-half pound of air at 30 psia and 40 F is compressed isothermally until its volume is one-half the original volume. Find the original volume and the final pressure and temperature. What would be the final pressure and volume if it had been compressed adiabatically?

14. Ten cubic feet of air at 65 F and 90 psia are expanded to four times the original volume, the law of expansion being $pV^{1.25} = C$. If $c_v = 0.17$, find the change of entropy.

15. Nitrogen is heated at constant atmospheric pressure from a temperature of 500 F to 2500 F. Calculate the increase of internal energy and the work done per pound considering a variable specific heat. If the gas is expanded adiabatically to 1000 F, what would be the work done per pound by the gas?

16. What is the change of entropy when 5 lb of carbon dioxide are heated from 200 F to 1200 F, considering variable specific heat?

17. The value of R for a gas is 53.7, $k = 1.4$ and 0.6 lb of it expands in a jacketed cylinder from 95 psia and 250 F to 16 cu ft. Find the heat flow between the jacket and gas if expansion is (a) isothermal, (b) $pV^{1.25} = C$.

18. Four cubic feet of air are expanded isothermally from 100 psia to a final pressure of 14.7 psia. Find (a) final volume, (b) work done, (c) heat added.

19. The temperature of 1 lb of air falls from 600 F to 300 F while expanding adiabatically doing 40,000 ft-lb of work. Find c_p and c_v .

20. One pound of a gas having $c_p = 0.24$ and $c_v = 0.17$ is expanded at constant pressure from a pressure of 60 psia and 300 F to three times its volume. Then it is compressed isentropically to its original volume, and finally the pressure is lowered to the original pressure at constant volume. Calculate R for the gas. Draw a pV diagram, and find the p , V , and T

at the beginning of each operation and the work done and heat added during each operation.

21. Air at 60 F and 15 psia is expanded adiabatically to 2 psia. The air then receives heat at constant pressure until the temperature is 60 F. It is then compressed adiabatically to 15 psia and exhausted at that pressure. Find the temperature of the exhaust and heat rejected in the exhaust per foot-pound of work done.

22. a. One cubic foot of a perfect gas at 40 psia and 40 F is heated at constant pressure until the final temperature is 540 F. Represent graphically on a TS diagram the heat added, the work done, and the change of internal energy if any.

b. If the original mixture had expanded isothermally to the same final volume, would the work performed by the gas be greater or less than the above? Prove your results graphically using the TS diagram of part (a).

23. A vertical cylinder, 3.5 in. in diameter, open at the top, contains a piston resting upon a charge of air whose temperature is 80 F. The bottom of the piston is 9 in. from the bottom of the cylinder. The weight of the piston and its rod is 15 lb. The barometric pressure is 14.5 psia. By means of a rope, connected to the piston rod and running over a pulley above, a man can pull up on the piston. Now suppose that he does this, pulling very slowly so that the charge of air remains at practically 80 F temperature, until the piston is raised 12 in. from its original position.

a. Calculate the force with which the man must be pulling (neglecting friction) when the piston has reached its new position.

b. Calculate the work in foot-pounds done by, or on, the air in the cylinder during the operation.

c. Calculate the work done by the man during the operation.

d. Draw a pV sketch to show areas that represent the total work done in lifting the piston, the work done by, or on, the air in the cylinder, and that done by the man.

24. An inverted cylinder, 6 in. internal diameter with lower end open to atmosphere, contains a piston, with a rod and hook "hanging" therefrom, the hook serving as a means of attachment for a load. With only the weight of piston and rod, which is 25 lb, the piston is 3 in. from the head, at the top of the cylinder. Atmospheric pressure 14.5 psia, temperature 60 F.

a. What load, applied to the hook and eased down gradually, will it take to displace the piston 40 in. from its first position.

b. What load, applied quickly, so that but little transfer of heat through cylinder walls takes place, will it require to displace the piston 40 in. from its initial position, and what will be the probable temperature of the air in the cylinder at the instant when the piston reaches its terminal position?

c. Is work done *by* or *on* the charge of air in the cylinder, and (for both cases *a* and *b*) how many foot-pounds? Make pV sketch to represent the work.

25. A horizontal cylinder 5 in. in diameter contains a charge of air held by a piston. The piston is restrained by a spring whose constant is 100; i.e., it requires a force of 100 lb to compress the spring 1 in. The initial state of the air is 20 psia and 70 F. The initial position of the piston is 6 in. from

the end of the cylinder. Now let heat be added to the air until the piston has moved out 4 in. from its original position. Determine:

- a. The pressure in pounds per square inch absolute and the temperature of the air at the end of the operation.
 - b. The work done by the air during the operation.
 - c. The number of Btu of heat that had to be added to the air.
- Make a pV sketch to illustrate the operation.

26. Compressed air is supplied to an air turbine at 120 psia and 80 F temperature. The air is exhausted from the turbine at 15 psia and 60 F below zero. If the turbine is developing 20 bhp, as shown by test:

- a. Illustrate the operation by a temperature-entropy sketch.
- b. What weight of air in pounds per hour is being consumed?
- c. Calculate the available heat per pound of air for the conditions given.
- d. Calculate the change of entropy per pound of air.
- e. What amount of heat is wasted by the turbine per pound of air? What area represents it?

27. A cylindrical tank 10 in. in diameter by 5 ft long contains compressed air at 140 psia pressure and 90 F temperature. If a part of the air escapes very quickly through a valve, what is the temperature of that remaining in the tank when the pressure is 70 psia?

28. Air is compressed from 15 psia pressure to 100 psia at the constant temperature of 80 F. Determine the change in enthalpy and entropy per pound of air. Illustrate by sketch.

29. Air is throttled in a compressed-air line from 100 psia and 80 F to 50 psia. Determine the temperature after throttling, the change in entropy, and the loss of available energy. Illustrate by sketch.

30. Air is compressed in a turbine compressor from 14.5 psia and 80 F to 87 psia. On account of frictional effects in the machine, 20 per cent more energy is expended upon the air by the compressor rotor than the theoretical amount necessary. If no provision is made for cooling, at what temperature will the compressed air be delivered? Represent the change of state of the air on a temperature-entropy sketch.

31. Air is supplied to a single-pressure-stage turbine at 150 psia and 440 F temperature and exhausts at 15 psia.

On a temperature-entropy sketch, let A represent state of air at entrance to turbine, B at discharge end of nozzle, and C at turbine exhaust.

a. If 95 per cent of the available energy is turned into kinetic energy by the nozzles, and 70 per cent transformed into work at the shaft, determine temperatures at B and C . What area represents the 30 per cent waste?

b. Determine value of n for a polytropic path directly from A to C .

c. For polytropic path AC , determine the work done per pound of air; also the amount of heat that will have to be supplied to, or rejected from, the air, and indicate the area that represents it.

d. Explain why work for polytropic AC is not the same as it is for actual (adiabatic) path ABC , although the initial and terminal states are alike in both cases.

CHAPTER IX

COMPRESSION AND EXPANSION OF GASES

120. The Compression of Air.—The operations of the compressor of Fig. 7 are illustrated by the pV diagram of Fig. 80, where NA represents intake, AD compression, and DM delivery of compressed air into pipes and receiver.

During compression, energy comes into the air as *mechanical work* which tends to increase the temperature. At the same time the air usually loses energy by direct *transfer of heat*. Since this operates to reduce the work required, loss of heat is usually encouraged by the use of a

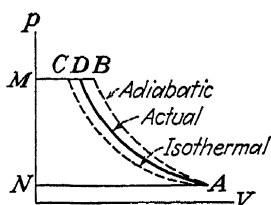


FIG. 80.

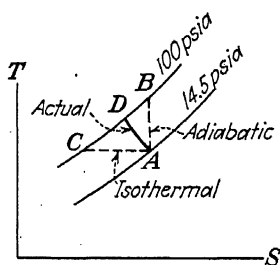


FIG. 81.

water jacket. It is possible to transfer heat into air as from a hot jacket, during compression, but as no useful effect would be accomplished, this case will not be considered now. Again, it is possible to abstract heat at such a rate that the temperature after compression would be lower than before. But if a medium of sufficiently low temperature to do this is available, it could be advantageously used to cool the air to that low level before it entered the compressor.

Two practical limiting cases are therefore established: (a) no loss of heat during compression—an *adiabatic operation B*, Fig. 80—(b) abstraction of heat at a rate just

sufficient to counteract temperature rise due to mechanical work, thus maintaining temperature constant—an *isothermal operation C*, Fig. 80.

The actual-compression line of an air compressor lies between the adiabatic and isothermal polytropics and is considered to be also a polytropic. The value of n is then somewhere between 1.00 and 1.40, and the path is represented by AD in Figs. 80 and 81.

If the value of n is known either by assumption or by the analysis of the compression line of an indicator diagram from an actual compressor, then the unknown properties, specific volume, and temperature at D can be determined by the two equations:

$$p_D V_D^n = p_A V_A^n \quad (35)$$

and

$$p_D V_D = \bar{W} R T_D \quad (26)$$

Or, the final temperature may be found by the equation

$$\frac{T_D}{T_A} = \left(\frac{p_D}{p_A} \right)^{\frac{n-1}{n}} \quad (42)$$

121. "Suction" of a Compressor.—With the piston of a compressor at its extreme position at the end of its stroke, the suction valve starts to open the moment the return stroke is begun, and air at atmospheric conditions (14.5 psia and 70 F) pushes in against the moving piston, and work is thus done by the atmospheric air because of its barometric head. This operation is usually spoken of as *suction*, but the term is apt to prove misleading where accurate conceptions are essential. It makes no difference whether the pressure of the atmospheric air is sufficient to move the piston against whatever resistances may be opposing its progress, or not; an amount of work is done upon the piston by an external agency, represented by the area $M'NAA'$, Fig. 82. The temperature-entropy diagram of Fig. 83 does not picture external work; it can only show

heat quantities. During the operation NA , Fig. 82, the air that has been pushed into the cylinder has experienced no enthalpy change. It has done work upon the piston, but only in the sense that it afforded material communication between the yielding piston and the external source of energy *i.e.*, flow work (Art. 10). Its function has been like that of the connecting rod of an engine. It neither gains nor contributes any energy itself; it experiences no change in any of its properties. As a consequence, the

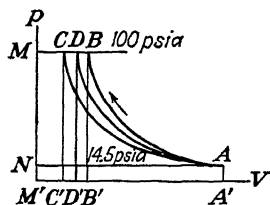


FIG. 82.

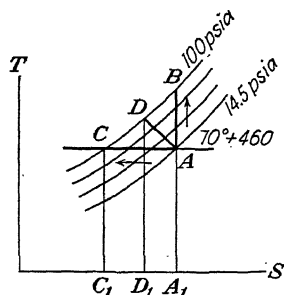


FIG. 83.

operation of filling the cylinder cannot be shown on the temperature-entropy diagram.

Upon completion of the suction stroke, the volume thus pushed into the cylinder for each pound of air is 13.55 cu ft. $V_A = (53.34 \times 530)/(14.5 \times 144) = 13.55$ cu ft. The work done *upon* the piston is:

Work $M'NAA' = 14.5 \times 144 \times 13.55 = 28,270$ ft-lb per lb of air.

122. Compression.—During compression, work is done by the piston upon the air. Three possibilities are considered (Figs. 82 and 83):

- a. Adiabatic along AB (constant-entropy polytropic).
- b. Isothermal along AC (constant-temperature polytropic).
- c. General polytropic along AD (assume $n = 1.25$).

For the case of the adiabatic, the specific volume at B is computed and found to be 3.41 cu ft and the absolute temperature, 921 F (Art. 112).

The work done is represented by the area $A'ABB'$.

$$\begin{aligned} A'ABB' &= \frac{p_2 V_2 - p_1 V_1}{1 - k} \\ &= \frac{(100 \times 144 \times 3.41) - (14.5 \times 144 \times 13.55)}{-0.40} \\ &= -52,070 \text{ ft-lb} \end{aligned} \quad (44)$$

For the case of the isothermal, the final specific volume is determined as 1.96 cu ft (Art. 113) and the temperature is 70 F, or 530 F abs. The work done is represented by the area $A'ACC'$, Fig. 82.

$$\begin{aligned} A'ACC' &= p_1 V_1 \log_e \frac{V_2}{V_1} = 14.5 \times 144 \times 13.55 \log_e \frac{1.96}{13.55} \\ &= -54,590 \text{ ft-lb} \end{aligned} \quad (46)$$

For the case of the general polytropic, where it was assumed that $n = 1.25$, the volume at D is found to be 2.89 cu ft. The work done is represented by the area $A'ADD'$.

$$\begin{aligned} A'ADD' &= \frac{p_2 V_2 - p_1 V_1}{1 - n} \\ &= \frac{(100 \times 144 \times 2.89) - (14.5 \times 144 \times 13.55)}{-0.25} \\ &= -53,400 \text{ ft-lb} \end{aligned} \quad (36)$$

During the compression of the air, its properties vary in their values, and energy interchanges take place. The operation of compression is therefore capable of being pictured on the temperature-entropy diagram, and the three cases are represented by the three lines AB , AC , and AD , Fig. 83.

123. Delivery of the Compressed Air.—After the air has been compressed to the desired pressure, it must be pushed out into the receiver space. The piston must do this work, but the air which is being pushed out is again only a medium of communication between the piston, which is now the source of energy, and the reservoir of compressed

air, which fills the receiver space and acts as the recipient of the energy expended by the piston. The charge of air being expelled is crowded into this reservoir space. If the receiver has no outlet open, then the new access of energy is manifested by a rise of pressure. If the receiver is furnishing air to an air engine or other utilizer, then the new charge, which has just been expelled from the cylinder into the receiver, may be thought of as displacing an equal amount into the utilizer. The work of the piston of the compressor is thus transmitted directly through the entire system to the utilizer, *i.e.*, flow work (Art. 10).

The delivery of the compressed air is assumed to take place at constant pressure. Hence, the work of delivery is the product of the pressure times the change in volume; or, in the cases assumed, where no clearance is specified, it is $-p_2V_2$, since the change in volume is $(0 - V_2)$.

The work of delivery is computed for the three cases:

a. Adiabatic, $V_2 = 3.41$:

$$-p_2V_2 = -(100 \times 144 \times 3.41) = -49,000 \text{ ft-lb}$$

b. Isothermal, $V_2 = 1.96$:

$$-p_2V_2 = -(100 \times 144 \times 1.96) = -28,270 \text{ ft-lb}$$

c. Polytropic ($n = 1.25$), $V_2 = 2.89$:

$$-p_2V_2 = -(100 \times 144 \times 2.89) = -41,620 \text{ ft-lb}$$

The negative signs mean, of course, that the air has done negative work on the piston; *i.e.*, the piston has done work upon the air.

Obviously, the operation of delivery is not representable on the temperature-entropy diagram, for the reasons referred to in Art. 121.

124. The Net Work of the Cycle.—Let the results of the computations of the preceding three articles be summarized in Table 8. The net work of each cycle is expressed in the last item.

The negative sign again indicates that the piston does more work *upon* the working substance than is done *by* the working substance, which, of course, fulfills the purpose of a compressor.

TABLE 8.—NET WORK OF AIR-COMPRESSOR CYCLES

	Adiabatic	Isothermal	$n = 1.25$
Initial pressure, psia.....	14.5	14.5	14.5
Final pressure, psia.....	100.0	100.0	100.0
Initial specific volume at A, Fig. 82.....	13.55	13.55	13.55
Final specific volume.....	3.41	1.96	2.89
Work of admission, area, Fig. 82.....	$M'NAA'$	$M'NAA'$	$M'NAA'$
Work of compression, area, Fig. 82.....	$A'ABB'$	$A'ACC'$	$A'ADD'$
Work of delivery, area, Fig. 82.....	$B'BMM'$	$C'CMM'$	$D'DMM'$
Net work of the cycle, area, Fig. 82.....	$NABM$	$NACM$	$NADM$
Work of admission, ft-lb.....	+28,270	+28,270	+28,270
Work of compression, ft-lb.....	-52,070	-54,590	-53,400
Work of delivery, ft-lb.....	-49,100	-28,270	-41,620
Net work of the cycle, ft-lb.....	-72,900	-54,590	-66,750

125. Expressions for the Net Work of the Cycle.—The *net work* for the general polytropic cycle is represented by the area $ADMN$, Fig. 82. It is desired to derive an expression for this work quantity:

$$ADMN = M'NAA' + A'ADD' + D'DMM'$$

It is best to apply the + sign to all the quantities at the outset, and when the equivalent working expressions are introduced, the direction of the work energy will then be indicated by the appropriate arithmetical sign.

$M'NAA' = p_1(V_1 - 0) = p_1V_1$. In this case, the final volume is V_1 , and the initial volume is zero.

$A'ADD' = \frac{p_2V_2 - p_1V_1}{1 - n}$, an expression that is known to

result in a negative arithmetical value when the operation is one of compression; and positive, when expansion.

' $D'DMM'$ ' = $p_2(0 - V_2) = -p_2V_2$. Here the final magnitude of the volume is zero, and the initial is V_2 .

Substituting the three expressions for the areas that represent them,

$$\begin{aligned}
 ADMN &= p_1V_1 + \frac{p_2V_2 - p_1V_1}{1-n} - p_2V_2 \\
 &= \frac{p_2V_2 - p_1V_1}{1-n} - (p_2V_2 - p_1V_1) \\
 &= \left(\frac{1}{1-n} - 1 \right) (p_2V_2 - p_1V_1) \\
 &= \frac{n}{1-n} (p_2V_2 - p_1V_1) = \frac{n}{1-n} \bar{W}R(T_2 - T_1)
 \end{aligned} \tag{52a}$$

and from Eq. (42)

$$ADMN = \frac{n}{1-n} \bar{W}RT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \tag{52b}$$

$$= \frac{n}{1-n} p_1V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \tag{52c}$$

The reversible adiabatic or constant-entropy change is a particular case of the general polytropic where n has the special value k . Hence, for this case,

$$\text{Net work} = ABMN = \frac{k}{1-k} (p_2V_2 - p_1V_1) \tag{53}$$

The isothermal or constant-temperature change is also a particular case of the polytropic, but the general expression for net work fails here, because it becomes indeterminate since both $1 - n = 0$ and $(p_2V_2 - p_1V_1) = 0$ (see Art. 113). Referring again to Fig. 82, for this case,

$$\begin{aligned}
 \text{Net work} &= ACMN = M'NAA' + A'ACC' + C'CMM' \\
 &= p_1(V_1 - 0) + p_1V_1 \log_e \frac{V_2}{V_1} + p_2(0 - V_2) \\
 &= p_1V_1 + p_1V_1 \log_e \frac{V_2}{V_1} - p_2V_2
 \end{aligned}$$

hence

$$\text{Net work} = ACMN = p_1 V_1 \log_e \frac{V_2}{V_1} = p_1 V_1 \log_e \frac{p_1}{p_2} \quad (54)$$

126. Water-jacketing of Air Compressors.—Comparing the work required of an ideal compressor to compress and deliver 1 lb of air at a given pressure, it is evident from the results of the examples summarized in Art. 124 that it requires much less energy when the compression is isothermal than when it is adiabatic, although it is interesting to note that in the single operation of compression it takes more work to compress isothermally. The question may now be asked: Has the pound of air which has been compressed adiabatically anything more to show for the extra work, 18,310 ft-lb, which has been expended upon it, than the pound which has been compressed isothermally? The answer depends entirely upon what happens to the air between the time it leaves the compressor and the time it enters the utilizer. If it is transmitted without loss or gain of heat, the adiabatically compressed pound of air, because of its higher temperature and greater volume, can return more work in the utilizer, thus accounting for the excess of energy required in compressing it. In an actual plant, however, the air is cooled in traveling through the pipes and in pausing in the receiver, nearly or quite to the atmospheric temperature. In the case of the adiabatically compressed air, some mechanical energy has thus been dissipated as heat in an irreversible operation, which means an irreparable loss of available energy. In the compressed-air plant, therefore, isothermal compression, although not usually attainable by a considerable margin, is the goal to be aimed at.

Water-jacketing of the compressor cylinder is the practical means of accomplishing a considerable approach toward isothermal compression. In the summary of Art. 124, the polytropic $pV^n = C$, where $n = 1.25$, represents what might

reasonably be expected of a water-jacketed cylinder and shows a saving of 6150 ft-lb over adiabatic compression.

In Figs. 84 and 85, AB , AC , and AD represent, respectively, adiabatic, isothermal, and a practically attainable water-jacketed compression. The area ABD , Fig. 84, is the saving in work due to water jacketing as compared with adiabatic compression; it represents the 6150 ft-lb alluded to above. The area ADC is the work lost because of the imperfect action of the water jacket as compared with isothermal compression—the ideal case.

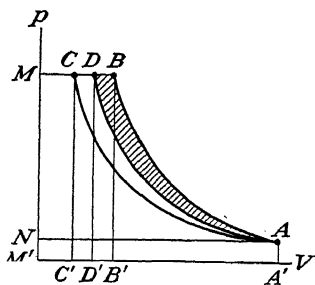


FIG. 84.

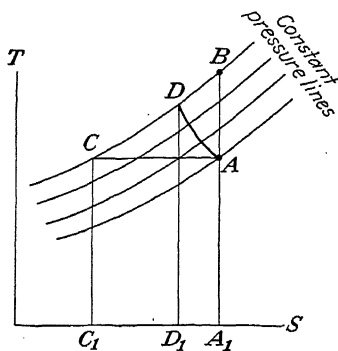


FIG. 85.

On the temperature-entropy diagram of Fig. 85, the adiabatic compression is along AB , and the subsequent cooling in the transmission system is at constant pressure along BC . The water-jacketed compression is along AD . Isothermal compression is represented by AC .

The heat that would be removed by atmospheric cooling, after adiabatic compression, is A_1BCC_1 . The heat that would be removed by a perfect water jacket, resulting in isothermal compression, is A_1ACC_1 . The heat removed by the actual water jacket is A_1ADD_1 . The area ABD , Fig. 85, is the heat equivalent of the work saved by water-jacketing. The area ADC is the heat equivalent of the work lost because the water-jacketing is not perfect.

127. Interstage Cooling of Air Compressors.—Cooling the air by means of water-jacketing the cylinder is only

partially effective because of the limitation of wall surface through which the heat must be transmitted and the brief period of time allowed for the transmission. Further reduction of temperature can be effected by the employment of *interstage cooling*, where compression is divided into two or more operations or stages. In the first cylinder, the air is compressed to an intermediate pressure and delivered into a receiver, which contains tubes through which cold water circulates. From this receiver the air passes to a second cylinder, in which the compression is completed. The interstage receiver, or cooler, is large

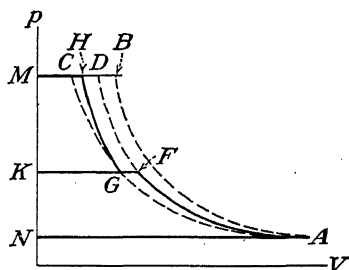


FIG. 86.

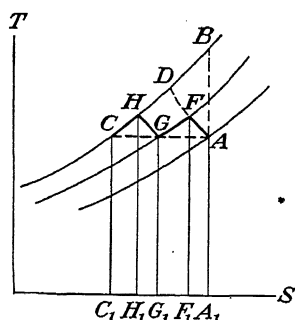


FIG. 87.

in volume compared with the size of the cylinders, and the cooling coils present relatively large surfaces for the transmission of the heat.

The operation of a compound compressor with interstage cooler is represented in Figs. 86 and 87.

On the pV plane, Fig. 86, the admission of air to the low-pressure cylinder is along NA . It is compressed along AF to the intermediate pressure KF . AF lies between the adiabatic AB and the isothermal AC . The air is then delivered from the first cylinder along FK into the interstage receiver, where it is cooled at constant pressure, suffering a reduction of volume. It enters the second cylinder along KF , but on account of shrinkage in volume its state is now represented by G , which, being shown on the iso-

thermal through A , means that the interstage cooler has reduced the temperature to its original value. From G the air is compressed in the second cylinder to the final pressure at H , along the line GH , after which it is delivered to the piping system or to the high-pressure receiver.

On the temperature-entropy plane, Fig. 87, AF represents compression in the first stage; FG , constant-pressure cooling in the interstage cooler; and GH , compression in the second stage.

The area $AFGHB$ (both figures) represents the work saved by the employment of water-jacketed cylinders and interstage cooling, as compared with complete adiabatic compression along AB .

The area $FGHD$ (both figures) represents the work saved by the use of the interstage cooler, as compared with complete compression along AD as would be the case with a single-stage water-jacketed cylinder.

On Fig. 87, the areas A_1AFF_1 , F_1FGG_1 , and G_1GHH_1 represent, respectively, the heat removed from the air by the jacket of the first-stage cylinder, the intercooling receiver, and the jacket of the second-stage cylinder.

128. Clearance in Air Compressors.—In the preceding discussion, clearance has been disregarded in order that attention might be concentrated upon other points that were being developed. As clearance is a factor that enters into the performance of all actual compressors, its influence must now be considered. Referring to Fig. 88, NA represents the total volume of air in the cylinder at the end of the suction stroke. Compression occurs along AD and delivery along DM . The clearance volume is MK ; hence, the delivery operation ceases at K , leaving the volume MK of compressed air still confined in front of the piston. What becomes of this residual quantity of air that the piston cannot force out?

The valves of a compressor are usually automatic in their operation—*i.e.*, they open and close because of slight differences in pressure upon their surfaces. In principle,

they are really check valves. When the piston starts back upon its next stroke, the delivery valve closes immediately, leaving the volume MK of compressed air in the cylinder cut off from all communication with other quantities, to reexpand along the line KL , whose law is similar to that of AD . It is not until the volume behind the piston has grown to NL that the pressure within falls to the intake pressure. Immediately thereafter the intake valve opens, and new air follows the piston, filling the cylinder at the end of the stroke by the volume LA , together with the reexpanded clearance air of volume NL .

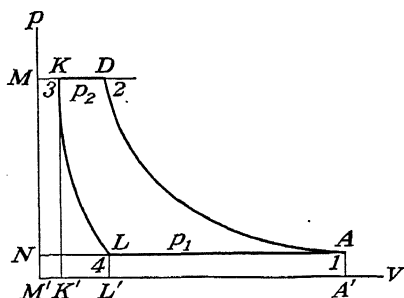


FIG. 88.

The compressor may be thought of as handling two kinds of air: a certain amount of new air which is drawn in, compressed, and delivered each stroke; and a certain amount of clearance air which remains in the cylinder and is compressed and expanded over and over again. The detrimental effect of clearance is that it permits the lodgment of compressed air, which, when reexpanded, occupies space that ought to be available for new air. It decreases the capacity of a cylinder by reducing the quantity of air that might be delivered in a unit time.

In Fig. 88 the distance $K'A'$ represents the piston displacement; the distance $L'A'$ represents the effective displacement. The ratio $L'A'/K'A'$ is called the *volumetric efficiency* of the cylinder. The volumetric efficiency is dependent upon the clearance; the greater the clearance, the less the volumetric efficiency.

Volumetric efficiency,

$$\eta_{\text{vol}} = \frac{L'A'}{K'A'} = \frac{K'A' + M'K' - M'L'}{K'A'} = 1 + \frac{M'K'}{K'A'} - \frac{M'L'}{K'A'}$$

Clearance is always expressed as a proportion of the cylinder displacement volume.

$$\text{Let } \frac{V_K}{V_A - V_K} = \text{clearance, } C = \frac{M'K'}{K'A'}$$

Then

$$\begin{aligned} \frac{M'L'}{K'A'} &= \frac{V_L}{V_A - V_K} = \frac{V_K}{V_A - V_K} \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} = C \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \\ \eta_{\text{vol}} &= 1 + C - C \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \end{aligned} \quad (55)$$

As an example, take the suction pressure as 14.5 psia, the compression pressure as 100 psia, $n = 1.30$ and clearance as 5 per cent, which means 5 *per cent of the piston displacement*, not of the total volume contained in the cylinder at the end of the piston's stroke. Then

$$\begin{aligned} \eta_{\text{vol}} &= 1 + 0.05 - 0.05 \left(\frac{100}{14.5} \right)^{\frac{1}{1.3}} \\ &= 1 + 0.05 - 0.221 = 0.829 \end{aligned}$$

The volumetric efficiency (as influenced by clearance) is 82.9 per cent. Volumetric efficiency decreases with increase of clearance and range of compression. In Fig. 88, if the clearance is as much as MD , no air will be delivered, and the volumetric efficiency is zero. Again, if the range of compression is great enough, the compression line AD will ultimately cut the clearance line $K'K$ extended, and the volumetric efficiency will then be zero.

Clearance does not affect the theoretical amount of work to be done in compression per pound of air delivered. The clearance air returns to the piston the work which the

piston does upon it. In a secondary way, energy is lost because, with clearance, a compressor would be compelled to make more strokes and would suffer more frictional effects in delivering a specified quantity of air than would an ideal compressor having no clearance.

The work performed on the air during compression is, Fig. 88,

$$\text{Area } ADMN = \frac{n}{1-n} p_1 V_A \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad (52c)$$

The work performed by the clearance air during the reexpansion KL is

$$\text{Area } KLMN = \frac{n}{1-n} p_1 V_L \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad (52c)$$

Assuming n to be the same for the compression and reexpansion lines, the net work of the cycle with clearance is

$$\begin{aligned} \text{Area } ADMN - \text{area } KLMN &= \text{area } ADKL \\ &= \frac{n}{1-n} p_1 V_A \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] - \frac{n}{1-n} p_1 V_L \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \\ &= \frac{n}{1-n} p_1 (V_A - V_L) \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad (56) \end{aligned}$$

$(V_A - V_L)$ is the volume of air actually taken into the cylinder per cycle. Let V represent the volume of air actually taken into the cylinder per minute. If V replaces $(V_A - V_L)$ in Eq. (56), we have

$$\text{Work per minute} = \frac{n}{1-n} p_1 V \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \frac{\text{ft-lb}}{\text{min}}$$

and

$$\text{Air horsepower, } P = \frac{n}{1-n} \frac{p_1 V}{33,000} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] \text{ hp} \quad (57a)$$

The adiabatic horsepower becomes

$$\begin{aligned} \text{Adiabatic horsepower, } P_s &= \frac{k}{1-k} \frac{p_1 V}{33,000} \\ &\quad \left[\left(\frac{p_2}{p_1} \right)^{\frac{k-1}{k}} - 1 \right] \text{ hp} \quad (57b) \end{aligned}$$

Similarly from Eq. (54)

$$\text{Isothermal horsepower } P_T = \frac{p_1 V}{33,000} \log_e \frac{p_1}{p_2} \text{ hp} \quad (57c)$$

Problems

1. A compressor has a clearance of 5 per cent and a suction of 14.7 psia. What is the maximum pressure of discharge when $n = 1.25$?

2. Air enters a compressor at 60 F and 15 psia. What is the temperature at discharge with polytropic compression $pV^{1.3} = C$ for discharge pressures of 30, 60, 120, and 240 psia? Plot discharge temperature vs. discharge pressure.

3. If there is danger of the lubricating oil exploding when the temperature of the discharge air of an air compressor is 800 F, what is the lowest safe intake pressure allowed beyond a throttle valve placed in the intake line? Discharge pressure 200 psia, $n = 1.35$, inlet temperature 60 F. This problem illustrates the inadvisability of placing a valve on the intake to an air compressor.

4. If p_1 , p_x , and p_2 represent intake, intermediate, and delivery pressures (absolute) for a two-stage compressor (no clearance), prove that, for equal work in the two stages, $p_x = \sqrt{p_1 p_2}$:

a. When $n = 1$ for both stages.

b. When $n =$ any value (but alike for the two stages), and an interstage cooler brings air at entrance to high-pressure stage to same temperature as it had at entrance to low pressure.

5. Find the ratio of the work done by a two-stage compressor to the work done by a single-stage compressor when air is compressed from 14.7 psia to 400 psia, $n = 1.33$. The air is cooled to the initial temperature between stages of the two-stage compressor. Determine the intermediate pressure by the equation of Prob. 4.

6. An air compressor takes in air at 14 psia and 80 F and delivers at 150 psia.

a. Calculate temperature of air delivered if compression takes place without change of entropy.

b. Would it be possible for this compressor, operating as above ($p_1 = 14$, $t_1 = 80$ F, $p_2 = 150$) to deliver air at a temperature *higher* than that calculated in a, assuming no "hot" jacket? Explain.

c. If b is a possible case, could the operation be said to be an *adiabatic* one? Explain.

7. A vacuum pump, used in connection with a condenser of a steam power plant, is really an air compressor whose suction is below atmospheric, and whose discharge pressure is that of the atmosphere. Assuming that the pump is of the piston type and runs at constant speed; that the substance handled by the pump is air; that the compression is isothermal, and the clearance is zero; and that the atmospheric pressure is 30 in., determine the relative amount of work done by the pump when the vacuum in the condenser is 5 in.; 10, 15, 20, 25, and 28 in. Plot these values for work against

the vacuum (as abscissas) to scale and draw the curve, and note vacuum at which the work is a maximum.

8. *a.* A 12 by 12-in. air compressor, 250 rpm, operating at sea level where the barometric pressure is 30.00 in. and temperature 70 F, delivers air at 90 psi gage pressure. Assuming the compression line to be a polytropic for which $n = 1.28$, and neglecting clearance, calculate the temperature of the air at the end of compression, and the number of foot-pounds of work per pound of air delivered.

b. Calculate the same quantities for this compressor when operating at an altitude where the barometric pressure is 24.00 in., all other conditions being the same as those specified in *a.*

9. Derive Eq. (52), using the general energy equation for steady flow. When is the kinetic energy term negligible?

10. A centrifugal blower raises the pressure of air in a pneumatic conveying system from 14.7 psia to 16 psia. Air is drawn into the blower from the atmosphere at 60 F and discharged to the system through a 10-in. tube at a rate of 5000 cfm. What is the adiabatic air horsepower of the blower?

11. An air compressor is required to compress 2000 cfm at 14.7 psia to 200 psia. Clearance 3 per cent, speed 250 rpm, double acting, $n = 1.35$. Assume a mechanical efficiency of 85 per cent.

a. What is the horsepower required to drive the compressor?

b. What is the cylinder displacement?

12. An air compressor is to deliver 360 cu ft of free air per minute compressed to 120 psi gage. The barometric pressure of the atmosphere is 29.50 in. Hg, and the temperature is 80 F. Zero clearance is assumed. Illustrate all operations of the solution by sketches on pV and TS planes.

NOTE:—The delivery of an air compressor is expressed in terms of the number of cubic feet of "free air" per minute, which means the volume that would be occupied by the compressed air delivered per minute if reexpanded to the pressure and temperature condition at the inlet.

a. What theoretical horsepower would be required to drive the compressor, if the air were to be compressed adiabatically?

b. If the compressed air of *a* after leaving the compressor is cooled back to 80 F at constant pressure, how many heat units will have to be withdrawn per minute?

c. If the air could be compressed isothermally, what horsepower would be required, and how many heat units per minute would have to be withdrawn from the air by water-jacketing?

d. If a compression line could be attained such that $n = 1.25$, what horsepower would be saved over adiabatic compression? How many pounds of cooling water should be supplied to the cylinder jacket per minute if the water enters at 65 F and leaves at 85 F?

13. A 15- by 12-in. double-acting air compressor runs at 200 rpm, taking air at 14.5 psia and 70 F temperature, and delivering at 160 psia. The compressor has a clearance of 3 per cent, and the compression and reexpansion lines are represented by the equation $pV^{1.25} = C$.

a. What is the volumetric efficiency of the cylinder as affected by the clearance?

b. Calculate the theoretical horsepower required for the above compressor with clearance and also without clearance.

c. Determine the number of cubic feet of free air delivered per minute, with and without clearance.

14. A two-stage compressor takes in air at 14.5 psi barometric pressure and 80 F. Delivery pressure is 390 psia, and intermediate pressure is 75 psia. Assume $n = 1.25$ for both cylinders, and neglect clearance. Calculate foot-pounds of work required for the cycle, per pound of air:

a. If an interstage cooler is used in which temperature of air delivered from the low-pressure cylinder is reduced to 80 F.

b. If no interstage cooler is used.

15. An air engine operates on a cycle like that of a compressor, but of course reversed in direction. Such an engine is to operate on compressed air, furnished at 95 psia and 100 F, and exhausts into the atmosphere at 15 psia. The temperature of the exhaust is observed to be 40 F below zero. How much work in foot-pounds has been obtained from each pound of compressed air furnished?

CHAPTER X

CYCLES FOR GASES

129. The Internal-combustion Engine.—The *internal-combustion engine* is a heat engine in which the three essential operations of compression, heating, and expansion are all performed in a single mechanical unit. The work of expansion is performed upon the same piston that does the work of compressing the charge. The heating is done by the combustion of a fuel mixed with the working air itself. It is the latter feature that has made this type of air engine a success. All attempts to build and operate air engines as heat engines for primary sources of power by introducing the heat from an external source through separating walls have proved failures. The rate of heat transfer per unit of area is too slow for large capacities, and the temperatures desirable for good efficiency are apt to be destructively high for the material of the heat-transmission surfaces.

The working substance of the internal-combustion engine is, of course, not pure air. For accurate analysis, the substance is to be treated as a mixture. On the other hand, since the predominating element of the mixture under all conditions is nitrogen, which is also the case with air, the properties of the mixture will never diverge very greatly from those of air. For preliminary or tentative calculations, it is therefore sufficiently accurate to consider the working substance as air.

130. The Otto Cycle.—The ordinary four-cycle, or more correctly speaking, four-stroke-cycle engine, draws in its charge (consisting of air with which the fuel is mixed) on the first stroke; compresses the charge on the second stroke, after which ignition and development of the heat

take place; receives the work of expansion of the gas on the third stroke; and ejects the residue of the charge, after release, on the fourth stroke, thus completing the entire cycle in four strokes or operations of the piston.

The ideal indicator diagram of a four-stroke-cycle Otto engine is illustrated in Fig. 89. MA represents the introduction of the charge; AB is adiabatic compression; BC is combustion of the fuel and consequent heating of the charge, at constant volume; CD is adiabatic expansion of the hot gas; DA represents constant-volume cooling; and AM is the ejection of the charge.

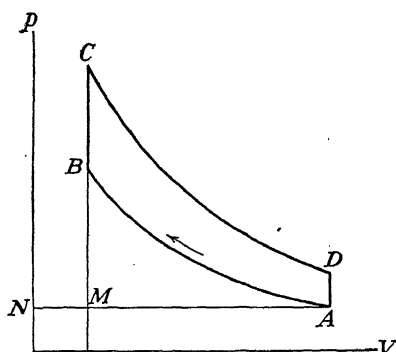


FIG. 89.

The actual indicator diagram departs from the ideal chiefly because the cylinder must be water-jacketed to keep the temperature of rubbing surfaces sufficiently low to allow effective lubrication. Hence, the compression and expansion lines are not adiabatics, and the theoretical temperature of combustion, represented by C , cannot be reached.

The ideal indicator diagram of a two-stroke-cycle Otto engine is essentially like that of the four-stroke cycle. The only difference is the absence of the intake and discharge operations MA and AM , Fig. 89, in the two-stroke cycle. Since those operations just balance each other in energy terms, they have no significance in the theoretical diagram.

131. Pressure-volume and Temperature-entropy Diagrams of the Otto Cycle.—Although Fig. 89 is a representation of the changes of pressure and volume that would take place *within the cylinder* of an ideal Otto engine, it is not a correct representation of all the changes in pressure and volume of the working substance, which are pictured more nearly correctly in Fig. 90. The temperature-entropy changes are shown in Fig. 91.

Figure 90 is just like Fig. 89, following the cycle around to *D*, the point of release. At the state *D*, the cylinder may be considered as a reservoir of compressed air or gas, not in communication with any source. When the exhaust

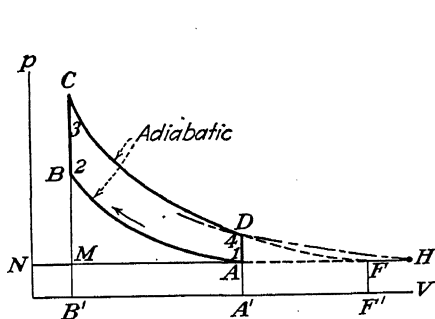


FIG. 90.

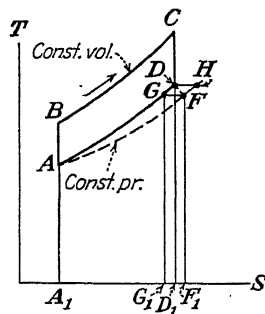


FIG. 91.

valve opens, there is an expansion like that described and discussed in Art. 117. There is a growth of volume from *D*, not an actual removal of heat at constant volume. Work is done by the substance in displacing the atmosphere to make room for itself. The energy for the work done must come from the internal energy, originally possessed by the charge at *D*. Consequently, the average temperature of the charge immediately following release is represented by *F*, lower than *H*, which is on an isothermal *DH* through *D*, Figs. 90 and 91, and on the atmospheric-pressure line *AF*.

Let it be supposed that the volume of the exhaust piping from the cylinder to the place of escape into the atmosphere is large in comparison to that of the cylinder. Upon release at *D*, the charge, which has been confined com-

pletely within the cylinder, now occupies both the cylinder volume and a part of the exhaust piping. Its state at this moment is represented by F , Figs. 90 and 91. If a sufficient length of time is allowed, the charge will cool by loss of heat through the walls of the exhaust piping at constant atmospheric pressure to the point A , which is the state of the charge at the beginning of the cycle. The heat lost during this cooling, which is the heat-rejection operation of the cycle, is represented by the expression

$$\bar{W}c_p(T_F - T_A)$$

In the ideal Otto cycle, the heat-rejection operation is *represented* as a cooling at constant volume from D to A . If it can be shown that

$$\bar{W}c_v(T_D - T_A) = \bar{W}c_p(T_F - T_A)$$

then the representation of the heat-rejection operation of the cycle as a constant-volume cooling may be accepted and used because of its simplicity.

When the charge is released at D , work is done by it in displacing the atmospheric air, to the extent of

$$\text{Area } A'AFF' = p_A(V_F - V_A)$$

T_F is the average temperature of the whole charge immediately after release. Let \bar{W} = the weight of the charge.

Then

$$\text{Loss of internal energy} = \bar{W}c_v(T_D - T_F) \quad (31)$$

but

$$Jc_v\bar{W}(T_D - T_F) = p_A(V_F - V_A)$$

In Fig. 91, take the point G on the constant-volume line through D , at the same temperature as F .

Then

$$\bar{W}c_v(T_D - T_F) = \bar{W}c_v(T_D - T_G)$$

hence

$$\frac{1}{J} p_A(V_F - V_A) = \bar{W}c_v(T_D - T_G) = \text{area } D_1DGG_1$$

Also

$$\begin{aligned}\bar{W}c_p(T_F - T_A) &= \bar{W}c_v(T_F - T_A) + \frac{1}{J} p_A(V_F - V_A) \quad (19) \\ &= \bar{W}c_v(T_G - T_A) + \frac{1}{J} p_A(V_F - V_A)\end{aligned}$$

Therefore

$$F_1FAA_1 = G_1GAA_1 + D_1DGG_1 \text{ (Fig. 91)}$$

or

$$\bar{W}c_p(T_F - T_A) = \bar{W}c_v(T_D - T_A)$$

The heat that must be given up by the charge following *free* expansion at D , in order that it may be cooled to the original state A , at constant pressure, is therefore *equal to the heat that would be withdrawn in cooling the charge at constant volume* to the original state of the cycle. The simple concept of constant-volume heat rejection in the fourth operation of the Otto (or Diesel) cycle may therefore be employed as the equivalent of the actual process of heat rejection.

In an actual engine employing the Otto cycle, following release, when a part of the charge escapes from the cylinder by its own elasticity, the piston promptly pushes out the remainder into the atmosphere. Without waiting for the identical charge to be cooled to its original state, the piston takes in another from the atmospheric reservoir, while the first is cooled at leisure at constant pressure by commingling with the outside air.

132. Efficiency of the Otto Cycle.—The Otto cycle, like the Rankine and also the Diesel cycles, to be described presently, is an *ideal* cycle. The area $ABCD$, Fig. 93, represents the available part of the entire amount of heat A_1BCD_1 , supplied during the cycle. The performance of the actual utilizer employing the Otto cycle as its pattern will depend upon the thermal efficiency of its ideal and upon its own capability in attainment of its ideal. The thermal efficiency of the Otto cycle, like that of the Rankine and

all other ideal cycles, is not a fixed and definite quantity but is dependent upon limiting external conditions between which it is compelled to work.

The energy changes for the various operations of the Otto cycle are shown in the following table.

Operation	Heat quantity— Q			Internal energy change ($U_2 - U_1$)	Work quantity— W		
	Supplied	Rejected	Area		By the charge	On the charge	Work area
AB	0	0	$\bar{W}_{cv}[T_2 - T_1]$	0	$\frac{p_2 V_2 - p_1 V_1}{(k-1)}$	$A'AB B'$
BC	$\bar{W}_{cv}[T_1 - T_2]$	0	$A_1 B C D_1$	$\bar{W}_{cv}[T_3 - T_2]$	0	0	
CD	0	0	$\bar{W}_{cv}[T_4 - T_3]$	$\frac{p_3 V_3 - p_4 V_4}{(k-1)}$	0	$B' C D A'$
DA	0	$\bar{W}_{cv}[T_4 - T_1]$	$D_1 D A A_1$	$\bar{W}_{cv}[T_1 - T_4]$	0	0	
Total	$\bar{W}_{cv}[T_3 - T_2]$	$\bar{W}_{cv}[T_4 - T_1]$	$\frac{ABCD}{\text{(Fig. 93)}}$	0	$\frac{p_3 V_3 - p_4 V_4}{(k-1)}$	$\frac{p_2 V_2 - p_1 V_1}{(k-1)}$	$\frac{ABCD}{\text{(Fig. 92)}}$

The thermal efficiency of the ideal Otto cycle, Fig. 93, is $ABCD/A_1 B C D_1$, or $(A_1 B C D_1 - A_1 A D D_1)/A_1 B C D_1$. From Art. 109,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}, \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{k-1}, \quad \text{and} \quad \frac{T_x}{T_y} = \left(\frac{V_y}{V_x}\right)^{k-1} \quad (41)$$

where XY is any vertical on the temperature-entropy sketch, Fig. 93, between AB and CD . Since BC and AD are constant-volume lines,

$$\left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{V_4}{V_3}\right)^{k-1} = \left(\frac{V_y}{V_x}\right)^{k-1} = r^{k-1}$$

where r is the compression ratio V_1/V_2 . Hence,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_x}{T_y}$$

and

$$\frac{(T_2 - T_1)}{T_2} = \frac{(T_3 - T_4)}{T_3} = \frac{(T_x - T_y)}{T_x}$$

This means that the ratio of the distance XY to the distance X_1X (Fig. 93) is the same for any ordinate that may be chosen and is therefore equal to the area ratio $ABCD/A_1BCD_1$. But thermal efficiency,

$$\eta_{th} = ABCD \div A_1BCD_1.$$

Hence,

$$\eta_{th} = \frac{(T_2 - T_1)}{T_2} \text{ or } \frac{(T_3 - T_4)}{T_3}$$

Also, in terms of pressures or volumes (see Art. 109),

$$\eta_{th} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} \text{ or } 1 - \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}} \quad (58)$$

$$= 1 - \frac{1}{(V_1/V_2)^{k-1}} = 1 - \frac{1}{r^{k-1}} \quad (59)$$

Hence, efficiency of an Otto cycle is determined by either the *compression ratio* or (what in this case is the same thing) the *expansion ratio*.

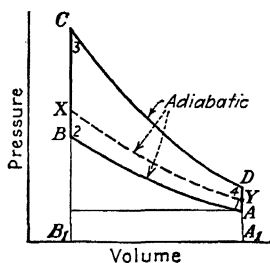


FIG. 92.

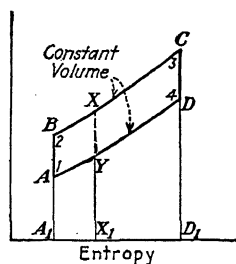


FIG. 93.

It will be apparent that the efficiency of an Otto cycle is wholly independent of the amount of heat supplied during the series of operations and of the maximum temperature attained. Thus, in Fig. 93, the efficiency of the cycle $ABCD$ is no higher than that of a cycle $ABXY$, although the heat supplied to the former and the maximum temperature are much greater. As more heat is added along BC , the temperature at which the heat is supplied becomes greater, but, owing to the character of the cycle, so also does the maximum temperature at which heat is rejected.

That the areas of a cycle represented on a pressure-volume plane offer no clue to *thermal efficiency* of a cycle is clearly demonstrated by Fig. 92. There the diagram would suggest that the efficiency of the cycle $ABCD$ is greater than that of $ABXY$, although we have just shown that they are actually the same.

133. The Diesel Cycle.—In an engine embodying the Otto cycle, burning must be extremely rapid so that the process may be completed while the piston is “dwelling” at its dead-center position. In order to realize the necessary rapidity of combustion, the fuel should be mixed with its air in the most complete manner possible. This is accomplished by drawing the liquid or gaseous fuel into the cylinder with the air on the suction stroke, which is a feature of all Otto engines. With a charge consisting of a combustible mixture, the compression ratio is limited to a value that will not result in a temperature of compression so high that *pre-ignition* will occur. Hence an Otto engine is not free to take advantage of the possibility of increasing the thermal efficiency of its cycle by an increase in the compression ratio.

The Diesel is an ideal cycle definitely adapted to offer higher thermal efficiencies through its ability to employ any compression ratio desired. In this cycle, represented by Figs. 94 and 95, a charge of *air only* is compressed adiabatically from A to B . Compression is carried up to the *maximum pressure of the cycle*, which is limited only by the mechanical strength of the engine and is usually about 500 psia. At or near the end of compression, the injection of liquid fuel begins. This may be done by atomizing with air separately compressed to a much higher pressure, or more usually by “solid injection” through a small nozzle under a high-pressure differential. Burning begins the moment fuel enters the combustion space of the cylinder because of the high temperature of compression of the air. The ideal of the second operation of the cycle is that the rate of injection of fuel shall be so adjusted to the rate of

the piston's motion that the pressure of the charge in the cylinder will remain constant until the injection is cut off, and combustion ceases at C , to give the constant-pressure line BC , Fig. 94. The third and fourth operations of the Diesel cycle are like those of the Otto cycle; CD is adiabatic expansion, and DA is equivalent constant-volume heat rejection. In an actual engine, it would be a matter of extreme difficulty to obtain a constant-pressure burning line BC , as this would require an impossibly precise control of the rate of oil injection to compensate for the varying rate of piston movement, as well as the variable rate of combustion.

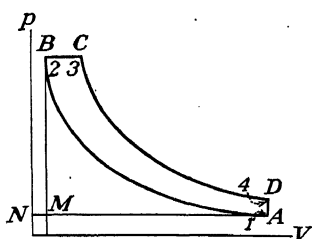


FIG. 94.

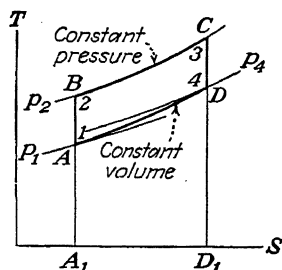


FIG. 95.

134. Efficiency of the Diesel Cycle.—The available heat of the Diesel cycle is represented by the area $ABCD$, Fig. 95, and its thermal efficiency is $ABCD/A_1BCD_1$. An inspection of Fig. 95 shows that the efficiency of the cycle grows less as the amount of heat supplied (corresponding to the load) increases. The greater the amount developed, the higher is the maximum temperature T_c ; but, at the same time, the maximum temperature of heat rejection T_d is increased relatively more than T_c . In Fig. 95, let

$$\frac{V_1}{V_2} = \text{the compression ratio } r$$

and let $V_3/V_2 =$ the cutoff ratio d . From Eq. (41),
Art. 109

$$\begin{aligned}
\frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1} & \text{and} & & T_2 &= r^{k-1}T_1 \\
\frac{T_3}{T_2} &= \frac{V_3}{V_2} = d & \text{and} & & T_3 &= dT_2 = dr^{k-1}T_1 \\
\frac{T_4}{T_3} &= \left(\frac{V_3}{V_4}\right)^{k-1} & \text{and} & & T_4 &= \left(\frac{V_3}{V_4}\right)^{k-1} T_3 \\
& & & & &= \left(\frac{V_3}{V_2} \times \frac{V_2}{V_4}\right)^{k-1} dr^{k-1}T_1 \\
& & & & &= d^{k-1}r^{1-k} dr^{k-1}T_1 \\
& & & & &= d^k T_1 \\
\eta_{th} &= \frac{A_1BCD_1 - A_1ADD_1}{A_1BCD_1} \\
&= \frac{\bar{W}_{c_p}(T_3 - T_2) - \bar{W}_{c_v}(T_4 - T_1)}{\bar{W}_{c_p}(T_3 - T_2)} \\
&= 1 - \frac{(T_4 - T_1)}{k(T_3 - T_2)} \\
&= 1 - \frac{d^k - 1}{k(d r^{k-1} - r^{k-1})} \\
&= 1 - \frac{1}{r^{k-1}} \left[\frac{(d^k - 1)}{k(d - 1)} \right] \tag{60}
\end{aligned}$$

Analysis of Eq. (60) shows that the efficiency of the Diesel cycle increases with an increase in the compression ratio r and with a decrease in the cutoff ratio d . At light loads (small d) the efficiency is higher than at full load.

The Diesel cycle was conceived and developed by Dr. Rudolph Diesel, a German engineer, about 1895.

135. The Low-compression Oil Engine.—Prior to the present widespread development of the Diesel engine in this country, a transitional type of oil engine, operating on a sort of modified Diesel cycle, appeared in large numbers. In these engines, the automatic ignition feature of the Diesel was employed, but by means of a hot spot (an area in the combustion-chamber wall uncooled by water), ignition temperature could be obtained with a compression pressure as low as 200 or 300 psia. Hence these engines

were called "low-compression oil engines" or sometimes, "semi-Diesels."

No great attempt was made to time the injection charge accurately, which was introduced in a "lump" quantity early enough in the cycle to ensure that burning would be well started at dead-center. The result was to produce an indicator diagram that had no particular ideal line upon which the heat was to be supplied. In Figs. 96 and 97, the combustion line BC first rises in pressure, owing to the excess of heat generated in rapid burning, over the rate at which the piston does work; and then later, as the burning

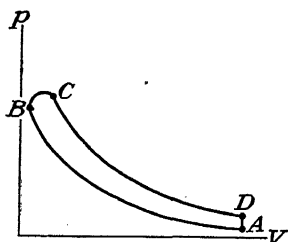


FIG. 96.

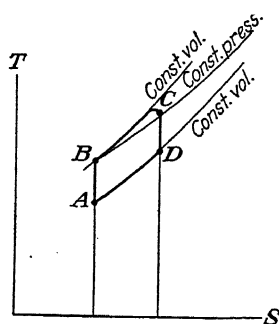


FIG. 97.

nears completion, the line BC gradually droops to merge with the expansion line CD .

There is nothing distinctive, from a thermodynamic standpoint, in the cycle of these low-compression oil engines, although as actual heat engines they gave very good results. In all respects other than the combustion line, their cycle is like that of the Diesel.

136. Diesel and Otto Cycles Compared.—As embodied in an actual engine, the Diesel cycle is more efficient than the Otto. The reason for the higher efficiency of the Diesel, however, is the higher compression ratio that its plan of operation permits rather than any inherent advantage in the cycle itself.

Figures 98 and 99 show an Otto cycle $ABDF$ superimposed upon a Diesel cycle $ABCDF$ on both the pressure-

volume and temperature-entropy planes. For the same amount of heat rejected, the Diesel cycle gives an available heat BCD , Fig. 99, greater than that of the Otto cycle. If, however, the compression in the Diesel were carried only as high as in the Otto cycle, *viz.*, to B , then the available heat

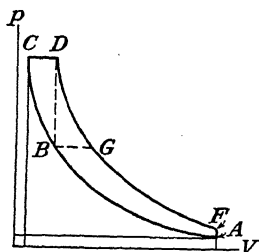


FIG. 98.

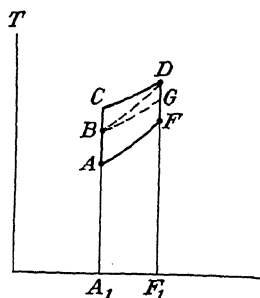


FIG. 99.

of the Otto cycle would be greater than that of the Diesel by the area BDG .

Assuming that the same maximum pressure is permissible in both cycles—the case represented in Figs. 98 and 99

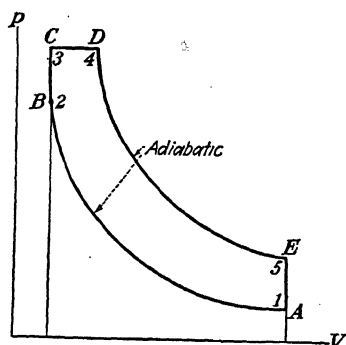


FIG. 100.

by the cycles $ABCDF$ and $ABDF$ —the Diesel reaps the advantage of having elevated the temperature of its charge by compression, so that when burning takes place its heat is supplied at a higher temperature level than is that of the Otto.

Comparing Eqs. (59) and (60), for equal compression ratios the Otto cycle is more

efficient than the Diesel cycle since the bracketed term of Eq. (60) is always greater than one.

137. The Dual-combustion Cycle.—The dual-combustion or limited-pressure cycle is a modification of the Otto and Diesel cycles. The cycle is approximated in the solid-

injection compression-ignition engine. Combustion begins and is partially completed at constant volume followed by completion of combustion at constant pressure. The cycle is shown in Figs. 100 and 101. Heat is added at constant volume along BC and at constant pressure along CD . Heat is rejected along EA .

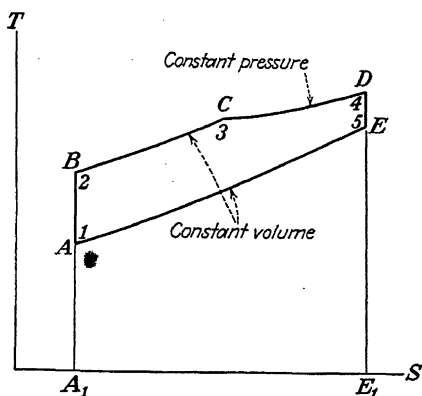


FIG. 101.

Let $V_1/V_2 =$ compression ratio r .

$V_4/V_3 =$ cutoff ratio d

$p_3/p_2 =$ pressure ratio of constant volume combustion, a

From Eq. (41), Art. 109

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1} \quad \text{and} \quad T_2 = r^{k-1}T_1$$

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = a \quad \text{and} \quad T_3 = aT_2 = ar^{k-1}T_1$$

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} = d \quad \text{and} \quad T_4 = dT_3 = dar^{k-1}T_1$$

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{k-1} = \left(\frac{V_4}{V_3} \times \frac{V_3}{V_5}\right)^{k-1} \quad \text{and} \quad T_5 = \left(\frac{d}{r}\right)^{k-1} T_4 = ad^k T_1$$

$$\begin{aligned} \eta_{th} &= \frac{A_1BCDE_1 - A_1AEE_1}{A_1BCDE_1} = 1 - \frac{A_1AEE_1}{A_1BCDE_1} \\ &= 1 - \frac{\bar{W}c_v(T_5 - T_1)}{\bar{W}c_v(T_3 - T_2) + \bar{W}c_p(T_4 - T_3)} \end{aligned}$$

$$\begin{aligned}
 &= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + k(T_4 - T_3)} \\
 &= 1 - \frac{(ad^k - 1)}{(ar^{k-1} - r^{k-1}) + k(dar^{k-1} - ar^{k-1})} \\
 &= 1 - \frac{1}{r^{k-1}} \left[\frac{ad^k - 1}{(a - 1) + ak(d - 1)} \right] \quad (61)
 \end{aligned}$$

When a equals 1, Eq. (61) reduces to Eq. (60) and the cycle becomes identical with the Diesel cycle. When d equals 1, Eq. (61) reduces to Eq. (59) and the cycle becomes identical with the Otto cycle.

138. The Brayton Engine.—Practically every internal-combustion engine at the present time operates upon

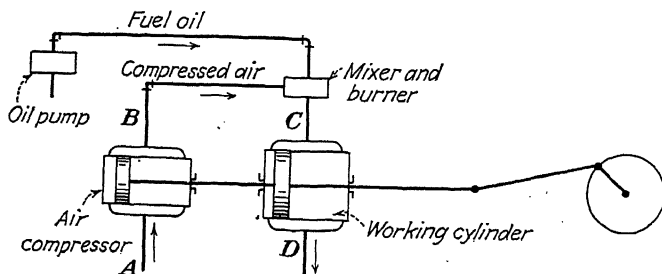


FIG. 102.

either the Diesel or the Otto cycle, or possibly upon a cycle that is a combination of the two. The first Otto engine was built in 1876 and was known as the Otto "silent" engine. The word "silent" is a relative term and was used presumably to emphasize the relief at not having to listen to the Otto and Langen free-piston engine, which was the immediate forerunner of the Otto engine. The Otto and Langen engine operated upon the Otto cycle. As a matter of fact, the Otto cycle was not invented by Otto, but was suggested and discussed about 1856 by Beau de Rochas, and is sometimes known by his name.

A few years before the Otto silent engine was developed, an American named Brayton developed an engine that met with some success commercially until it was displaced by

the Otto engine, which proved superior to it. The cycle of the Brayton engine is sometimes called the Joule cycle, because Joule had suggested it before the engine was developed by Brayton.

Figure 102 gives some idea of the mechanical elements of the engine. The air is compressed in the small cylinder. The compressed air and fuel oil are mixed and burned and then admitted to the working cylinder, where work is done

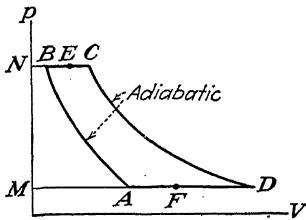


FIG. 103.

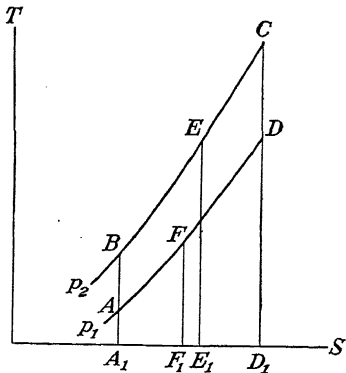


FIG. 104.

both by the pushing in of the charge and by the further expansion of the charge after cutoff, after which the spent gases are exhausted into the atmosphere.

The cycle of the Brayton engine is represented on the pV and TS planes in Figs. 103 and 104. $MABN$ is the work done in compressing the air; $NCDM$ is the work done by the charge; and $ABCD$ is the net work delivered by the engine.

139. Efficiency of the Brayton Cycle.—From Fig. 104

$$\begin{aligned}\eta_{th} &= \frac{A_1BCD_1 - A_1ADD_1}{A_1BCD_1} \\ &= \frac{c_p(T_C - T_B) - c_p(T_D - T_A)}{c_p(T_C - T_B)} \\ &= 1 - \frac{T_D - T_A}{T_C - T_B}\end{aligned}$$

From Eq. (42), Art. 109

$$\frac{T_D}{T_C} = \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = \frac{T_A}{T_B} = \frac{T_D - T_A}{T_C - T_B}$$

Let $V_A/V_B = r$, the compression ratio

$$\begin{aligned}\eta_{th} &= 1 - \frac{T_A}{T_B} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}} = 1 - \left(\frac{V_B}{V_A}\right)^{k-1} \\ &= 1 - \frac{1}{r^{k-1}}\end{aligned}\quad (62)$$

Equation (62) is identical with Eq. (59) for the efficiency of the Otto cycle; hence the efficiency of the Brayton cycle without regeneration is the same as the efficiency of the Otto cycle.

The efficiency of the Brayton cycle may be improved by regeneration. Since the temperature at D is higher than the temperature at B when the compression ratio is not too large, the exhaust gases leaving the prime mover may be used to partially heat the compressed gas going to the prime mover. Let $T_E = T_D$. Then, with regeneration

$$\begin{aligned}\eta_{th} &= \frac{c_p(T_C - T_B) - (T_D - T_A)}{c_p(T_C - T_E)} \\ &= \frac{T_C - T_B - T_D + T_A}{T_C - T_B} = \frac{T_C - T_D - T_B + T_A}{T_C - T_D} \\ &= 1 - \frac{T_B - T_A}{T_C - T_D}\end{aligned}$$

From Eq. (42), Art. 109

$$\frac{T_B}{T_C} = \frac{T_A}{T_D} = \frac{T_B - T_A}{T_C - T_D}$$

Let $V_C/V_B =$ the cutoff ratio, d .

$$\eta_{th} = 1 - \frac{T_B}{T_C} = 1 - \frac{V_B}{V_C} = 1 - \frac{1}{d}\quad (63)$$

The regenerative Brayton cycle is more efficient at full load (large d) than at light load. As d becomes smaller, T_D approaches T_B for a given compression ratio. When T_D equals T_B , regeneration becomes impossible and Eq. (63) no longer applies. The area A_1BEE_1 will equal area

F_1FDD_1 . Since the constant-pressure lines converge downward, the entropy change F_1D_1 will be smaller than the entropy change A_1E_1 and T_F will be above T_B .

$$T_B = T_A \left(\frac{V_A}{V_B} \right)^{k-1} = T_A r^{k-1}$$

$$\eta_{th} = 1 - \frac{T_A}{T_C} r^{k-1} \quad (64)$$

For a given inlet temperature T_A and fixed maximum temperature T_C , the efficiency decreases with an increase in compression ratio, a result opposite to the effect of compression ratio on the nonregenerative Brayton cycle. As the compression ratio is increased, T_B approaches T_D and less regeneration is possible. For a given compression ratio and inlet air temperature, the efficiency increases with maximum temperature of the cycle.

140. The Gas Turbine.—The Brayton cycle engine was a failure largely because the maximum temperature

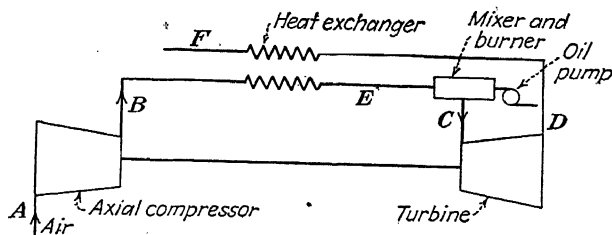


FIG. 105.

of the cycle was not high enough and the efficiency of the compressor was so low that no excess energy was available for doing work external of the cycle. The gas turbine has overcome these difficulties. With the development of alloys for turbine blades to stand high temperatures and improved design of a compressor, the Brayton cycle has proved successful. A high-speed axial turbocompressor which will compress efficiently large volumes of air is directly connected to a reaction-type gas turbine. Figure 105 shows a line diagram of the unit using regeneration.

At the present time (1943) the gas turbine employs a maximum temperature of 1000 F and is used in the Houdry process for cracking oil, as an auxiliary of the Velox boiler, and for driving the supercharger of internal combustion engines. At maximum temperatures of 1500 F and using regeneration, the gas turbine can be expected to have efficiencies comparable to the efficiency of Diesel engines. Low-grade fuels and waste heat can be used with the gas turbine.

141. The Lenoir Cycle.—The first commercially successful internal-combustion engine was built in 1860 by Lenoir, a Frenchman. The Lenoir cycle is pictured in Figs. 106 and 107. The engine drew in a mixture of fuel

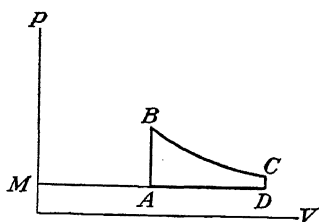


FIG. 106.

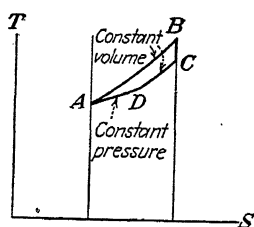


FIG. 107.

and air along MA , Fig. 106. At about half stroke the charge was ignited, producing, theoretically, constant-volume heating AB . Adiabatic expansion then occurred along BC , followed by the equivalent of constant-volume cooling after release at C and then constant-pressure cooling to A . With no compression of the charge, it is evident that the efficiency could not have been very high. The Lenoir engine could not compete in economy with the Otto and Langen free-piston engine, developed in 1867, notwithstanding its superiority over the latter in mechanical operation. The importance that attaches to the Lenoir engine is the fact that it stands as the first successful internal-combustion engine emerging from a long period of study and experiment devoted to the air heat engine by scientists and inventors.

142. The Heat-rejection Line.—In the operation of heat rejection, the use of a single cylinder, for both compression and expansion, results in some unavoidable loss. The lower portions of three different kinds of air-engine cycles are shown in Figs. 108 and 109. The constant-volume heat-rejection line of the Diesel and Otto cycles, in which but one cylinder is used, is represented by DA . In the Brayton cycle, which has a separate compressor for compression, adiabatic expansion of the working stroke can be continued to atmospheric pressure at F . Area ADF , Fig. 108, represents work sacrificed and ADF , Fig. 109, the larger amount of heat rejected in the exhaust, when

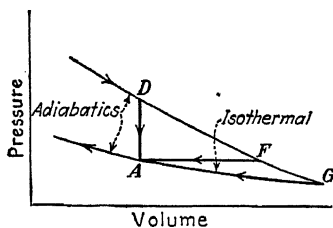


FIG. 108.

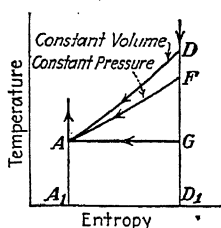


FIG. 109.

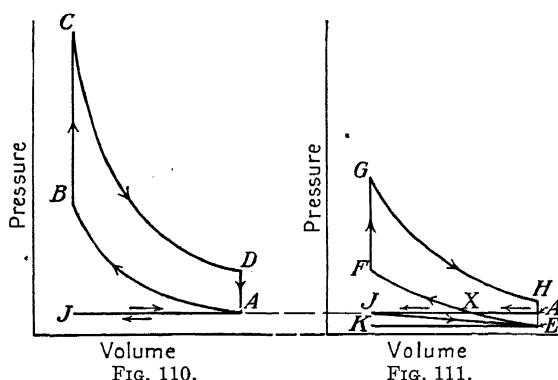
compression and expansion must both be done in one cylinder.

For maximum efficiency, all the heat to be rejected in the exhaust should be disposed of at the lowest temperature level T_A of the cycle. This would require a continuation of the expansion line to G , which is on a level with A , Fig. 109, but falls far below the atmospheric pressure, as will be seen in Fig. 108. It would be conceivably possible to do this by allowing the piston to continue to G , Fig. 108, and then compress isothermally back to the starting point of the cycle at A , cooling the charge by transferring heat from it through the walls of the cylinder to the atmosphere—an impossibly slow process from the mechanical standpoint. The heat rejected would then be represented by the area A_1AGD_1 , Fig. 109, which is the minimum amount possible.

In respect to the heat-rejection operation, the steam (or any vapor) heat engine has a fundamental advantage over the air heat engine, because the constant-pressure heat-rejection line, which is *mechanically* practicable, is coincident with the constant-temperature line, which offers the greatest *thermodynamic* advantage.

143. Effect of Throttling on the Otto-cycle Engine.—

Otto-cycle engines are governed by throttling the charge on the intake stroke. In Fig. 110, JA represents atmospheric pressure and $JABCDAJ$ the normal four-stroke full-power cycle. When the suction is throttled, the cycle becomes



$J'EF'GH'AJ'$, as represented in Fig. 111. Supposing, for illustration, that the pressure at E' is just half that at A , in which case the weight of air contained in the cylinder at E' is one-half that at A . If the weight of fuel is regulated to correspond with the amount of air, the pressures at all points of the cycle of Fig. 111 will be one-half that of corresponding points of the cycle of Fig. 110. The figure $E'F'G'H'E'$, Fig. 111 (completed by drawing in the line $A'E'$, which, however, represents no real operation), is therefore one-half the area of the figure $ABCD$, Fig. 110. Hence, the work developed in the cylinder under this condition of throttling is one-half that for the unthrottled condition, and the thermal efficiency would be alike for the two cases except that some of the work is lost in throttling. The work

usefully developed in the cylinder with throttling is the net area of the cycle $J E F G H A J$, Fig. 111, or $X F G H A - J E X$, and this is equal to $E F G H E$ (= one-half $A B C D$, for condition of throttling assumed) less $J E A$, the work of throttling.

144. Supercharging.—When an airplane ascends into higher altitudes, the effect upon the power developed by its engine is exactly the same as that of throttling at ground level. For example, at an altitude of about 20,000 ft, the atmospheric pressure is about half that at sea level and the cylinder fills with half the weight of air that would be contained at ground level. In Fig. 111, the line $K E$ is drawn at half the height $J A$, which represents atmospheric pressure at ground level, so that the point E at the beginning of compression coincides with the location of the similar point in the cycle illustrating the effect of throttling as described in Art. 143. The cycle for the higher altitude is then $K E F G H E K$. The net area of the cycle is $E F G H E$, which, if it is assumed that the fuel is properly adjusted to the air, is one-half the area of the full-power cycle $A B C D$, Fig. 110, at ground level. In this case, there is nothing to be deducted for throttling, so that the work developed in the cylinder at the altitude assumed is one-half that developed at ground level, and the thermal efficiency is the same. Based upon propeller power, however, the thermal efficiency in the rarefied atmosphere is less, because the mechanical losses remain about the same in absolute amount but are relatively greater per pound of fuel consumed.

At high levels, the deficiency in power developed may be overcome by the use of a *supercharger*, which consists of a centrifugal compressor that will compress the rarefied air of high altitudes up to a value which may here be considered to equal atmospheric pressure at ground level. The compressor may be driven from the engine shaft or by a turbine operated by the gas exhausted from the engine.

A *mechanically driven supercharger* is illustrated in diagrammatic form, in Fig. 112. The engine draws its charge from the supply of compressed air and exhausts into

the atmosphere of lower pressure. The cycle of the compressor is $MFGN$, Fig. 113, while that of the engine is $NGHJKLMN$. The work developed in the engine cylinder is $GHJKG + GLMNG$. The theoretical work to drive the

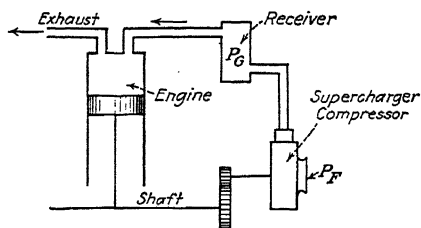


FIG. 112.

compressor is $MFGNM$. The net power developed by the system (considered ideally) is $GHJKG - FGL$, which is nearly that of the full-power cycle of ground level.

An *exhaust-gas turbine-driven supercharger* is illustrated by the sketch of Fig. 114. Here, the engine cylinder has no

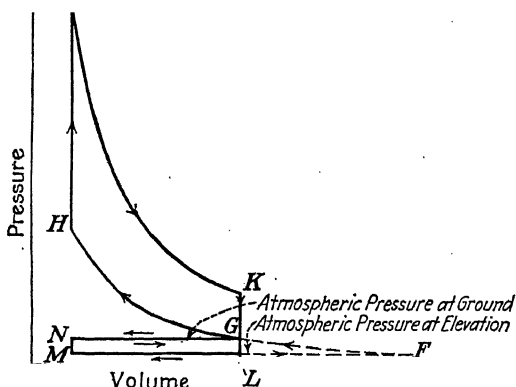


FIG. 113.

communication with the surrounding atmosphere. The cycle of the engine is $GHJKG$, Fig. 115, and that of the compressor is $MFGN$. At release, the exhaust gas is delivered into the nozzle chest of the turbine, where the nearly constant pressure GR is maintained. The volume

of the gases is then NR (approximately), and the Rankine cycle of the turbine is $NRSM$. The line KR is not a continuation of the adiabatic JK , but is merely a line joining K with R , which represents the state of the gas after free

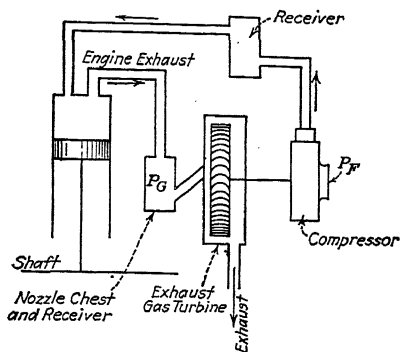


FIG. 114.

expansion from the pressure at K to that at G . The theoretical work of the turbine, $NRSM$, is somewhat greater than that required by the compressor, $MFGN$. The addition of a supercharger of this kind to an engine

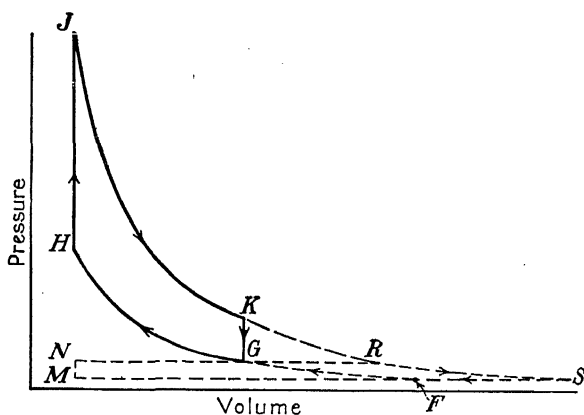


FIG. 115.

is the near equivalent of compounding the engine by the addition of a large low-pressure cylinder.

145. Nature of Losses in an Otto-cycle Engine.—As in the case of the steam engine or turbine, the actual

Otto (or similar) engine is, on account of losses, not capable of transforming the whole of the available heat represented by its cycle into useful work. The general character of the principal thermodynamic losses encountered is indicated by the temperature-entropy sketch of Fig. 116, in which $ABCD$ represents an ideal Otto cycle, and the area enclosed is the available heat.

Starting at A , it seems probable that the relatively cool charge will receive some heat from cylinder and piston near the beginning of compression, so that the path will

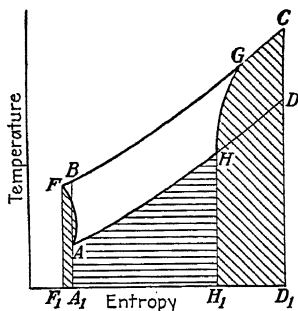


FIG. 116.

incline somewhat to the right from A . After a time, the heat flow will be reversed owing to the rise of temperature of the charge with compression, thus causing the path of the state point to incline to the left, reaching the constant-volume burning line at F . The area A_1AFF_1 represents the net amount of heat withdrawn during compression AF . Assuming that constant-volume burning takes place, the final temperature of the charge, which should have reached C , will fall far short of this value owing to the transfer of heat to the cylinder walls and thence to the jacket water, the actual maximum temperature being represented by some point G . At the beginning of expansion, while the piston is moving slowly, there will be further loss of heat to the cylinder walls, represented by the sharp leftward trend of the expansion line GH . As expansion proceeds, and the gas cools to a temperature more nearly equal to that of cylinder and piston surface, the path will become more nearly vertical, finally intersecting the constant-volume line AD at H . Heat rejection by way of the constant-volume line HA completes the cycle. Piston friction would have the effect of causing the two lines AF and GH to trend a little more to the right.

Area $AFGH$ represents the useful work developed from the heat supplied. Heat given up to the water jacket is represented by $A_1AFF_1 + D_1CGHH_1$; and heat rejected in the exhaust, by H_1HAA_1 .

146. The Stirling "Hot-air" Engine.—Forty-four years before the appearance of the Lenoir internal-combustion-air engine, the first external-combustion-air engine or, as it is commonly called, hot-air engine, was built by Stirling, an English preacher.

The engine consisted primarily of two cylinders A and B , Fig. 117. In the cylinder A , the working charge received and rejected heat; while in the cylinder B , the power was developed. A furnace G is represented as the source of heat which is applied at the bottom of cylinder A . C is a displacer piston, whose function is to move the charge from one end of the cylinder A to the other, at the proper time. The air, upon being displaced from one end of A , passed through the checkerwork heat regenerator R to the other end. In the upper end of A was a cooling coil D . All the necessary elements of a heat engine are thus accounted for. G is the source of heat energy; D is the refrigerator into which unavailable and waste heat is rejected; B is the heat utilizer; and the working substance is air, of which a charge is permanently confined within the engine.

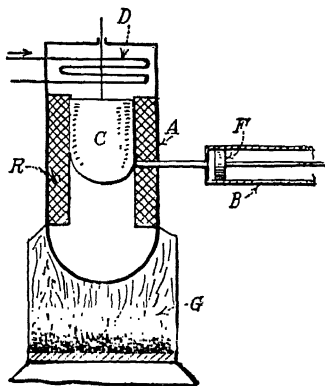


FIG. 117.

Assume, to begin with, that the displacer piston C is at the top of its stroke, as shown in Fig. 117. Nearly all the air in cylinder A is now at the bottom, in contact with the heating surface. The working piston F is at the beginning of its stroke, at the left. With the heating of the charge, the air expands, pushing out the piston F , and the

expansion, which is supposed to be isothermal, continues until F reaches the end of its stroke. At that moment C descends, displacing the air below it to the space above. The heated air, in passing through R , parts with some heat, which is stored up to await the opportunity to return to the working substance. The thermodynamic operation with respect to the air is one of constant-volume cooling. It is to be understood that during the movement of C the working piston F is dwelling at its extreme position and that the quantity of air in A is much larger than in B , so that it is approximately true to say that the heating operations which take place in A apply to the entire charge of air.

The air, which is now mostly confined in the upper part of A , is subjected to the action of the refrigerator and begins to contract in volume. The working piston is moved back to the left, and isothermal compression is supposed to take place. When F reaches the end of its stroke, C ascends and displaces the air above it to the bottom of A . The temperature of the air is relatively low as it enters the passage R , and it now picks up the heat that it stored on its upward passage. This is the principle of the heat regenerator, and the Stirling engine was the first apparatus to make use of that principle.

147. The Cycle of the Stirling Engine.—The ideal cycle of the Stirling engine consists of two isothermals and two constant-volume lines, as shown in Figs. 118 and 119. AB is the working isothermal; BC is the constant-volume cooling, resulting from the air giving up its heat to the regenerator; CD is the isothermal compression; and DA is the constant-volume heating. The heat supplied is represented by the area $D_1DAA_1 + A_1ABB_1$, Fig. 119; the heat rejected by area $B_1BCC_1 + C_1CDD_1$, and the heat equivalent of the work done, by the difference, *viz.*, $ABCD$. However, the heat quantity B_1BCC_1 is not rejected to the refrigerator but is temporarily stored up in the regenerator; nor does that portion of the heat supplied, D_1DAA_1 , come

from the heat source. Ideally, these heat quantities are equal.

$$C_1 = D_1 D A A_1$$

The length DC = the length AB (Fig. 119).

Hence

$$DC = GH \quad \text{and} \quad \text{area } D_1 D C C_1 = A$$

The efficiency of the ideal cycle then is

$$\eta_{th} = \frac{T_1 - T_2}{T_1}$$

which is the same as that of the Carnot cycle.

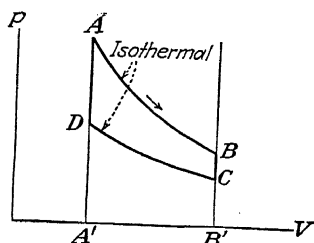


FIG. 118.

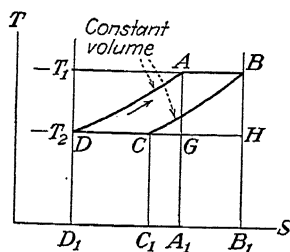


FIG. 119.

148. The Ericsson Hot-air Engine.—Another type of hot-air engine was developed by John Ericsson, a Swede by birth, who left his native country while still a young man to practice as an engineer in England and later became a naturalized American. He was the inventor and builder of the *Monitor* of Civil War fame. While in England, he undertook to equip a 2200-ton vessel with hot-air engines for the United States Navy. There were to be four engines, each having a single-acting working cylinder 14 ft in diameter and 6-ft stroke. The engines ran at 9 rpm and developed 300 hp each. They were unsuccessful for the same reasons that caused the failure of the Stirling engine. The heating surfaces deteriorated under the high temperatures to which they were subjected. In any external-combustion engine, large quantities of heat must be transferred

from one substance to another through a separating wall. This is accomplished without difficulty in a steam-boiler plant, but the receiving substance is water whose relatively low temperature protects the transmitting material from undue heating. In the hot-air engine much higher temperatures of the working substance were necessary under normal operating conditions, and at the same time the possibility of accidental overheating was much greater.

149. The Cycle of the Ericsson Engine.—The cycle of the Ericsson engine is shown on both the pressure-volume and temperature-entropy planes, Figs. 120 and 121. It is com-

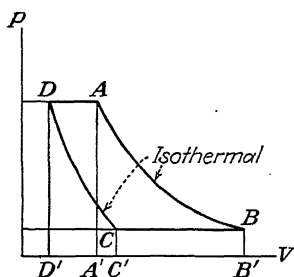


FIG. 120.

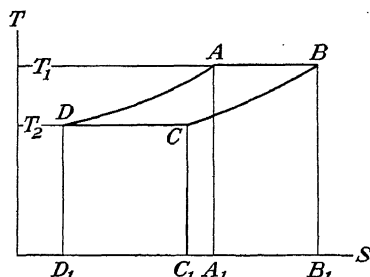


FIG. 121.

posed of two isothermals and two constant-pressure lines. The heat A_1ABB_1 , Fig. 121, is supplied along AB , and the work $A'ABB'$, Fig. 120, is done on the piston. Constant-pressure cooling takes place along BC , as the air is transferring the heat B_1BCC_1 to the regenerator; the work $B'BCC'$ is done by the piston upon the charge. The air is then compressed isothermally, having the further work $C'CDD'$ done upon it, while the heat C_1CDD_1 is rejected. It then passes through the regenerator again, picking up the heat D_1DAA_1 ($= B_1BCC_1$, which it had previously deposited) and doing the work $D'DAA'$ by its growth in volume.

It is apparent that the expression for efficiency of the cycle is the same as that derived for the Stirling engine.

$$\eta_{th} = \frac{T_1 - T_2}{T_1}$$

The hot-air engine as a heat motor is now all but obsolete. Our present interest in it is the recognition of the place that it has had in the development of the science of thermodynamics. Both types of hot-air engine were brought out at a time contemporaneous with the developments of Carnot in working out the second law of thermodynamics and the experiments of Joule in establishing the first. Men were thinking deeply and carefully at that time, and whatever may be the shortcomings of the hot-air engine as a result of the limitations of working mechanisms, thermal conductivity, and refractoriness of material, it cannot be denied that the conception of these cycles reveals a knowledge on the part of the inventors that is truly wonderful.

150. Comparison of Gas Cycles.—Figures 122 and 123 show pV and TS diagrams for several gas cycles operating

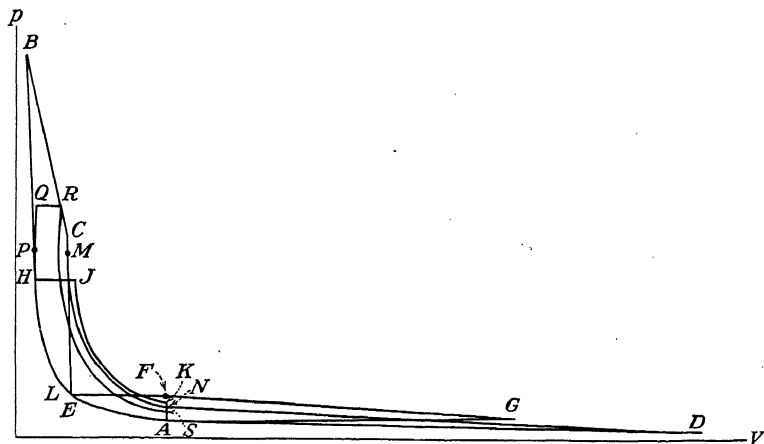


FIG. 122.

between the same temperature limits. Pertinent data for these cycles are shown in Table 9.

The Carnot cycle is impractical because of the extreme pressure and volume ranges. A maximum pressure of 1560 psia would necessitate a very heavy and strong prime mover, and the large increase in volume would require a

large cylinder. The Carnot cycle has a very low mep (mean effective pressure) in contradistinction to the very high maximum pressure. The Brayton cycle has a small pressure range and a large volume range thereby making centrifugal compressors and turbines desirable. The efficiency of the regenerative Brayton cycle is comparable to those of the Diesel and Otto cycles. The Diesel and Otto cycles have the advantage of small volume ranges with moderate pressures, lending themselves to advantageous use in a reciprocating machine in which compression and

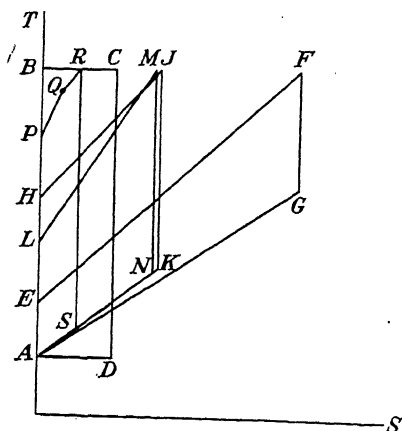


FIG. 123.

expansion may take place with a high mechanical efficiency. In steady-flow machines such as a turbine, the temperature of the parts in contact with the gases are not readily cooled, and thus the blades, for example, assume the temperature of the working substance. In reciprocating machines using nonflow processes, cooling of the parts is practical and the temperature of the working substance may be considerably higher than that of the cylinder walls. The efficiency of all cycles increases with increase of the temperature at which heat is supplied to the working substance. Hence the efficiency and mep of the Otto and Diesel cycles may be considerably above those shown in the table,

the table values being limited by a maximum temperature of 1500 F for all cycles.

TABLE 9.—COMPARISON OF GAS CYCLES

Cycle	Area	Work/ lb, Btu	Com- pres- sion ratio	p_{\max} , psia	p_{\min} , psia	V_{\max} , cu ft	V_{\min} , cu ft	Effi- ciency,	
Carnot.....	<i>ABCD</i>	109.5	27.8	1560	5	38.6	0.462	15.4	72.5
Dual combustion...	<i>APQRS</i>	57.7	14	728	15	12.85	0.918	26.1	64.9
Diesel.....	<i>AHJK</i>	92.2	10	375	15	12.85	1.285	43.1	57.8
Regenerative Bray- ton.....	<i>AEEFG</i>	95.2	2.7	60	15	32.4	4.75	18.6	61.2
Brayton.....	<i>AEEFG</i>	95.2	2.7	60	15	32.4	4.75	18.6	32.5
Otto.....	<i>ALMN</i>	78.1	7	393	15	12.85	1.836	38.4	53.5

Air, $k = 1.4$, minimum temperature 60 F, maximum temperature 1500 F for all cycles.

Problems

In the solution of the following problems, it is suggested that certain assumptions be made for the sake of simplicity.

Assume the working substance to be air undiluted with fuel or other gases, unless otherwise stated.

Take $c_p = 0.241$, $c_v = 0.171$, and $k = 1.40$.

While the results calculated in this manner will not be strictly correct, the principles involved will be illustrated as well as if the solutions were more exact.

1. *a.* An Otto cycle engine operates on a compression ratio of 6.5. What is the percentage clearance?

b. A Diesel cycle engine operates on a compression ratio of 16. What is the percentage clearance?

2. In an ideal Otto cycle, as represented by Figs. 92 and 93, the temperature at *A* is 110 F, and the volume at *A* is four times that at *B*. What is the thermal efficiency of the cycle?

3. The temperature at the beginning of compression in an Otto cycle is 70 F, and at the end of compression it is 600 F. Find the clearance, compression ratio, and thermal efficiency of the cycle.

4. Calculate the thermal efficiency of the Otto cycle for compression ratios of 4, 6, 8, 10 and of the Diesel cycle for compression ratios of 6, 8, 10, 14, 18. Cutoff ratio equals 2. Plot efficiency vs. compression ratio.

5. Consider an Otto-cycle gas engine in which natural gas is used as the fuel. The clearance is 30 per cent. Of the charge drawn in on the suction stroke the proportions are 12 cu ft of air to 1 cu ft of gas. The pressure of the charge in the cylinder, at the end of the suction stroke, is 14 psia and the temperature is 120 F. The heating value of the gas is 900 Btu/cu ft

at 14.7 psia pressure and 60 F. Assuming the charge to have the properties of air, and considering the ideal cycle of the Otto engine (Figs. 92 and 93):

a. Calculate the heat and work quantities for each of the operations of the cycle and insert the numerical values (for 1 lb wt.) in a table like that of Art. 132.

b. What is the efficiency of the cycle?

6. The actual efficiency of an engine is 0.6 of the efficiency of an Otto cycle using air as the working substance. The engine is a four-stroke cycle engine with four cylinders, 3-in. bore, $4\frac{1}{2}$ -in. stroke, has a compression ratio of 5, and consumes 15.4 lb of gasoline per hour at 2000 rpm. Calorific value of fuel is 19,000 Btu/lb. Estimate the mean effective pressure.

7. In an ideal Diesel cycle (Figs. 94 and 95), the pressure at *A* is 14.0 psia; and that at *B*, 500 psia. The temperature at *A* is 120 F.

a. What is the temperature at *B*?

b. What percentage clearance must be provided in order that the pressure of 500 psia may be attained?

c. If 800 Btu/lb of air is applied from *B* to *C*, what will be the temperatures at *C* and *D*, and what is the thermal efficiency of the cycle?

d. What is the thermal efficiency of the cycle if 400 Btu per lb of air is applied from *B* to *C*?

8. A Diesel engine operates with a gas having $c_p = 0.22$ and $c_v = 0.156$. The pressure at the end of compression is 550 psia. The volume is 0.1 cu ft, and the temperature is 2000 F abs. At the beginning of the adiabatic expansion, the temperature is 2400 F abs, and at the end it is 1200 F abs.

a. Find *T*, *p*, *V* for each point.

b. Draw *pV* and *TS* diagrams.

c. Find the heat change in each process.

d. Find the internal energy change in each process.

e. Find the work done during each process.

f. Find the efficiency of the cycle.

9. Air enters a Diesel engine at 14 psia and 110 F. The air is compressed along the line $pV^{1.32} = C$; the compression ratio is 14. Calculate the maximum temperature of the cycle, allowing for variable specific heat of the gas if 300 Btu of heat are released during combustion. Neglect the heat absorbed by the water jacket during combustion, and assume the gas has properties of nitrogen during combustion.

10. Find the thermal efficiency of each of the following cycles, Otto, Brayton, Diesel. In each cycle the suction pressure is 14 psia, the temperature is 80 F, and 600 Btu of heat are supplied per pound of mixture. Compression pressure of the Otto cycle is 105 psia. In Brayton and Diesel cycles, the compression pressure is the same as the explosion pressure of the Otto cycle.

11. Air at 14.7 psia and 60 F enters an engine operating on a dual-combustion cycle. Compression ratio is 8.5, cutoff ratio 1.8, pressure ratio 1.2. What is the temperature at the end of compression, end of constant-volume combustion, and end of constant-pressure combustion? What is the efficiency of the cycle?

12. Inlet conditions of a dual-combustion cycle are 14 psia and 60 F. The compression ratio is 8. If the maximum pressure of the cycle is 800 psia and half of the heat released during combustion occurs at constant volume and half at constant pressure, what is the efficiency of the cycle?

13. A gas turbine operating on a regenerative Brayton cycle has a suction pressure of 15 psia and an inlet temperature of 60 F. The compression pressure is 60 psia. $c_p = 0.26$. $R = 53.3$. Find the thermal efficiencies for maximum temperatures of 1000, 1200, 1400, 1600, 1800 F. Plot efficiency vs. temperature.

14. What is the thermal efficiency of the gas turbine of Prob. 13 operating without regeneration? Does the maximum temperature affect the efficiency?

15. An Otto cycle aircraft engine admits 0.356 lb of air at ground level at 14 psia and 120 F. Compression ratio is 7.5, and 400 Btu of heat are supplied per cycle. What is the work per cycle? At 10,000-ft altitude the suction pressure without a supercharger is 10 psia and the inlet temperature 80 F. A gear-driven supercharger compresses the air isentropically to 14 psia. What is the ratio of the net theoretical work output at 10,000 ft to that at ground level (a) without supercharging? (b) with supercharging? Assume constant air-fuel ratio.

CHAPTER XI

REFRIGERATION

151. The Air Refrigerating Machine.—A diagrammatic sketch of an air refrigerating machine is shown in Fig. 124. The air enters the compressor at *A*, is delivered at *B* into the receiver which is also a cooler, expands in the engine or expander, and after passing through the cold room begins its circuit over again.

The essential element of the system is the expander. In it, the working substance is to be lowered in temperature from that at *C*, which is the minimum obtainable by agents

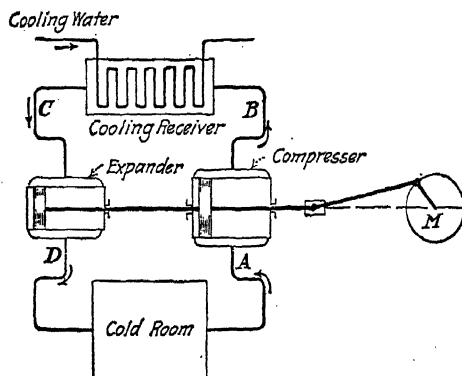


FIG. 124.

at natural temperature, to that at *D*, a temperature low enough to effect the purpose of the machine. Cooling is effected by the reduction of the internal energy of the air at *C*, *by the doing of external work*. Here is seen the fundamental difference in principle between the air refrigerating machine and the vapor refrigerating machine. In the former, lowering in temperature of the working agent must be attained *by the withdrawal of energy in the form of work*; in the latter, lowering of temperature is attained *by merely dropping the pressure, i.e., by throttling*.

It is seen that the air refrigerating machine is after all nothing more than a compressed-air system, such as was illustrated in Art. 25, Fig. 7, with the exception that the circuit of the system is closed so that the same quantity of working substance is used over and over again, instead of being exhausted into the atmospheric reservoir as in the general case of Fig. 7, and with the further exception that in the refrigerating machine the compressor and utilizer are combined into a unit mechanism for convenience. The compressed-air system, generally speaking, utilizes air as a vehicle whereby mechanical energy at one point may be conveyed to a more or less distant station; the compressed-air refrigerating machine—a special case of a compressed-air system—utilizes the air to obtain the thermal effects desired; the transmission of the energy of the compressor to the utilizer is incidental.

152. Diagrams for the Air Refrigerating Machine.—

Figure 125 represents the operation of compression on the pV plane. The compressed air, at state B , is delivered into the receiver, the line BM picturing the process. In the receiver the air is cooled; its pressure remains constant, however, so that the withdrawal of heat results in a reduction of volume. The cooled compressed air is now admitted to the expander at M , Fig. 126, filling the cylinder to C , from which point expansion to D occurs with consequent lowering of temperature. The charge of cold air is then exhausted into the cold room, where it gradually takes up heat from the surrounding objects, cooling them, and itself becoming heated, finally reentering the compressor cylinder along the admission line NA , Fig. 125, at a larger volume than it had upon discharge from the expander, by reason of the heat acquired in the cold room.

The operation of the entire machine may be represented by a combination of the diagrams of Figs. 125 and 126 into the diagram of Fig. 127. AB represents compression of the air; BC , reduction of volume by cooling in the receiver; CD , expansion in the working cylinder; and DA , growth of

volume due to the warming of the cold air by the objects that it is intended to cool.

The area $NABM$ represents the work done by the compressor upon the air; and $MCDN$ the work done by the air in the expander. The work $ABCD$ must be furnished by the motor M , Fig. 124.

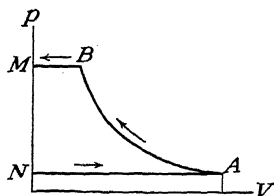


FIG. 125.

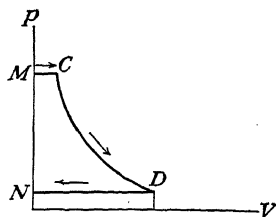


FIG. 126.

In Fig. 128, the cycle of the system is shown on the temperature-entropy plane. AB is compression; BC is the cooling in the receiver, at constant pressure; CD is expansion in the working cylinder; and DA is the warming of the air in the refrigerating chamber at constant pressure.

From what has already been said concerning compressors, it is obvious that the ideal compression should be

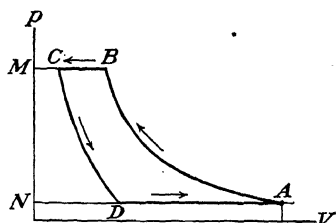


FIG. 127.

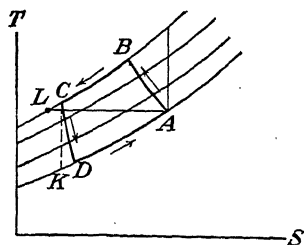


FIG. 128.

isothermal, *i.e.*, along AL , to require the least work. The actual compression with water-jacketed cylinder is represented by AB . The temperature at C , after cooling, will depend upon the temperature of the cooling water and the effectiveness of the coils in the receiver as a heat exchanger. The expansion in the working cylinder should be such as will result in the maximum amount of work done

and consequent minimum exhaust temperature. It is apparent that adiabatic expansion is to be desired here. Practically, however, adiabatic expansion is unattainable on account of the warming effect of the cylinder's environment, together with the reheating due to frictional losses. The actual expansion is represented by CD as distinguished from the isentropic CK .

Attention is directed here again to the fundamental difference in principle between the vapor refrigerating machine and the air refrigerating machine. In the vapor machine, the lowering of the temperature of the working medium is secured merely by reduction of pressure; in the air machine, the necessary low temperature must be secured by the withdrawal of some of the internal energy in the form of work. In the vapor machine, large quantities of heat are absorbed by the working medium without a rise of temperature, *i.e.*, by evaporation and latent heat; in the air machine, heat can be absorbed only as internal energy with rising temperature.

153. The Ammonia Compression Machine.—In Fig. 129, ammonia in the vapor state enters the compressor at A . Its pressure, in this particular case, is shown to be 25 psia and the temperature 30 F. The compressor, which is mechanically exactly the same thing as an air compressor, delivers the ammonia at B , at 200 psia pressure and 210 F. The ammonia is conveyed through the pipe to C , where the temperature is 190 F, the drop of 20 degrees being due to loss of heat to the atmosphere. On its way, the vapor is passed through an oil separator to remove particles of oil or water that may be carried along the cylinder.

The ammonia next traverses the condenser and emerges at D at 80 F in the liquid state. Condensation is accomplished by the circulation of water at whatever may be the "natural" temperature of the locality and the season. The receiver is a reservoir for storage of the liquid, which pauses here until it is called upon to make another circuit of the system. From the receiver, the ammonia passes to

the expansion valve V , through which the pressure is reduced. The valve V is nothing more than a throttling, or pressure-reducing, valve. The pressure following the reducing valve is shown to be 25 psia by the gage near the compressor.

In the expansion coils, the liquid ammonia is evaporated, an operation which demands a supply of heat from some source. The source here is the brine. The brine thus cooled returns to the ice cans and gets another load of heat to convey to the ammonia. This operation goes on indefinitely, the brine acting as a heat vehicle, loading up at

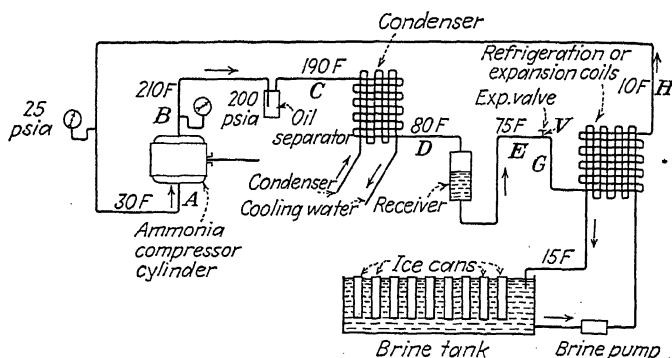


FIG. 129.

one point and discharging its load at another. In place of the ice-making equipment, the brine may serve to cool a room by taking heat from the air; or the ammonia coils themselves may be placed directly within a room which is to be kept cool.

The ammonia finally leaves the expansion coils at H , completely evaporated, with a temperature, as shown by the thermometer, of 10 F.* On its way back to the compressor, the effect of the atmosphere is to warm the ammonia; the "radiation" effect is negative. The ammonia is returned to the compressor at A with a temperature of 30 F, thus making the complete circuit of the system.

154. The Properties of Anhydrous Ammonia.—Just as was the case in the study of the steam power plant, an

investigator is at once confronted with the necessity of a knowledge of the properties of the working substance. As used in the refrigerating system described, ammonia is a substance which is sometimes a liquid, sometimes a vapor, and sometimes a mixture of the two. In that respect it is like the working substance of the steam power plant. It will, therefore, behave in a similar manner, and the properties can be worked out from experimental data in a form parallel to that of the Steam Tables. The properties of ammonia in an abbreviated form are given in Tables 10 and 11.¹

TABLE 10.—PROPERTIES OF SATURATED ANHYDROUS AMMONIA

Absolute pressure, psia p	Temperature, F t	Specific volume, cu ft		Enthalpy above -40°F			Entropy from -40°F		
		Saturated liquid v_f	Saturated vapor v_g	Saturated liquid h_f	Evaporation h_{fg}	Saturated vapor h_g	Saturated liquid s_f	Evaporation s_{fg}	Saturated vapor s_g
5.0	-63.1	0.0227	49.31	-24.5	612.8	588.3	-0.0599	1.5458	1.4857
10.0	-41.3	0.0232	25.81	- 1.4	598.5	597.1	-0.0034	1.4310	1.4276
15.0	-27.3	0.0235	17.67	13.6	588.8	602.4	0.0318	1.3620	1.3938
20.0	-16.6	0.0238	13.50	25.0	581.2	606.2	0.0578	1.3122	1.3700
25.0	- 8.0	0.0240	10.96	34.3	574.8	609.1	0.0787	1.2728	1.3515
30.0	- 0.6	0.0242	9.24	42.3	569.3	611.6	0.0962	1.2402	1.3364
35.0	5.9	0.0244	7.99	49.3	564.3	613.6	0.1113	1.2123	1.3236
40.0	11.7	0.0245	7.05	55.6	559.8	615.4	0.1246	1.1879	1.3125
45.0	16.9	0.0247	6.31	61.3	555.6	616.9	0.1366	1.1661	1.3027
50.0	21.7	0.0248	5.71	66.5	551.7	618.2	0.1475	1.1464	1.2939
120.0	66.0	0.0262	2.48	116.0	512.4	628.4	0.2452	0.9749	1.2201
130.0	70.5	0.0263	2.29	121.1	508.1	629.2	0.2548	0.9584	1.2132
140.0	74.8	0.0265	2.13	126.0	503.9	629.9	0.2638	0.9430	1.2068
150.0	78.8	0.0266	1.99	130.6	499.9	630.5	0.2724	0.9285	1.2009
160.0	82.6	0.0268	1.87	135.0	496.1	631.1	0.2804	0.9148	1.1952
170.0	86.3	0.0269	1.76	139.3	492.3	631.6	0.2881	0.9019	1.1900
180.0	89.8	0.0271	1.67	143.3	488.7	632.0	0.2954	0.8896	1.1850
190.0	93.1	0.0272	1.58	147.2	485.2	632.4	0.3024	0.8778	1.1802
200.0	96.3	0.0273	1.50	150.9	481.8	632.7	0.3090	0.8666	1.1756
210.0	99.4	0.0275	1.43	154.6	478.4	633.0	0.3154	0.8559	1.1713

¹ From "Thermodynamic Properties of Ammonia," U. S. Bur. Standards, Circ. 142. For an excellent compilation of the thermodynamic properties of various refrigerants and brines, see *Amer. Soc. Refrig. Eng. Circ. 9*.

With the pressures and temperatures indicated by the gages and thermometers, and with the ammonia tables at hand, all the properties of the working substance at the several points in the circuit can now be obtained.

TABLE 11.—PROPERTIES OF SUPERHEATED AMMONIA

Absolute pressure, psia		Temperature, F						
		-20°	-10°	0°	10°	20°	30°	40°
15	Specific volume.....	18.01	18.47	18.92	19.37	19.82	20.26	20.70
	Enthalpy.....	606.4	611.9	617.2	622.5	627.8	633.0	638.2
	Entropy.....	1.4031	1.4154	1.4272	1.4386	1.4497	1.4604	1.4709
25	Specific volume.....			11.19	11.47	11.75	12.03	12.30
	Enthalpy.....			613.8	619.4	625.0	630.4	635.8
	Entropy.....			1.3616	1.3738	1.3855	1.3967	1.4077
35	Specific volume.....				8.078	8.287	8.493	8.695
	Enthalpy.....				616.1	622.0	627.7	633.4
	Entropy.....				1.3289	1.3413	1.3532	1.3646
45	Specific volume.....					6.363	6.530	6.694
	Enthalpy.....					618.8	624.9	630.8
	Entropy.....					1.3068	1.3193	1.3313
		130°	150°	170°	190°	210°	230°	250°
140	Specific volume.....	2.460	2.569	2.675	2.779	2.880	2.981	3.080
	Enthalpy.....	667.4	679.9	692.0	704.0	715.8	727.5	739.2
	Entropy.....	1.2738	1.2945	1.3141	1.3328	1.3507	1.3679	1.3846
160	Specific volume.....	2.125	2.224	2.319	2.411	2.502	2.591	2.679
	Enthalpy.....	664.4	677.2	689.7	701.9	713.9	725.8	737.6
	Entropy.....	1.2542	1.2757	1.2958	1.3148	1.3331	1.3506	1.3675
180	Specific volume.....	1.865	1.955	2.042	2.126	2.208	2.288	2.367
	Enthalpy.....	661.3	674.6	687.3	699.8	712.0	724.1	736.1
	Entropy.....	1.2364	1.2586	1.2792	1.2987	1.3172	1.3350	1.3521
200	Specific volume.....	1.656	1.740	1.820	1.897	1.972	2.046	2.118
	Enthalpy.....	658.1	671.8	684.9	697.7	710.1	722.4	734.5
	Entropy.....	1.2200	1.2429	1.2641	1.2840	1.3029	1.3209	1.3382

155. Representation of Cycle on the Temperature-entropy Plane.—Beginning with the ammonia at A, Fig. 130, as it is about to enter the compressor, the temperature is 30 F and the pressure 25 psia. Referring to Table 10, the saturation temperature for 25 psia pressure is found to be

-8.0 F, and the ammonia is therefore superheated 38.0 degrees at A . The entropy at A is 1.3967 . The zero of enthalpy and entropy for these tables is -40 F, so chosen to avoid negative values for enthalpy of liquid and entropy of liquid in the temperature range associated with refrigeration work.

At B , the pressure is 200 psia and the temperature is 210 F. The saturation temperature is found to be 96.3 F, so that there is superheat here also, to the extent of 113.7 degrees, and the entropy is 1.3029 . The entropy at B is seen to be less than that at A , which is consistent with the

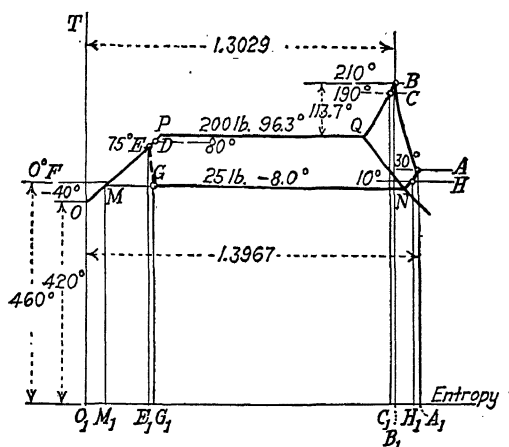


FIG. 130.

nature of the compression from A to B , because the compressor cylinder is water-jacketed so that, although the energy content is increased, heat is actually withdrawn during the operation by the amount A_1ABB_1 . Only *heat quantities* can be pictured directly on the T - S diagram, and A_1ABB_1 represents the heat carried away by the jacket water per pound of ammonia. From B the ammonia is lowered in temperature to C by loss of heat, B_1BCC_1 , to the atmosphere and is then cooled and condensed, leaving the condenser at state D , as a liquid at 80 F temperature but still at 200 psia pressure. Point D is on the liquid line

at a temperature below the saturation temperature; and the point E is 5 degrees lower, on account of the atmospheric cooling. The next change is a reduction of pressure through the expansion valve V . This will be recognized as a purely throttling action. Hence, the enthalpy is the same at G as at E .

$$h_{f1} = h_{f2} + x_2 h_{fg2}$$

$$x_2 h_{fg2} = h_{f1} - h_{f2} = O_1 OEE_1 - O_1 OMM_1 = M_1 MEE_1$$

or

$$M_1 MGG_1 = M_1 MEE_1$$

From the ammonia tables,

$$h_{f1} = 126.2; \quad h_{f2} = 34.3; \quad \text{and} \quad h_{fg2} = 574.8$$

whence

$$x_2 = 0.160 \text{ or } 16.0 \text{ per cent}$$

The entropy at M with respect to O is, from the tables, 0.0787. The entropy from M to G is

$$x_2 h_{fg2} \div T_2 = 0.160 \times 574.8 \div 452.0 = 0.2033.$$

The coordinates of the point G are thus established.

From G to H , the ammonia receives heat from the brine. Evaporation takes place and finally some superheating. A small amount of heat is further absorbed from the air, represented by the area $H_1 HAA_1$. From A , the ammonia begins the cycle of operations over again.

156. Heat Quantities.—Referring to Fig. 130, the area $A_1 ABB_1$ is the heat removed by the compressor cylinder jacket water. $B_1 BCC_1$ is the heat lost to the surrounding air. In this case, such loss of heat is a gain in economy, since heat lost to the air does not have to be removed by the condenser. The condenser removes the heat represented by the area under $CQPD$, and a small amount (under DE) is again advantageously lost to the air from the condensed ammonia. It has already been shown

that there is no change of enthalpy in passing through the expansion valve. The ammonia receives from the brine the amount of heat G_1GHH_1 , and it is the value of this area that expresses the refrigerating capacity for each pound of ammonia circulated. The area H_1HAA_1 is another atmospheric effect, and, although it is a gain in heat, it represents a loss in efficiency, since whatever heat is gained here must be taken out elsewhere in the restoration process.

157. Work of the Cycle.—The heat equivalent of the mechanical work of the cycle of Fig. 130 is represented by the area $ABQPMNA$, which may be shown as follows:

Let h_A = enthalpy of ammonia at beginning of compression = area O_1OMNAA_1

h_B = enthalpy at end of compression = area O_1OPQBB_1

W/J = heat equivalent of work done by compression

Then

$$h_A + \frac{W}{J} = h_B + \text{heat lost to water jacket}$$

or

$$\frac{W}{J} = h_B - h_A + \text{heat lost to water jacket} \quad (65)$$

In terms of areas:

$$\begin{aligned} \frac{W}{J} &= O_1OPQBB_1 - O_1OMNAA_1 + B_1BAA_1 \\ &= O_1OPQBAA_1 - O_1OMNAA_1 = ABQPMNA^* \end{aligned}$$

The work of the compressor is seen to be the same no matter what the state of the ammonia entering the expansion valve: whether at E , as shown in Fig. 130, or at a lower temperature; or at P , or even at a partially condensed state represented by some point along PQ . However, the lower

* It is here tacitly assumed that there is no heat developed by friction in the presence of the working substance, and hence the path AB of Fig. 130 is determined wholly by the compression and by the water-jacket cooling.

the temperature of the liquid as it comes to the expansion valve, the greater is the heat-absorbing capacity or refrigerating effect per pound of ammonia.

158. Refrigerating Capacity.—The capacity of a refrigerating plant is expressed in terms of the rate at which it is able to remove heat. Refrigerating equipment which can remove heat at the rate of 200 Btu per min, or 12,000 Btu per hr, is said to have a refrigerating capacity of 1 ton of refrigeration. Thus a 1-ton plant operating 24 hr per day could absorb $24 \times 12,000 = 288,000$ Btu. This is the quantity of heat to be transferred when 1 ton of water at 32 F freezes to 1 ton of ice at 32 F.

The refrigeration developed by a plant will depend upon two things, *viz.*, the amount of heat removed per pound of ammonia (the area G_1GHH_1 , Fig. 130), and the rate at which ammonia is handled.

In the example represented by Fig. 130, the enthalpy of ammonia at H (at 25 psia and 10 F) is 619.4 Btu.¹ The enthalpy at G (which is equal to that of the liquid at 75 F at point E) is 126.2. Hence, the refrigerating effect per pound of ammonia

$$(\text{area } G_1GNHH_1) = 619.4 - 126.2 = 493.2 \text{ Btu.}$$

To develop 1 ton of refrigerating capacity, ammonia must be circulated at the rate of $12,000 \div 493.2 = 24.3$ lb per hr.¹

159. The Refrigerating Coil.—When allusion is made to that section of the ammonia refrigerating plant in which the brine is cooled by the evaporation of ammonia, as the *refrigerating coil*, the apparatus is viewed from the standpoint of the effect which it is desired to produce. But viewed from the standpoint of the ammonia, the refrigerating coil is a boiler. Heat is absorbed by the liquid from a surrounding warmer substance, and the liquid is thereby converted into a vapor, which is exactly what happens in a

¹ The ratio of the area G_1GHH_1 to the area $ABQPMNA$ of Fig. 130 is called the "coefficient of performance." It expresses the relation of useful thermal effect to the heat equivalent of the work expended.

steam-boiler plant. The brine corresponds to the hot gases. Both are cooled in the operation which takes place. Either apparatus is a *boiler* when the effect upon the confined volatile substance is considered; either is a *refrigerator* when the operation is viewed from the standpoint of the substance which furnishes the heat.

Liquid ammonia is fed into the ammonia boiler, just as water is fed into a steam boiler, but with this difference: the temperature of the ammonia liquid is higher than that of the vaporization pressure carried. It is as though water at a temperature of 400 F were fed to a steam boiler carrying 100 psia steam pressure. In place of heat having to be applied to raise the temperature of the feed to that of vaporization, the feed liquid possesses an excess amount of energy which it contributes toward vaporizing itself. It is a common error, or at least an incomplete statement, to say that the liquid ammonia "flashes" into vapor on passing through the expansion valve. It is true that the excess enthalpy of liquid is there applied to vaporizing some of the liquid itself. But the proportion of vapor by weight is the ratio $MG \div MN$, Fig. 130, from which it is evident that, of the entire weight after passing the expansion valve, liquid constitutes by far the greater proportion.

160. How Low Temperature Is Attained.—The first and most essential requirement for any refrigerating system is the production and maintenance of a temperature lower than that of natural surroundings. States of the working substance containing both liquid and gaseous phases possess the peculiar property that temperature and pressure are rigidly related so long as gas and liquid are in intimate contact. Thus, the temperature at which water boils, or steam is condensed, is determined by the pressure imposed during the operation.

In the sketch of Fig. 131, *A* represents a vessel containing liquid ammonia, open to the atmosphere whose pressure is assumed to be 15 psia. In Table 10, it is found that

the boiling temperature of ammonia is -27.3°F , which is the highest temperature it can assume under the pressure of 15 psia. It might be possible for the liquid to be at a lower temperature, just as it is possible and usual to find water at temperatures far below its boiling point under atmospheric pressure. In the case of Fig. 131, the temperature of surroundings is much greater than the highest that can possibly be assumed by the ammonia (-27.3°F). The resultant inflow of heat will maintain this maximum temperature, and the liquid will boil more or less rapidly,

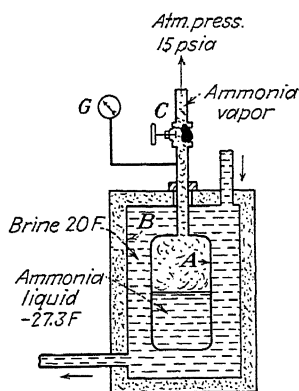


FIG. 131.

depending upon the rate of heat inflow. Thus, by the simple process of imposing a convenient pressure upon a liquid, adequately low temperatures for refrigerating purposes can easily be produced and maintained.

If the vessel containing the ammonia is placed in an insulated chamber through which brine is circulated at say 20°F , as indicated in Fig. 131, heat will flow continuously from brine to ammonia under a thermal head of 47.3 degrees. Ammonia liquid will be evaporated, and brine will be cooled at a rate corresponding to the amount of transmission surface provided.

In case the temperature of -27.3°F is lower than necessary, the escape of gas may be throttled by means of the valve *C*, which will cause a higher pressure than 15 psia to be built up in *A*. If, for example, a temperature of -8.0°F is desired, the operator has only to refer to a table of saturated ammonia properties (such as Table 10) and ascertain what pressure is associated with -8.0°F , and then adjust the valve *C* until the gage *G* registers a value corresponding to that absolute pressure which, in this case, is 25 psia. If a temperature lower than -27.3°F is desired, it would be necessary to connect a suction pump to the

vapor line from *A*, by means of which pressures less than atmospheric could be maintained. Again, the gage *G* becomes the index to guide in the operation of the pump to produce the desired temperature. The necessary low temperatures for refrigeration are produced solely by the control of pressure upon a liquid. Hence, the term *mechanical refrigeration* is aptly applied to such a system.

161. The Maintenance of Low Temperature.—In the course of time, the ammonia liquid of vessel *A*, Fig. 131, would become exhausted, and the apparatus would no

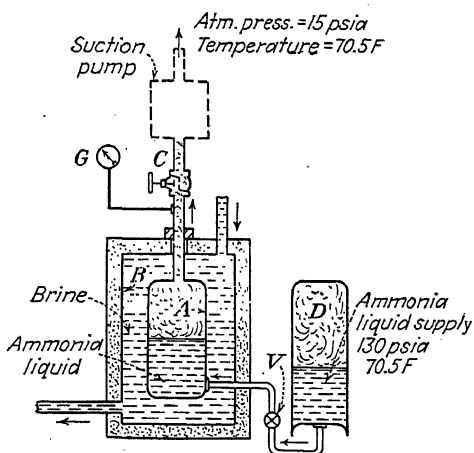


FIG. 132.

longer function as a refrigerator. To operate continuously, some source of liquid supply must be provided, as indicated in Fig. 132, where an external supply is contained in a drum *D*, such as those used in the storage and transportation of liquid ammonia. With the exception of brief intervals of time necessary to replace an empty drum by a full one, refrigeration could now be carried on continuously with the apparatus of Fig. 132.

The liquid in drum *D* necessarily assumes the temperature of the atmosphere, which is indicated as 70.5 F in Fig. 132. By reference to Table 10, it is found that the pressure corresponding thereto is 130 psia, which means that

vapor will be formed and fill all the space not occupied by liquid, at this pressure. It therefore becomes necessary to provide the valve V in the liquid feed line from reservoir D to evaporator A , by means of which the rate of liquid flow may be adjusted to the rate of evaporation. This valve performs the same function in the elementary apparatus of Fig. 132 as does the valve V in the diagram of Fig. 129, and the process of throttling is represented by the operation EG , Fig. 130.

While the expansion valve is the mechanical element of a vapor refrigerating machine in which, apparently, low temperature is produced, the more accurate idea is that the valve is merely a convenient means of introducing the new supply of liquid into a region where a lower pressure prevails which results in the lower temperature. The low temperature in the evaporator is not at all dependent upon the *range* of pressure or temperature through which the supply is throttled, but is determined solely by the terminal pressure. Thus, in Fig. 130, the terminal point G of the expansion operation EG will fall on the line MN (-8.0°F), no matter if E should be located farther down on the liquid line, say at 60; or at P ; or somewhere on PQ ; or even higher upon the liquid line OMP extended, corresponding to some pressure higher than 200 psia.

162. Recovery of the Vapor.—While the simple apparatus of Fig. 132 includes the essential elements of a vapor refrigerating plant, the cost of the liquid and the objectionable character of the vapor discharged into the atmosphere render it necessary to introduce additional elements to effect the *recovery of the vapor and its restoration to the liquid state*, so that it may be used over and over again. This is accomplished by the compressor F (the compressor also performs the function of a suction pump) and the condenser E of Fig. 133. By means of the compressor, the condensing temperature of the vapor is elevated from -8.0 to 96.3°F by increasing the pressure from 25 to 200 psia. At 96.3°F , there is a sufficient margin of temperature

over that of cooling water available to drain the vapor of its latent heat and restore it to the liquid state, in which form it is returned to reservoir *D*. This closes the ammonia cycle within the system, and the elementary refrigerating apparatus of Fig. 131 has now evolved through Figs. 132 and 133 into the plant represented by the diagram of Fig. 129 and the cycle of Fig. 130.

163. Pressures in the System.—With respect to pressure, the entire circuit of the refrigerant is separated into two

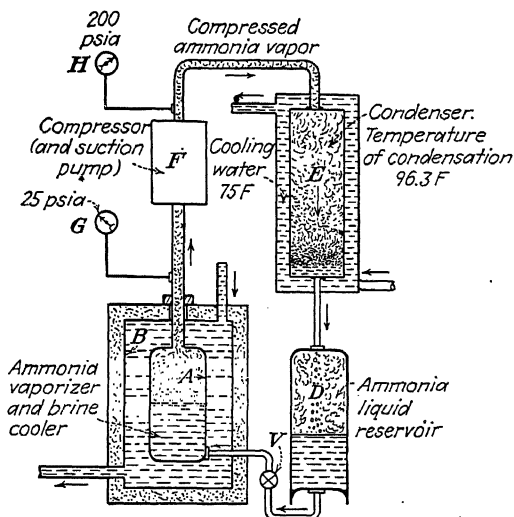


FIG. 133.

divisions. From compressor to expansion valve (Figs. 129 and 133), the working medium is at the condensing or *compression pressure*, while from expansion valve back to the compressor, the evaporating or *suction pressure* prevails. These pressures are determined by the temperature conditions under which the plant is to operate and by the properties of the refrigerant employed.

In Fig. 134, *RR* represents the average temperature of cooling water available and *SS* the average temperature of the cold body (brine, for example) to be maintained. An ammonia cycle represented by the dotted lines is to be constructed appropriate to these temperatures. The

liquid and saturation lines may be drawn in by plotting entropy vs. temperature from Table 10. The vaporizing temperature of the ammonia must be lower than that of the brine by a sufficient amount to cause heat to flow ex-

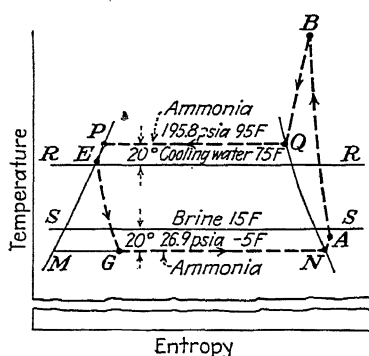


FIG. 134.

editiously through the transmitting surface. If this thermal head is taken as 20 degrees, the line MN is established at -5 F for which the corresponding pressure is 26.9 psia.

If a drop of 20 degrees is also allowed between condensing ammonia and cooling water, the temperature of condensation will be 95 F, and the corresponding pressure is 195.8 psia. Compression, starting at A , must be carried to intersection with the constant-pressure line PQB .

The maximum practicable temperature of vaporization and the minimum practicable temperature of condensation are determined, respectively, by the temperature of refrigeration desired and by the temperature of water available, which is the final cooling agent. These are independent of the kind of vaporous refrigerant employed. On the other hand, the pressures will depend upon the peculiar properties of each refrigerant.

The temperature differences of 20 degrees indicated in Fig. 134 between the lines PQ and RR , and SS and MN , respectively, could be reduced somewhat by the use of larger surfaces of heat transfer, which would increase the size of condenser and evaporator, with consequent increase in first cost. The gain to be expected is a saving in the work of the compressor, represented by the resultant decrease of the work area $ABQPMNA$.

164. Refrigerants.—Temperature-enthalpy diagrams for several refrigerants are shown in Fig. 18. Carbon dioxide

TABLE 12.—COMPARISON OF REFRIGERANTS

Evaporation temperature 5 F, liquification temperature 86 F, no liquid subcooling, no suction superheat

Refrigerant	Ammonia	Carbon dioxide	Freon F-12	Methylene chloride
Chemical formula.....	NH ₃	CO ₂	CCl ₂ F ₂	CH ₂ Cl ₂
Suction pressure, psia.....	34.3	332	26.5	1.17
Compression pressure, psia.....	169.2	1043	107.9	10.02
Specific volume at A, Fig. 135, cu ft/lb	8.15	0.266	1.485	51.3
Refrigerating effect, Btu/lb.....	474.4	55.5	51.1	134.6
Refrigerant per ton, lb/min.....	.422	3.6	3.91	1.49
Volume of refrigerant per ton, cu ft/min.....	19.32	.0738	0.379	76.3
Work per ton, Btu/min.....	42	77	42.5	42.6
Coefficient of performance.....	4.77	2.59	4.71	4.71

requires very high pressures for saturation temperatures near the temperature of cooling water usually available. Freon, F-12, has a low latent heat compared with ammonia, NH₃. A comparison of several refrigerants is shown in Table 12. The cycle for the data of the table is shown in Fig. 135.

The work per ton is about the same for all refrigerants excepting carbon dioxide. The work for carbon dioxide is high because *G* on the carbon dioxide *TS* diagram falls

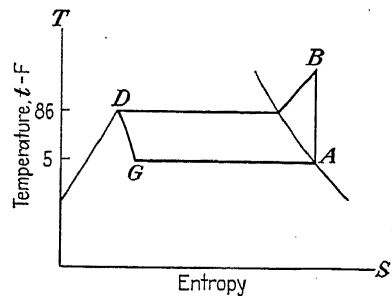


FIG. 135.

close to *A*. The coefficient of performance is likewise about the same for all refrigerants of the table except carbon dioxide. Methylene chloride requires low pressures and a very high volume rate per ton, making centrifugal compressors desirable.

Vapor substances offer two remarkable characteristics as refrigerants: (a) any desired low temperature can be produced by imposing upon the liquid the saturation pres-

sure corresponding thereto; and (b) heat can be absorbed from things to be cooled without rise of temperature.

165. Steam (H_2O) as a Refrigerant.—If water is placed in an open vessel, the pressure and also the temperature (nearly) will be those of the environing atmosphere, say 14.7 psia and 70 F, and there is no necessary relation between the two. Suppose it is desired to reduce the temperature of the water to 40 F. Let the vessel containing the water be closed, except for a connection from the space above, to an air pump, which is put into operation and begins to produce a vacuum. As the vacuum increases, no change in temperature of the water will be observed until the pressure falls below 0.3631 psia (corresponding to 70 F, from the Steam Tables). After that, each further increment in vacuum will be accompanied by a decrease in temperature, until, when the pressure reaches 0.1217 psia (Steam Tables), the desired temperature of 40 F will have been attained. Thus, by a purely mechanical means, a temperature lower than that of environment can be produced, using water as the working medium.

The experiment with water differs in no fundamental way from that illustrated in Figs. 132 and 133 in connection with ammonia. The pressures to be imposed upon the liquids in the two cases, to obtain the temperature of 40 F, are, however, widely different—0.1217 psia (about 29.75 in. vacuum) for water, and 73.3 psia for ammonia. At 40 F, however, 1 lb of ammonia vapor occupies 3.97 cu ft, while 1 lb of steam (water vapor) occupies 2444 cu ft. It is evident that in order to bring water to 40 F and maintain that temperature, a very good vacuum pump and one of high volumetric capacity is required.

Notwithstanding this difficulty, the water-vapor refrigeration machine has had a remarkable development, and its principle is now successfully employed in commercial machines in capacities up to 1000 tons and more. Two factors are responsible, (a) the development of the steam-jet compressor; and (b) the creation of a new field

of refrigeration, by the widespread demand for conditioned air, for which temperatures of refrigerant in the range of 40 F to 55 F are perfectly satisfactory.

A water-vapor system is illustrated by Fig. 136. The main feature is the kinetic compressor or ejector, consisting of a battery of small steam nozzles, *B*, and Venturi tube *D*. The ejector maintains a suction pressure of 0.1475 psia (45 F) in the evaporator *A* and discharges into the condenser where a pressure of 1 psia is maintained by an ordinary vacuum pump. Some of the returning water, upon passing the float-controlled valve *V*, is "flashed" into

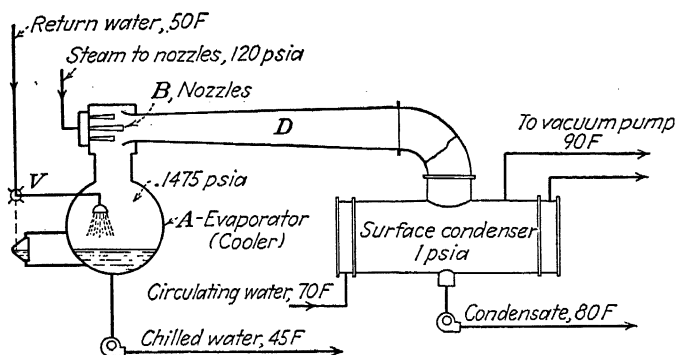


FIG. 136.

steam, thus cooling the remainder from 50 F to 45 F, an operation like that represented by *EG*, Fig. 130. The water thus chilled is itself circulated to the place of use.

The temperature of the steam (resulting from evaporation) is raised to 102 F by compression, thus making it possible to condense it (together with the motive steam used in the nozzles) by circulating water supplied at 70 F. The condensate is then pumped into the atmosphere with little expenditure of work.

A centrifugal steam compressor has been developed by one manufacturer to serve the same purpose as the steam-jet compressor of Fig. 136. With mechanical compression, only the steam evaporated in *A*, Fig. 136, must be condensed, whereas, with the jet compressor, the nozzle

steam (several times the amount of that from the evaporator) must also be condensed, thus requiring more cooling water.

166. The Ammonia Absorption Machine.—Referring to the description of the ammonia compression plant, Art. 153 and Fig. 129, it is apparent that the real effect of the whole system is accomplished in the refrigerating coils. All the rest of the apparatus is auxiliary to this and may be considered as merely participating in the restoration of the ammonia to the liquid state, at low pressure, so that it can perform its function over again in the refrigerating coils.

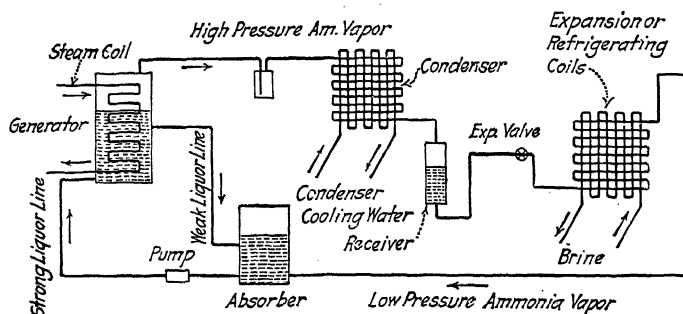


FIG. 137.

In the *absorption machine*, compression is effected almost wholly by thermal processes, by taking advantage of the remarkable capacity of water to absorb ammonia at moderate temperatures and the rapid reduction of this capacity with increase of temperature. Figure 137 illustrates the absorption plant in a general way. Heat, applied in the generator by the steam coil, drives ammonia vapor out of water in which it has been dissolved. The condenser, expansion valve, and refrigerating coils are no different from those of the compression system. The ammonia vapor, after evaporation in the refrigerating coils, is conducted to the absorber, where it goes into solution in water at low pressure. This solution, called the *strong liquor*, is pumped into the generator, where the high pressure

prevails, and the cycle is begun over again. In the meantime, the water in the generator, from which the ammonia has been driven off, now called the *weak liquor*, is conveyed to the absorber, to receive its charge of ammonia again.

The pressures in an absorption system are like those of Fig. 129 because ammonia is the refrigerant. Compression is effected by dissolving the gas, then raising the pressure of the liquid, and disengaging the ammonia by heat at the higher pressure. Little mechanical energy is needed, but a large amount of heat is required in the generator, which also adds to the burden of heat rejection at the absorber. In one type of domestic refrigerator, even the liquid pump is discarded and elevation of pressure is accomplished by the principle of *partial pressures*, an inert gas (hydrogen) being introduced to share total pressure with the ammonia during a part of its cycle.

167. The Solution Circuit.—In the absorption system, a double fluid circuit is involved, as may be readily seen by further reference to Fig. 137. There is first the *pure-ammonia circuit* from generator around to absorber. Then there is the *ammonia-solution circuit* from generator to absorber and return. The two circuits are common in that section from absorber to generator, where the ammonia of the first is merged with the solution of the second. The *solution circuit* is illustrated in greater detail in Fig. 138, where the necessary auxiliary apparatus is represented as well as the main elements.

The *analyzer* consists of a number of trays located in the vapor space of the generator. The strong solution returning to the generator at *R* is discharged upon the trays and trickles down through them. The down-flowing liquid, meeting with hot vapor coming up, loses a large part of its ammonia before joining with the liquid of the generator. The analyzer is a kind of preboiler, in which a considerable part of the ammonia is vaporized out of the rich solution by the heat released from the vapor mixture arising from the liquid below.

The *rectifier* is a precondenser whose purpose is to separate steam from ammonia vapor. The condensation temperature of steam is higher than that of ammonia vapor, so that the steam succumbs first to condensation, which is drained off by the pipe *L*, while the relatively steam-free ammonia vapor passes on to the condenser of Fig. 137, where it is reduced to the liquid state for refrigeration.

The generator is to be kept hot and the absorber cool. If the cool strong solution were returned to the generator

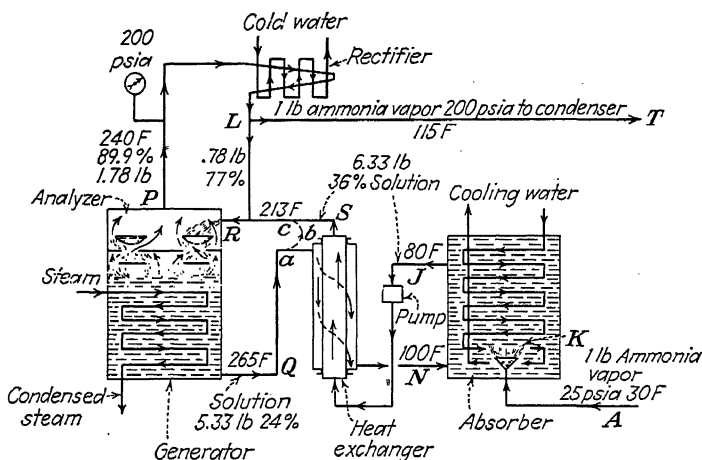


FIG. 138.

without preheating, a considerable amount of heat would have to be expended in warming the liquid to the boiling temperature. On the other hand, if hot weak solution were passed directly from generator to absorber without precooling, the latter would have to remove this extra enthalpy brought over from the generator. The counter-flow *heat exchanger* of Fig. 138 serves both as a preheater for the generator and a pre cooler for the absorber. The exchanger, in effect, strains the heat out of the liquid and returns it to the generator as suggested by the dotted line *abc*.

168. Temperatures and Pressures of the Solution.—

A solution of ammonia in water has a definite boiling (or

Pressure, psia	Concentration, per cent	Temperature of solution*	Saturation temperature of	
			Steam	Ammonia vapor
25	24	128.7		
25	36	88.4	240	-8
200	24	264.5		
200	36	216.7	382	96

* JENNINGS, B. H., and F. P. SHANNON, Tables of the Properties of Aqua-ammonia Solutions, *Refrig. Eng.*, May, 1938. Also published in "Science and Technology Series, No. 1," Lehigh University.

condensing) temperature for each different pressure, being in this respect similar to each of its pure constituents acting alone. As might be expected, this temperature, for a particular pressure, falls between those for anhydrous ammonia and water. The stronger the concentration of ammonia, the more nearly will the boiling temperature of the solution coincide with that of pure ammonia. The term *concentration* means the proportion of ammonia (by weight) in the mixture and is usually expressed in per cent. For the pressures and concentrations indicated in Fig. 138, the solution temperatures, together with those of saturated water vapor and ammonia vapor, are given in the table at the top of this page. By the term *temperature of solution* is meant the temperature at which ammonia will boil out of, or condense into, the solution.

169. Properties of Aqua-ammonia.—The properties of aqua-ammonia solutions may be read from the curves of Fig. 139. From the temperature curves, one may read the temperature at which a solution of the concentration and pressure given is saturated, *i.e.*, boils under the pressure in question. The enthalpy of the saturated liquid aqua, h_f , for various pressures and concentrations may be read from the bottom set of curves. A similar set of curves for the enthalpy of the saturated vapor, h_v , in equilibrium with the saturated liquid aqua appears at the top of Fig. 139. The other set of curves at the top shows

the weight percentage ammonia concentration, x_v , of the saturated vapor in equilibrium with the saturated liquid

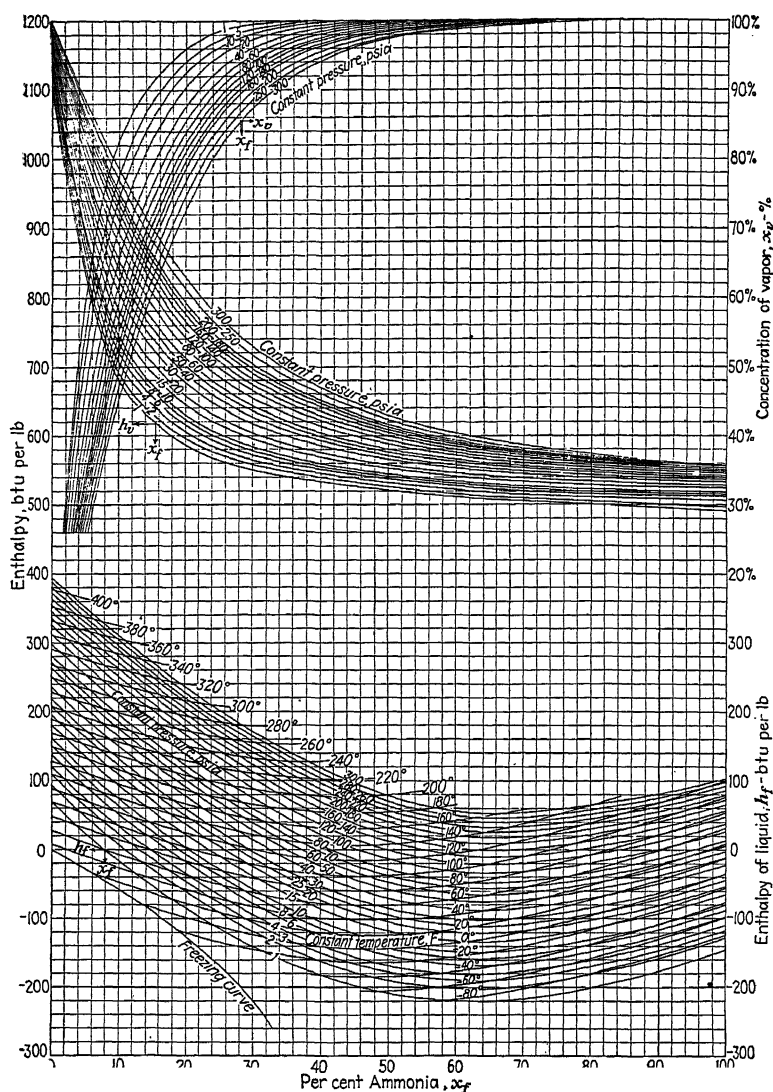


FIG. 139.

aqua. If a given aqua solution of per cent concentration x_f and pressure p is heated to the saturation temperature t ,

it will boil and have an enthalpy h_f . The vapor rising from this saturated solution will have an enthalpy h_v and an ammonia concentration x_v . Neglecting the difference of pump work, the enthalpy of a subcooled solution at a given concentration is the enthalpy of a saturated solution of lower pressure and same temperature. For example, the enthalpy of a 24 per cent aqua ammonia solution at 0 F is -109.5 Btu per lb under a pressure of 1 psia or any higher pressure.

170. Heat of Solution.—In Fig. 138, ammonia vapor at 25 psia pressure is directed by the distributor K into the liquid of the absorber where it is condensed, just as steam is condensed in a jet condenser. In this case, however, a vapor whose liquefying temperature is -8 F is condensed in a liquid whose temperature is approximately 100 deg higher, and we have the seemingly impossible thermal operation of causing heat to flow from a lower to a higher temperature. Not only this, but there is liberated in the process an additional amount of heat, besides the latent heat of condensation, called the *heat of solution*, which varies inversely in magnitude with the percentage of concentration.

Heat of solution liberated in the absorber was stored in the solution in the separation of ammonia from water in the generator. Primary heat supplied by steam is received by the solution at temperatures ranging from 216.7 F to 264.5 F, or an average of 240 F. The saturation temperature of ammonia at 200 psia is 96 F (Table 10). Hence we have the phenomenon of vapor (ammonia) rising from its liquid (the solution), not in the saturated state, but *superheated* 144 degrees (240 F - 96 F). The average concentration in the generator is 30 per cent. From Fig. 139 the concentration of the vapor leaving is 89.9 per cent. The enthalpy of the vapor leaving the generator at 200 psia, 89.9 per cent concentration, and 240 F is 703.7 Btu per lb, Fig. 139. At the same time, the dilute solution carries with it to the absorber an amount of energy that is a

chemical form of energy and is therefore independent of temperature. When solution and ammonia vapor meet again in the absorber, this energy manifests itself as an affinity of such power that the vapor is redissolved and condensed in a liquid whose temperature is far above that normally related to the pressure of the vapor, and, in the process, energy is liberated as heat of solution. This energy of solution is not heat energy and cannot, therefore, be properly represented on a temperature-entropy diagram. Its liberation in the form of heat in the absorber is akin to the process of combustion.

171. Weight of Solution per Pound of Ammonia.—In the example of Fig. 138, the concentrations of the solutions entering and leaving the absorber are 24 and 36 per cent, respectively.

Let M = rate of flow of weak solution from generator to absorber per pound of ammonia circulated through the refrigerating system

$M + 1$ = rate of flow of strong solution from absorber to generator

$0.24 M$ = weight of ammonia in M lb weak solution

$0.36(M + 1)$ = weight of ammonia in $(M + 1)$ lb strong solution

Then

$$0.36(M + 1) - 0.24 M = 1 \text{ lb} \quad (66)$$

Whence

$M = 5.33$ lb weak solution per pound working ammonia and

$$M + 1 = 6.33 \text{ lb strong solution}$$

In the rectifier the vapor is cooled to 115 F. At this temperature nearly all the water is condensed out. At a pressure of 200 psia and a saturation temperature of 115 F, the concentration of the solution is 77 per cent and the concentration of the vapor 99.8 per cent, Fig. 139. There are, therefore, \bar{W} lb of 77 per cent aqua-ammonia leaving the rectifier at L and mixing with 6.33 lb of 36 per cent

aqua-ammonia to enter the generator at R . One pound of nearly anhydrous ammonia goes to the condenser at T . Evidently $\bar{W} + 1$ lb of 89.9 per cent aqua-ammonia leaves the generator at P . Then

$$0.899(\bar{W} + 1) = 1 + 0.77\bar{W} \quad (67)$$

whence,

$$\bar{W} = 0.783 \text{ lb}$$

leaving at L per pound of ammonia leaving at T .

172. Heat Liberated in Absorber.—Using the values of Fig. 138 as an example, the heat, Q_{Ab} , to be transferred to the cooling water in the absorber may be determined by writing a heat balance for the absorber.

$$1 \times h_A + M \times h_N = (M + 1) \times h_J + Q_{Ab} \quad (68)$$

Enthalpy values may be read from the Ammonia Tables and Fig. 139.

$$\begin{aligned} Q_{Ab} &= 1 \times 552.5 + 5.33 \times (-6.2) - 6.33 \times (-53) \\ &= 855 \text{ Btu} \end{aligned}$$

for each pound of ammonia used in refrigeration.

It is evident that the absorber must be provided with a liberal supply of cooling water to carry away this large amount of heat.

173. Heat to Be Supplied in Generator.—The amount of heat to be supplied by the steam in the generator can now be determined by a heat balance of the generator.

$$\begin{aligned} Q_{\text{Gen}} + (M + 1)h_s + \bar{W} \times h_L &= (\bar{W} + 1)h_P + M \times h_q \quad (69) \\ Q_{\text{Gen}} &= -6.33 \times 95.8 - 0.783 \times 52.8 + 1.783 \times 703.7 \\ &\quad + 5.33 \times 170 \\ &= -606.4 - 41.4 + 1254.7 + 906.1 = 1513 \text{ Btu} \end{aligned}$$

which is the amount of heat to be supplied by the steam in the coils of the generator for each pound of ammonia used in refrigeration.

Problems

1. Make tentative calculations for an air refrigerating machine illustrated by the diagram of Fig. 124. The maximum temperature of the air circulated

through pipes at 40 psia in the cold room is to be 35 F. The compressor will compress the air along the line $pV^{1.2} = C$, delivering at 300 psia. The compressed air is reduced in temperature, in the cooling receiver, to 85 F. Expansion then takes place to the lower pressure following the equation $pV^{1.35} = C$.

a. If 40,000 Btu/hr are to be removed from the cold room, how many pounds of air per minute must be circulated?

b. Disregarding clearances, what theoretical horsepower is necessary to run the compressor? How much is furnished by the expander? How much remains to be furnished by the motor?

c. Make a TS sketch to show the cycle, indicate the areas that represent the heat quantities received and rejected by the working air during the cycle, and calculate the magnitude of these quantities in Btu per hour.

2. Referring to Fig. 129, suppose the following observations of an ammonia refrigerating plant have been recorded:

At A , the pressure is 35 psia; the temperature, 10 F.

At B , the pressure is 160 psia; the temperature, 170 F.

At C , D , E , and H , the temperatures are 160, 80, 77, and 8 F, respectively.

a. From these data, and with the help of the tables of Properties of Ammonia, construct to scale a temperature-entropy diagram similar to Fig. 130.

b. For convenience, assume the compression line (Fig. 130) to be straight. Determine the following heat quantities per pound of ammonia:

Heat removed from ammonia by compressor-cylinder jacket.

Heat removed from ammonia by "radiation" from pipe line from compressor to condenser.

Heat removed from ammonia in condenser.

Heat removed from ammonia by radiation from liquid pipe line and receiver on the way from condenser to expansion valve.

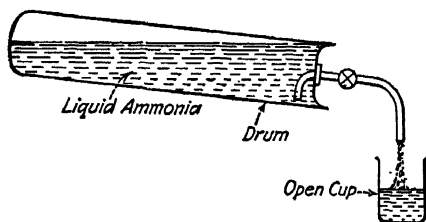
Heat received by ammonia in expansion coils, from brine.

Heat received by ammonia by warming effect of atmosphere on vapor pipe line from expansion coils to compressor.

Heat equivalent of work done by the compressor upon the ammonia.

c. If 625 lb of ammonia are circulated per hour, what is the refrigerating capacity of the plant in tons?

3. Ammonia liquid is drawn from a shipping drum into an open cup as illustrated by the accompanying figure.



What will be the temperature of the liquid in the cup, and the pressure in the drum, for each of the following conditions of atmospheric pressure and temperature in the room in which the apparatus is set up?

(a) 15 psia, 70 F; (b) 15 psia, 90 F; (c) 20 psia, 70 F.

For each case, determine the weight of liquid collected in the cup for each pound of liquid withdrawn from the drum.

4. In a region where the prevailing temperature is 280 F, it is desired to produce a temperature of 180 F by mechanical refrigeration using water as the refrigerant. If the water comes to the expansion valve at 285 F, evaporates at 160 F, and leaves expansion coils at 170 F, make a temperature-entropy sketch of the cycle and determine the Btu absorbed by each pound of steam.

5. A steam refrigerating system is to cool drinking water from 70 F to 50 F, at the rate of 200 lb/hr. Make diagram sketch of apparatus, and determine volume of vapor to be handled by the steam compressor in cubic feet per hour.

6. A steam-jet refrigeration machine is to chill water to 40 F, for cooling air, from which service the water returns at 50 F. How many pounds of "new" water for each 100 lb of chilled water will it be necessary to provide at 70 F to make up for the loss by evaporation?

7. In the example represented by Figs. 129 and 130, suppose, through inadequacy of cooling water supply, the ammonia gas is not completely liquefied in the condenser, and as a result approaches the expansion valve as a mixture of 60 per cent vapor and 40 per cent liquid by weight.

a. Illustrate cycle by temperature-entropy sketch.

b. Determine refrigeration in Btu per pound of ammonia for the above cycle, and compare with that for the cycle of Fig. 130 (see Art. 158).

c. In which of the two cycles is the more work required of the compressor per pound of ammonia? Explain.

8. In passing through the expansion valve, the ammonia changes in state from *E* to *G*, Fig. 130.

a. Is this an adiabatic operation? Why?

b. If, instead of the expansion valve, the pressure of the ammonia were reduced by letting it do work in a small engine in a Rankine cycle, in what way, if any, would the path be different from *EG*?

c. Would any useful work result (in case b) and if so, would the usefulness of the ammonia as a refrigerant be impaired?

d. Would the work of the compressor be increased or decreased?

9. a. What is the minimum generator steam gage pressure required for the data of the absorption system of Fig. 138?

b. If the temperature of the cooling water entering the condenser is 80 F, what is the minimum ammonia pressure on the high side of the expansion valve?

c. With the pressure of (b) and the concentrations of Fig. 138, what is the minimum steam-gage pressure required for the generator?

10. The following data were recorded during the test of an absorption refrigerating plant:

Pressure, ammonia in condenser.....	190 psia
Ammonia in absorber.....	40 psia
Ammonia in generator.....	190 psia
Steam in generator.....	25 psia
Temperature, ammonia gas into absorber.....	20 F
Ammonia to expansion valve.....	90 F
Strong aqua from absorber.....	95 F
Drip from rectifier.....	115 F
Weak aqua to absorber.....	115 F
Vapor from generator.....	220 F
Weak aqua from generator.....	220 F
Ammonia gas from rectifier.....	110 F
Cooling water entering condenser, absorber, rectifier.....	85 F
Cooling water leaving condenser, absorber, rectifier.....	95 F
Condensate from generator.....	230 F

Steam enters the generator dry and saturated. Assume no loss of heat to the surroundings of the heat exchanger. Ammonia passes through the expansion valve at a rate of 160 lb/hr. Concentration of weak aqua, 30 per cent; strong aqua, 40 per cent. Find the following:

- a. Pounds of steam per pound of ammonia.
- b. Pounds of cooling water to condenser per pound of ammonia.
- c. Pounds of cooling water to absorber per pound of ammonia.
- d. Pounds of cooling water to rectifier per pound of ammonia.
- e. Refrigerating effect per pound of ammonia.
- f. Tons of refrigeration.

CHAPTER XII

MIXTURES OF GASEOUS SUBSTANCES

174. Weight and Volume Relations.—Let a mixture of gaseous substances be made up of constituents a , b , c , etc. Then according to Dalton's law the total pressure, p_m , exerted by a mixture of gases is the sum of the partial pressures. A partial pressure is the pressure which one constituent, such as a , would exert if it alone occupied the whole volume.

$$\Sigma p = p_m = p_a + p_b + p_c + \dots \quad (70)$$

If a mixture of gases, a , b , c , etc., is contained in a volume V , each gas occupies the whole volume V , exerts a partial pressure p_a , p_b , p_c , etc., and all the constituents are at a temperature T . Then for any constituent such as a ,

$$p_a = \bar{W}_a R_a \frac{T}{V} \quad (71)$$

and from Eq. (70)

$$\begin{aligned} \Sigma p &= (\bar{W}_a R_a + \bar{W}_b R_b + \bar{W}_c R_c + \dots) \frac{T}{V} \\ &= \Sigma(\bar{W} R) \frac{T}{V} \quad (72) \end{aligned}$$

dividing Eq. (71) by Eq. (72)

$$\frac{p_a}{\Sigma p} = \frac{\bar{W}_a R_a}{\Sigma(\bar{W} R)} \quad (73a)$$

from Eq. (29)

$$R = \frac{1544}{M} \quad (29)$$

Then

$$\frac{p_a}{\Sigma p} = \frac{\bar{W}_a \bar{M}_a}{\Sigma \bar{W} \bar{M}} \quad (73b)$$

If R_m is a representative gas constant for the mixture and p_m is the total pressure of the mixture

$$\Sigma p = p_m = \bar{W}_m R_m \left(\frac{T}{V} \right) = (\Sigma \bar{W}) R_m \left(\frac{T}{V} \right) \quad (74)$$

Then from Eqs. (74) and (72)

$$(\Sigma \bar{W}) R_m \left(\frac{T}{V} \right) = \Sigma (\bar{W} R) \left(\frac{T}{V} \right)$$

and

$$R_m = \frac{\Sigma (\bar{W} R)}{\Sigma \bar{W}} \quad (75)$$

and

$$M_m = \frac{1544}{R_m} \quad (76)$$

If the molecules of a mixture could be sorted into compartments separated by movable partitions, Fig. 140, each constituent would have its own volume V_a , V_b , V_c , etc., and all constituents would be at the same pressure p and temperature T . Then for any constituent such as a ,

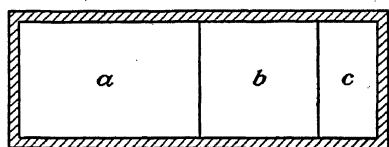


FIG. 140.

$$V_a = \bar{W}_a R_a \frac{T}{p} \quad (77)$$

and

$$\begin{aligned} \Sigma V &= V_a + V_b + V_c + \dots \\ &= (\bar{W}_a R_a + \bar{W}_b R_b + \bar{W}_c R_c + \dots) \frac{T}{p} = \Sigma (\bar{W} R) \frac{T}{p} \quad (78) \end{aligned}$$

dividing Eq. (77) by Eq. (78)

$$\frac{V_a}{\Sigma \bar{V}} = \frac{\bar{W}_a R_a}{\Sigma (\bar{W} R)} \quad (79a)$$

and from Eq. (29)

$$\frac{V_a}{\Sigma \bar{V}} = \frac{\frac{\bar{W}_a}{M_a}}{\Sigma \frac{\bar{W}}{M}} \quad (79b)$$

Solving Eq. (77) for \bar{W}

$$\bar{W}_a = \frac{V_a p}{R_a T}$$

Then

$$\begin{aligned} \Sigma \bar{W} &= \bar{W}_a + \bar{W}_b + \bar{W}_c + \dots \\ &= \left(\frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} + \dots \right) \frac{p}{T} = \Sigma \frac{V}{R} \left(\frac{p}{T} \right) \end{aligned} \quad (80)$$

and

$$\frac{\bar{W}_a}{\Sigma \bar{W}} = \frac{\frac{V_a}{R_a}}{\Sigma \frac{V}{R}} \quad (81a)$$

or

$$\frac{\bar{W}_a}{\Sigma \bar{W}} = \frac{V_a M_a}{\Sigma (V M)} \quad (81b)$$

Letting $\Sigma \bar{W} = \bar{W}_m$ and $\Sigma V = V_m$

$$\bar{W}_m = \frac{V_m}{R_m} \left(\frac{p}{T} \right) = \frac{\Sigma V}{R_m} \left(\frac{p}{T} \right) = \Sigma \bar{W} \quad (82)$$

From Eqs. (82) and (80)

$$\frac{\Sigma V}{R_m} \left(\frac{p}{T} \right) = \Sigma \frac{V}{R} \left(\frac{p}{T} \right)$$

and

$$R_m = \frac{\Sigma V}{\Sigma \frac{V}{R}} \quad (83)$$

or

$$\frac{1544}{M_m} = \frac{\Sigma V}{\frac{\Sigma(VM)}{1544}} \quad \text{and} \quad M_m = \frac{\Sigma(VM)}{\Sigma V} \quad (84)$$

Equation (79) enables one to convert from a weight or gravimetric basis to a volumetric basis, and Eq. (81) enables one to convert from a volumetric basis to a gravimetric basis.

Suppose the mixture, air, is considered in terms of its two principal constituents, nitrogen and oxygen. Its composition by volume is nitrogen 79.3 per cent and oxygen 20.7 per cent. What is its composition by weight, apparent molecular weight of the mixture M_m , gas constant R_m , and density ρ at 14.7 psia and 32 F? By Eq. (81b)

Gas	Per Cent by Volume, V	\times	M	$= VM$	$\frac{\bar{W}_{N_2}}{\Sigma \bar{W}} = \frac{2220}{2882} = 77 \text{ per cent}$
N ₂	79.3	\times	28	$= 2220$	
O ₂	20.7	\times	32	$= 662$	$\frac{\bar{W}_{O_2}}{\Sigma \bar{W}} = \frac{662}{2882} = 23 \text{ per cent}$
	$\Sigma V = 100.0$			$\Sigma(VM) = 2882$	

from Eq. (84)

$$M_m = \frac{2882}{100} = 28.82^*$$

from Eq. (29)

$$R_m = \frac{1544}{28.82} = 53.58^*$$

$$\rho = \frac{\bar{W}}{V} = \frac{p}{RT} = \frac{14.7 \times 144}{53.58 \times 492} = 0.0805^* \frac{\text{lb}}{\text{cu ft}}$$

175. Specific Heats of Mixtures.—The specific heat of a mixture is easily determined if the proportions of the several constituents by weight and their specific heats are known.

$$c_{p_m} = \frac{\Sigma(\bar{W}c_p)}{\Sigma \bar{W}} \quad (85)$$

For example, what is the specific heat at constant pressure

* This value differs slightly from the value in Table 6 because carbon dioxide, argon, and other rare gases were neglected.

of air, considered as a mixture of N_2 and O_2 , for temperatures not exceeding 400 F?

Gas	\bar{W} Per Cent by Weight	\times	Specific Heat c_p Table 6	$=$	$\bar{W}c_p$
	77	\times	0.247	$=$	19.02
O_2	23	\times	0.217	$=$	4.99
	$\Sigma \bar{W} = 100$			$\Sigma (\bar{W}c_p) =$	24.01

c_p for mixture $= 24.01 \div 100 = 0.240$.

176. Mixture of Air and Water Vapor.—Besides air, which is the chief constituent, the atmosphere always contains some water vapor. In a sample of the atmosphere, the total (barometric) pressure p_t is the sum of p_a , the partial pressure of the air, and p_H , that of the vapor, which is really superheated steam.

The partial pressure of the steam, p_H , is conveniently determined from observations taken with an instrument called a *psychrometer*, consisting of two thermometers—one with its bulb uncovered, to register actual or *dry-bulb temperature* of the atmosphere, and the other with its bulb covered with a thin cloth sack, moistened with water, to register *wet-bulb temperature*.

From readings of barometer and dry-bulb and wet-bulb thermometers, the partial pressure of the steam in the atmosphere can be determined by Eq. (86), which was developed by Carrier¹ from thermodynamic considerations (see Art. 186).

$$p_H = p_s' - \frac{(p_t - p_s')(t - t')}{2755 - 1.28t'} \quad (86)$$

where p_H = partial pressure of the steam

p_s' = pressure corresponding to temperature t' as found in Saturated Steam Tables

p_t = total (barometric) pressure

¹ *Trans. Amer. Soc. Mech. Eng.*, vol. 33, p. 1005. This article as well as other material of this chapter are based upon Carrier's excellent work. The constants (2755 and 1.28) of Eq. (86) are slightly different from those resulting directly from derivation but are given by Carrier to correct for a 1.6 per cent radiation error in the quantity $(t - t')$ as determined by the sling psychrometer.

These pressures are usually expressed in pounds per square inch, but any units may be used (often inches of mercury) provided all three are in like *absolute* units.

t and t' = dry-bulb and wet-bulb temperatures, F

With p_H determined by Eq. (86), p_a is obtained by the relation $p_a = p_t - p_H$. Then from the gas equations, the weight of each constituent in *one cubic foot* can be found.

$$\rho_H = \frac{(p_H \times 1)}{(R_s T)} \quad \text{and} \quad \rho_a = \frac{(p_a \times 1)}{(R_a T)} \quad (87)$$

T is the common temperature, while R_s and R_a are the gas constants whose values are given in Table 6, Art. 100. p_H and p_a must here be expressed in pounds per square foot.

In Eq. (87), the gas law is applied to steam, which we ordinarily would not expect to conform very closely to the perfect gas laws. But as a matter of fact, at the very low (partial) pressures at which it exists in the atmosphere, steam behaves like a perfect gas and the gas equations apply with accuracy.

177. Dew Point and Saturation Point.—With temperature known, and pressure determined by Eq. 86, the state of the *steam* may be represented on a temperature-entropy sketch as in Fig. 141. The state point must lie somewhere

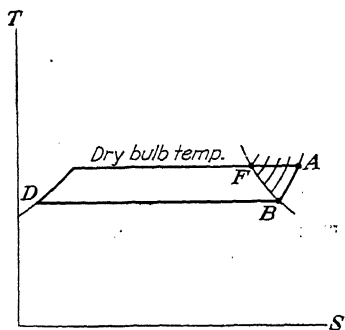


FIG. 141.

on the dry-bulb temperature line and also somewhere on the constant-pressure line p_H , represented by DBA . The actual state is indicated by the intersection of these two lines at point A , which is located in the superheat region. The vertical distance from DB to FA represents the degrees of superheat.

Imagine steam at state A to be confined in a cubic-foot vessel by itself, with a weighted lid acting as a piston. If heat is withdrawn, there will be a constant-pressure cooling,

represented on Fig. 141 by AB . Condensation will begin at B , which is called the *dew point*, and the temperature at B , which is that corresponding to pressure p_H , is the *dew-point temperature*.

If the piston lid of the vessel containing the steam at state A is pushed down while temperature is kept constant, the compression is isothermal and is represented by the path AF , which crosses successively higher and higher pressure lines in its progress to the left. Exactly the same effect will result if the volume of the vessel is kept fixed, while additional steam at the same (dry-bulb) temperature is crowded into it. In either case, the pressure of the steam (partial pressure in the mixture) keeps on increasing until it may equal that corresponding to the dry-bulb temperature (at F). Then any further compression, or crowding in of steam, *with temperature constant*, is represented by progress of the state point to the left from F , which means that condensation takes place. The point F represents the maximum weight of steam as *steam* (vapor) that can be contained in the cubic-foot volume at the dry-bulb temperature. The steam (and by general usage of the term the air that contains it) is then in the saturated state, and F of Fig. 141 is the *saturation point*.

178. Humidity.—The amount of steam in the atmosphere is designated by the term *humidity*. The *weight of steam per cubic foot*, ρ_H , at the actual observed state A of Fig. 141, is called the *absolute humidity* or *density of steam* and is expressed in pounds or more usually in grains. There are 7000 grains in a pound.

The maximum amount of steam that 1 cu ft can contain *at the dry-bulb temperature* is ρ_s , the density of steam at saturation or the weight of 1 cu ft of *saturated steam* (point F , Fig. 141). The ratio ρ_H/ρ_s is called the *relative humidity*.

For the same volume $V = 1$ cu ft of steam at states A and F (Fig. 141),

$$p_H V = \rho_H R_s T \quad \text{and} \quad p_s V = \rho_s R_s T$$

Dividing the first equation by the second,

$$\frac{p_H}{p_s} = \frac{\rho_H}{\rho_s}$$

which is relative humidity. Hence, *relative humidity* may also be defined as the *ratio of the steam pressures*, at the actual state *A* and at the saturated state *F* (Fig. 141). Or

$$\text{Relative humidity} = \frac{p_H}{p_s} \quad (88)$$

A few examples will illustrate how partial pressures, humidity, and weights of steam and air in the atmosphere can be determined.

From readings of psychrometer and barometer, the dry-bulb temperature is found to be 80 F; wet-bulb 70 F; and barometric pressure 14.45 psia.

a. What is the partial pressure of steam and air, and what is the dew-point temperature?

In Carrier's equation, Eq. (86), Art. 176,

$$t = 80 \text{ F}, \quad t' = 70 \text{ F}, \quad p_t = 14.45, \quad \text{and} \quad p_s' = 0.3631 \text{ psia}$$

(pressure corresponding to $t' = 70 \text{ F}$, from Steam Tables¹)

$$p_H = 0.3631 - \frac{(14.450 - 0.3631)(80 - 70)}{2755 - 1.28 \times 70} = 0.3108 \text{ psia}$$

and

$$p_a = 14.4500(p_t) - 0.3108(p_H) = 14.1392 \text{ psia}$$

Dew-point temperature is that corresponding to pressure $p_H = 0.3108$ and, from the Steam Tables, is found to be 65.5 F. The steam is superheated $80 \text{ F} - 65.5 \text{ F} = 14.5 \text{ deg}$.

b. What is the weight of steam, air, and mixture in 1 cu ft of the atmosphere?

From Eq. (87), Art. 176,

$$\rho_H = \frac{p_H}{(R_s T)} = \frac{(0.3108 \times 144)}{(85.7 \times 540)} = 0.000967$$

$$\rho_a = \frac{p_a}{(R_a T)} = \frac{(14.1392 \times 144)}{53.34 \times 540} = 0.070686$$

and ρ_s , the total weight of mixture in 1 cu ft = 0.071653.

¹ For cases involving temperatures below 32 F, a table of "Properties of Saturated Water Vapor at Low Temperatures" will be found in the *Amer. Soc. Heat. Vent. Eng. Guide*. At such temperatures, a wetted bulb becomes coated with ice, but equilibrium between the pressure of the vapor in the atmosphere and that of sublimation from the ice gives the wet-bulb temperature.

The *relative* weights of steam and air in the atmosphere (not the relative humidity) are:

$$\text{Air, } \left(\frac{\rho_a}{\rho_t} \right) = \frac{0.070686}{0.071653} = 0.9865 \text{ or } 98.65 \text{ per cent}$$

c. What is the absolute and the relative humidity? The absolute humidity is the weight of steam ρ_H , in 1 cu ft, and from (b) $\rho_H = 0.000967$ lb or $0.000967 \times 7000 = 6.77$ grains.

Relative humidity = ρ_H/ρ_s , where ρ_s = weight of saturated steam per cubic foot at $t = 80$ F, = $1/633.1 = 0.001578$ (from Steam Tables). Whence relative humidity = $(0.000967/0.001578) \times 100 = 61.3$ per cent. Also, from Eq. (88), relative humidity = p_H/p_s . $p_H = 0.3108$ (from a), and $p_s = 0.5069$ (from Steam Tables, corresponding to 80 F). Whence relative humidity = 0.613 , or 61.3 per cent as before.

It is often necessary to find the dew-point temperature from a given atmospheric temperature and relative humidity (H_R). Thus if $t = 75$ F, and $H_R = 50$ per cent, what is the dew-point temperature?

$$H_R = \frac{p_H}{p_s} \quad \text{or} \quad p_H = (H_R) \times p_s$$

p_s at 75 F = 0.4298 ; $p_H = 0.50 \times 0.4298 = 0.2147$. From Steam Tables, dew-point temperature, corresponding to $p_H = 0.2147$, is 55 F.

179. Definition of Terms.—The subject of mixtures of vapor and gas, as represented by that of steam and air, is rendered much more difficult than it should be because of the terms that are employed in connection therewith, which are undescriptive and often misleading. It will be helpful therefore, to define these terms in connection with their physical significance.

Atmospheric air consists of a mixture of which the two chief constituents are pure air and *steam*. “Water vapor” or “moisture” is the term usually employed to designate the steam constituent, but this, while technically correct, does not convey to the engineer the definite conception that the word “steam” conveys.

The steam constituent of the mixture is *superheated*. The evidence of this is the fact that water can be evaporated from an exposed wet surface. If the steam of the mixture were saturated, then no more steam could be contained in a given space at the same temperature with-

out a corresponding condensation or precipitation occurring at the same time.

Nearly all the terms employed in describing the state of the mixture of steam and air are really descriptive of, or applicable to, the *steam only*. "Saturated air" means a mixture of air and *saturated steam*. "Dew point" refers to the vaporizing temperature corresponding to the *partial pressure of the steam*. "Absolute humidity" is the *weight of the steam per cubic foot*. "Relative humidity" is the ratio of the *actual weight of the steam per cubic foot* to the *maximum weight of steam* (as *steam*, not a mixture of steam and water) that could be contained in the same space at the *same temperature*.

180. Evaporation vs. Boiling.—Water placed in an open vessel in the atmosphere at ordinary temperature will evaporate and disappear as steam. Accustomed as we are to associate temperature of evaporation with pressure exerted upon the surface of the water, it is somewhat difficult to understand how evaporation can take place at a temperature far below that corresponding to atmospheric pressure. The explanation depends upon the fact that when steam above a body of water is mixed with another gas *the vaporizing temperature is associated with the partial pressure of the steam and not with the total pressure of the mixture*.

As a first step in an approach to an explanation of this principle, let us imagine a closed chamber completely evacuated of air. In this chamber is placed an open vessel containing a quantity of water. If the temperature of the water is, let us say, 80 F, then we know that the chamber is filled with saturated steam at 80 F, and from the Steam Tables the pressure is found to be 0.5069 psia. Water surface and steam space are in exact equilibrium with respect to temperature and pressure. If any molecules escape from the water into the steam space, the temperature of the water surface (and, therefore, its molecular pressure) is slightly lowered by evaporation;

while, at the same time, the accumulation of more molecules in the steam space will slightly increase the vapor pressure therein. The result of such a momentary excess of vapor pressure would be to force some molecules back into the water and thus restore the equilibrium of pressure.

As a second step, suppose this vessel, in which the temperature is 80 F and the pressure 0.5069 psia, is opened to the atmosphere, whose temperature is also assumed to be 80 F and whose pressure is, say, 14.50 psia. Atmospheric air now crowds into the vessel, and the pressure exerted upon the surface of the water is then 14.50 psia. If we assume the atmosphere to consist of pure dry air, then the number of molecules of steam filling the vessel has not been changed in the least by the invasion of the swarm of air molecules. The steam pressure, which is now a *partial pressure*, is still 0.5069 psia, as before, and is still in exact equilibrium with the molecular pressure at the surface of the water, since the temperature of the latter has not been changed from 80 F. The steam, although mixed with air, is in the *saturated state* as it was before mixing. If the atmosphere admitted to the vessel already contains some steam, then the number of steam molecules in the vessel is greater than before, and the partial pressure of the steam is momentarily in excess of that of the water at 80 F. The equilibrium of pressure could be reestablished only by the condensation of some of the steam into the water or the coalescence of steam molecules into fine droplets or mist.

As a third step in our development, let another opening be made in our chamber, and let atmospheric air at 80 F be caused to circulate through the space above the water. The current will sweep out some of the steam molecules, thus reducing the partial pressure of the steam, which will then be lower than the vapor pressure of water at 80 F. As a result, the water will begin to evaporate in an endeavor, so to speak, to make up for the deficiency of vapor in the space above, with a consequent decrease in temperature of water. But as soon as the temperature of the water falls

below 80 F, it begins to receive heat from the air in contact with its surface, and in due time a new equilibrium temperature will be established at which the rate of heat reception by the water from the air will just balance the loss of heat by evaporation. This new temperature is the *wet-bulb temperature* for the conditions specified and is that which would be observed by a thermometer immersed in the water.

In this last case, we have the phenomenon of *evaporation* occurring at a temperature lower than 80 F, although the total pressure on the surface of the water is that of the atmosphere. It is purely a surface effect since no steam can be formed in the interior of the mass of water because the pressure there (the total pressure) is too great to permit the release of molecules and the formation of bubbles. At the surface, however, there is opportunity for molecular interchange between the water and steam because of the *porosity* of the superincumbent air.

In order to *boil* water under a total pressure of 14.50 psia, it is necessary to increase its molecular activity by the application of heat to bring its temperature up to about 211 F. The velocity of molecules will then be so great that escape from the liquid mass may occur anywhere in the body of the water despite the pressure, and bubbles of steam will be formed that rise to the surface and escape. This is the phenomenon of *boiling* or *ebullition*. When boiling occurs in the atmosphere, the outflow of steam from water sweeps away the air and the total pressure on the surface is that of the steam alone; or, more correctly speaking, the boiling maintains a zone of pure steam between the surface of the water and the environing atmosphere, and the pressure of the latter is transmitted through the steam zone to the water.

181. The Wet-bulb Temperature.—If a water surface is presented to the atmosphere by means of a wetted cloth sack surrounding the bulb of a thermometer, the reaction at the surface of the water will be exactly the same as that

described in the preceding article in which an open vessel containing water was used in illustration. The temperature finally attained will be that at which the vapor pressure of the water is in equilibrium with the pressure of the steam in the immediate vicinity; *i.e.*, when the condition of saturation exists there. Carrier calls this *the temperature of evaporation from a free surface, or the temperature of adiabatic saturation*, and these terms are synonymous with the true *wet-bulb* temperature. The term *adiabatic* as here used means that the thermal processes involved occur wholly within the interacting substances without any gain or loss of heat from or to an outside body. The true wet-bulb temperature of a free atmosphere depends entirely upon the state of the steam in the mixture. Thus, if the steam content is saturated, the temperature indicated by a wet-bulb thermometer will be the same as that shown by a dry bulb. On the other hand, if the steam content is in a highly superheated state, the wet-bulb depression (below the dry bulb) will be a large amount. The true wet-bulb temperature can be calculated from a knowledge of the *state* of the atmosphere and is therefore independent of factors that influence the *rate* of evaporation of water, such as velocity of current over wetted surface. Clothes hung out to dry will dry faster in a wind than in a quiet atmosphere, but the temperature of the fabric, if it were measured, would be found alike in the two cases, except for certain incidental factors that will be referred to later. The greater the velocity of current over a wetted surface, the greater the mass of air to give up heat and the greater the rate of evaporation. Rate of evaporation, however, does not determine the temperature of evaporation. This fact seems contrary to common personal experience, because when the human body is covered with perspiration, a very much lower temperature is perceived in a breeze than in quiet air. The temperature experienced in this case, however, is not the true wet-bulb temperature, because the operation is not adiabatic. The body itself is contributing

heat to the process: The temperature attained at the surface is, therefore, dependent upon the rate of evaporation. The greater this rate as produced by air movement, the less significant, relatively, is the heat contributed by the body, and the more nearly does the temperature of the surface approach that of a true wet bulb.

182. Steam Associated with 1 Lb of Dry Air.—Until now the cubic foot has been considered the basic quantity in our developments, and this is entirely satisfactory in the handling of problems like that illustrated by the example of Art. 178, where only one condition or state of the atmosphere is involved. But in those numerous practical problems in air conditioning and drying, where the atmosphere is employed as a thermal vehicle, a far more satisfactory basic quantity is the *pound of dry air*. When a current of atmospheric air is blown over a wetted surface, the weight of steam is increased by evaporation; the weight of air *in a cubic foot* changes because of a redistribution of partial pressures and change of temperature; and the total weight of air and steam is likewise altered. But a pound of *air* (usually designated as *dry air* to distinguish it from “atmospheric air”) is a quantity that remains unchanged throughout its progress. The weight of steam associated with a pound of dry air, at any moment, or particular condition, is called the *specific humidity*.

For the steam-air mixture:

$$\bar{W}_H = \frac{(p_H V_H)}{(R_s T_H)}$$

and

$$\bar{W}_a = \frac{(p_a V_a)}{(R_a T_a)}$$

Dividing the first expression by the second, and canceling common terms ($V_H = V_a$ and $T_H = T_a$), we get, for $\bar{W}_a = 1$ lb,

$$w_H(\text{lb}) \quad \frac{R_a p_H}{R_s p_a} = \left(\frac{53.34}{85.7} \right) \left(\frac{p_H}{p_a} \right) = 0.622 \left(\frac{p_H}{p_a} \right) \quad (89a)$$

where w_H is the weight of steam associated with 1 lb of dry air and p_H and p_a are partial pressures. If w_H is to be expressed in grains,

$$w_H(\text{grains}) = 7000 \times 0.622 \left(\frac{p_H}{p_a} \right) = 4354 \quad (89b)$$

Let w_s represent the weight of steam associated with 1 lb of dry air when the air is saturated. Then by definition, the ratio w_H/w_s is the *percentage humidity by weight*. It should be noted that the percentage humidity by weight w_H/w_s is not equal to the relative humidity $\rho_H/\rho_s = p_H/p_s$ defined in Art. 178.

183. Adiabatic Absorption of Moisture.—Imagine an insulated passageway or duct in which are placed a series of open pans containing water, not connected to a supply. Atmospheric air at t dry-bulb and t' wet-bulb temperature is drawn into the passage by a fan. If temperatures of air and water are measured at successive points along the duct, the following interesting facts will be observed:

a. Temperature of the water in all the pans will be t' —the wet-bulb temperature of entering air.

b. Wet-bulb temperature of the air is constant at t' .

c. Dry-bulb temperature decreases with progress along the passage, toward t' as a minimum, which will be reached if the passage is long enough, or if time is allowed, to permit complete saturation.

d. Dew-point temperature, if determined, will be found to increase toward the limit of t' .

e. The rate of evaporation of water from upstream pans is more rapid than that from those downstream.

We have here an example of *adiabatic absorption of moisture* by air, which means that all the heat used in evaporating water comes from the sensible heat¹ (enthalpy

¹ *Sensible heat* is a term referring to heat added or removed while the temperature is changing. For gases and vapors, it is enthalpy of superheat and for liquids, enthalpy of the liquid. The term *sensible heat* is commonly used in the thermodynamics of air conditioning.

of superheat) of the atmosphere, no other heat quantities participating. The case is like those described in Arts. 180 and 181, only here the entire amount of air flowing takes part in the thermal interchange. The water in the pans is really a great wet bulb.

Adiabatic absorption of moisture is closely approached when air is cooled and humidified by passing it through a series of water sprays. The water is recirculated at the constant temperature of the wet bulb, and (except for "make-up") the water which does not evaporate neither loses nor gains sensible heat (enthalpy of the liquid). The water which evaporates loses its latent heat.

The drying of material is not an adiabatic absorption of moisture, because a free water surface is not offered, and the sensible heat of the material being dried will become a factor. Likewise, the cooling of water in spray ponds and cooling towers is not an adiabatic operation, because the water contributes some of its sensible heat (enthalpy of the liquid) to the process.

184. Enthalpy of Steam-air Mixtures.—The enthalpy of 1 lb of air and the water vapor associated with it is the sum of the enthalpy of the air and the enthalpy of the water vapor. The datum for enthalpy of air is commonly taken as 0 F, and the datum for water vapor as 32 F. Then the enthalpy of dry air,

$$h_a = c_{p_a}(t - 0) = 0.24t \text{ Btu per lb of dry air}$$

and the enthalpy of the water vapor associated with 1 lb of dry air is

$$h_H = w_H(h_f'' + h_{fg}'' + h_{su}'') = w_H[h_f'' + h_{fg}'' + c_{p_H}(t - t'')] = w_H h_H''$$

where w_H = weight of steam associated with 1 lb of dry air

t = dry-bulb temperature, F

t'' = dew-point temperature, F

c_{p_a} = specific heat at constant pressure of dry air

c_{pH} = specific heat at constant pressure of superheated steam

h_f'' = enthalpy of liquid

h_{fg}'' = enthalpy of evaporation

h_{su}'' = enthalpy of superheat

h_H'' = total enthalpy of superheated steam at the dry bulb temperature t and pressure p_H (corresponding to t'')

The enthalpy of the mixture becomes

$$h = h_a + h_H = 0.24t + w_H h_H'' \quad (90a)$$

At the limiting state of complete saturation at wet-bulb temperature t' , in an adiabatic process (as in the insulated duct illustration, Art. 182) the steam-air mixture leaves with a dry-bulb temperature t equal to the wet-bulb temperature t' and there is no superheat. Then

$$h = 0.24t' + w_H'(h_f' + h_{fg}') \quad (90b)$$

where h_f' and h_{fg}' are the enthalpies of the liquid and evaporation at the wet-bulb temperature t' . The enthalpy as given by Eq. (90b) will vary with the wet-bulb temperature.

Another term which Carrier names the *sigma heat content* or *sigma function*, h_Σ , consists of the total enthalpy of the mixture minus the enthalpy of the liquid at the wet-bulb temperature per pound of dry air. Air entering the duct of Art. 183 will have a sigma heat content

$$h_\Sigma = 0.24t + w_H[h_{fg}' + c_{pH}(t - t')]^* \quad (91a)$$

Air leaving the duct adiabatically saturated will have a final temperature $t = t'$. Then

$$h_\Sigma' = 0.24t' + w_H'h_{fg}' \quad (91b)$$

* The bracketed term should be $[h_f'' + h_{fg}'' + c_{pH}(t - t'') - h_f']$ where '' values refer to the dew-point temperature. At low pressures the enthalpy of steam is a function of temperature only. Hence,

$$[h_f'' + h_{fg}'' + c_{pH}(t - t'')] = [h_f' + h_{fg}' + c_{pH}(t - t')]$$

and

$$[h_f'' + h_{fg}'' + c_{pH}(t - t'') - h_f'] = [h_{fg}' + c_{pH}(t - t')]$$

Equating Eqs. (91a) to (91b)

$$0.24(t - t') + w_H c_{p_H}(t - t') = (w_H' - w_H) h_{fg}' \quad (92)$$

The left side of Eq. (92) is the sensible heat lost by the air and water vapor in cooling from t to t' during adiabatic saturation, while the right side is the heat gained by the saturated air from the evaporation of $(w_H' - w_H)$ pound of water vapor added to the mixture. Since Eq. (92) was obtained by letting $h_s = h_s'$, it follows that adiabatic saturation of air is a process in which the sigma heat content remains constant. Also adiabatic saturation is a

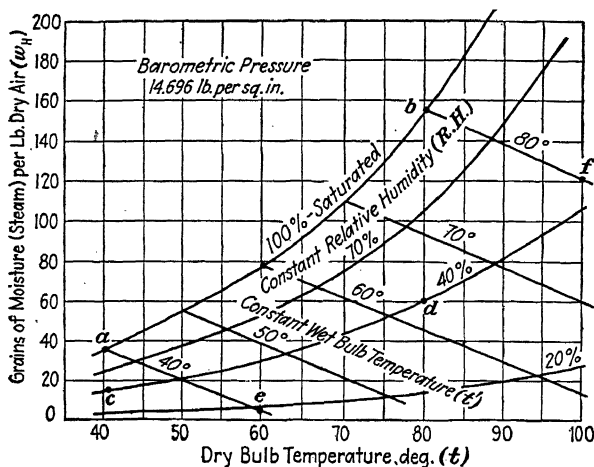


FIG. 142.

process in which the wet-bulb temperature remains constant. Hence on the psychrometric chart (Art. 185) lines of constant wet-bulb temperature are also lines of constant sigma heat content. Furthermore, since the sigma heat content is constant for all mixtures having the same wet-bulb temperature, Eq. (91b) evaluates the sigma heat content for any air-water vapor mixture with a wet-bulb temperature t' .

185. The Psychrometric Chart.—For the steam-air mixture, the solution of problems is greatly facilitated by the use of a *psychrometric chart*, on which the principal variables

involved are shown in their proper relation. The data for the construction of constant-value lines are obtained by the use of the Steam Tables and of the equations derived in the paragraphs immediately preceding. In the sketch of Fig. 142, the horizontal scale represents t , the dry-bulb temperature, and the vertical, w_H , the grains of moisture (steam) per pound of dry air. Lines ab and cd are examples of constant relative-humidity lines; and ae and bf of constant wet-bulb temperature and constant sigma heat content lines.

The method of procedure in the calculation of points for plotting constant relative-humidity lines and constant wet-bulb temperature lines is illustrated by the examples below.

Determine w_s for point a at $t = 40$ F and $H_R = 100$ per cent. Partial pressure of steam $p_s = 0.1217$ (Steam Tables), corresponding to 40 F. Partial pressure of the air $p_a = 14.696 - 0.1217 = 14.574$. Using Eq. (89b), Art. 182, $w_H = w_s = 4354(0.1217/14.574) = 36.3$ grains per pound of dry air. Hence point a is located at $t = 40$ F, and $w_s = 36.3$ grains. Any number of points for the 100 per cent relative humidity line can be determined in like manner.

Determine w_H for a point d , at $t = 80$ F and $H_R = 40$ per cent. From Eq. (88), Art. 178, $p_H = (H_R) \times p_s$ (p_s = saturation pressure at dry-bulb temperature). Hence $p_H = 0.40 \times 0.5069 = 0.2027$.

$$p_a = 14.696 - 0.2027 = 14.493.$$

$w_H = 4354(0.2027/14.493) = 60.9$ grains, Eq. (89b), Art. 182. In the same way, a number of other points for the 40 per cent H_R line, and a whole family of constant H_R lines can be determined.

Determine w_H for a point f at $t = 100$ F and $t' = 80$ F. From Eq. (86), Art. 176 (Carrier's equation),

$$p_H = 0.5069 - \frac{(14.696 - 0.5069)(100 - 80)}{2755 - 1.28 \times 80} \quad 0.3997$$

$$14.696 - 0.3997 = 14.296$$

From Eq. (89b), Art. 182,

$$w_H = 4354 \quad = 121.8 \text{ grains}$$

Other points for $t' = 80$ F at various chosen values of t can be determined in this way for the 80 F constant wet-bulb temperature line. On the satura-

tion line (100 per cent H_R), the dry-bulb and wet-bulb temperatures are alike, as at point b .

On this chart, constant-temperature lines are vertical; adiabatics are parallel to the constant wet-bulb temperature lines; and constant-moisture-content lines are horizontal.

Psychrometric charts are plotted in various ways, depending upon what quantity is represented by the vertical scale. Figure 142 illustrates the original or Carrier form, in which grains of moisture per pound of air are represented by the vertical scale. Partial pressure or dew-point temperature of the steam may also be plotted on the vertical, and sometimes logarithmic scales are employed.

With a chart such as that illustrated in Fig. 142, a user may determine *relative humidity* and *moisture content* directly from dry-bulb and wet-bulb temperatures. *Partial pressure of steam* can then be readily calculated by Eq. (88), Art. 178, $p_H = (H_R) \times p_s$; and *partial pressure of the air* determined by the relation $p_a = p_t - p_H$. The *weight of steam and air* in a given space (say 1 cu ft) can then be found by the characteristic equation and thence the density of the mixture. Sigma heat content can be determined by Eq. (91b), Art. 184. Working charts usually have supplemental features enabling the user to ascertain such additional quantities without resorting to the use of equations.

Values corresponding to only one particular barometric pressure can be represented on any one chart; but if that pressure is approximately the normal for a given locality, no serious error will result in the use of the chart for any barometric pressure within the ordinary range of variation from normal. However, a chart constructed for 29.92 in., for example, cannot be used without risk of serious error at altitudes of several thousand feet. Such error can be readily determined and corrected for by means of the equation developed in this chapter.

A number of problems are given at the end of this chapter involving the use of the psychrometric chart. It is desirable that each student should have a working chart available for this purpose.¹

186. Approximate Derivation of Carrier's Equation.—Equation (86) is of such unusual form as to suggest an empirical origin. In order to satisfy the reader that the equation is based upon fundamental physical laws, the following *approximate* derivation is offered.

The relation existing among the several variables involved in the wet-bulb phenomenon is based upon the necessary equality between heat given up by an atmosphere brought into contact with a wet bulb and the heat required in the vaporization of the water.

Let t and t' = the dry- and wet-bulb temperatures, respectively, F

T and T' = the dry- and wet-bulb temperatures, F abs

p_a and p_H = partial pressures of air and steam, psia

p_t = total pressure = $p_a + p_H$

p_s' = pressure corresponding to wet-bulb temperature, psia

w_H = weight of steam in 1 lb of air of the atmosphere

w_s' = weight of saturated steam per pound of air at wet-bulb temperature

c_{pa} = specific heat of air at constant pressure = 0.241

c_{pH} = specific heat of superheated steam, at constant pressure = 0.46

h_{fg}' = latent heat of steam at wet-bulb temperature, which is closely related to t' by the expression $h_{fg}' = 1091 - 0.56t'$

¹ The Carrier Engineering Corporation (Syracuse, N. Y.) publishes a psychrometric chart which is very generously furnished gratis for use in schools and colleges. The Trane Company (La Crosse, Wis.) can supply charts printed on 8½-by 11-in. paper put up in pads of 50 each, at a very reasonable price. The General Electric Psychrometric chart may be procured from the General Electric Company, Bloomfield, N. J.

Heat given up per pound of dry air = $(c_{pa} + c_{pH}w_H)(t - t')$.

Heat required to vaporize additional steam taken up per pound of dry air = $(w_s' - w_H)h_{fg}'$.

Hence

$$(w_s' - w_H)h_{fg}' = (c_{pa} + c_{pH}w_H)(t - t') \quad (A)$$

The weight of steam mixed with air is ordinarily quite small, so that in this approximate derivation its sensible heat (enthalpy of superheat) will be neglected.

Then

$$(w_s' - w_H)h_{fg}' = c_{pa}(t - t') \quad (B)$$

Using the gas law relation,

$$w_s' = \frac{144 p_s' V_s'}{R_s T'} \quad \text{and} \quad w_H = \frac{144 p_H V_H}{R_s T} \quad (C)$$

For 1 lb of air,

$$V_a = \frac{R_a T}{144 p_a} \quad (D)$$

The steam occupies the same volume as the air, so that

$$V_s' = V_H = V_a = \frac{R_a T}{144 p_a} \quad (\text{nearly}) \quad (E)$$

Substituting Eq. (E) into Eq. (C) and considering $T' = T$

$$w_s' = \frac{p_s' R_a}{p_a R_s} \quad \text{and} \quad w_H = \frac{p_H R_a}{p_a R_s} \quad (F)$$

Let $S =$ the ratio of constant R_a/R_s and substitute Eq. (F) into Eq. (B)

$$\begin{aligned} \left(\frac{p_s'}{p_a} - \frac{p_H}{p_a} \right) h_{fg}' S &= c_{pa}(t - t') \\ p_s' - p_H &= \frac{p_a c_{pa}(t - t')}{h_{fg}' S} \end{aligned} \quad (G)$$

$$p_a = p_t - p_s'; \quad c_{pa} = 0.241; \quad S = 53.34/85.78 = 0.622; \quad h_{fg}' = 1091 - 0.56t'$$

Substituting these values into Eq. (G)

$$\begin{aligned} p_s' - p_H &= \frac{(p_t - p_s')(t - t') \times 0.241}{[0.622 \times (1091 - 0.56t')]} \\ &= \frac{(p_t - p_s')(t - t')}{(2818 - 1.45t')} \end{aligned}$$

whence

$$p_H = p_s' - \frac{(p_t - p_s')(t - t')}{2818 - 1.45t'} \quad (H)$$

The difference between Eq. (H) and Eq. (86) is due to the approximations made in the derivation of Eq. (H) and also to certain modifications to correct for difference between true wet-bulb temperature and that registered by

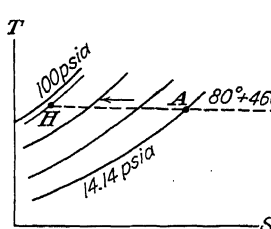


FIG. 143.

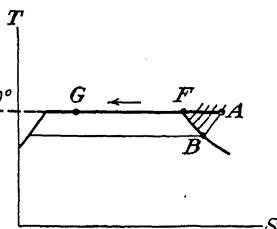


FIG. 144.

a wet-bulb instrument. An actual wet bulb receives a small amount of heat by radiation from surrounding objects at dry-bulb temperature and, possibly, by conduction along its stem. The purpose in deriving Eq. (H) is to show the rational basis of Carrier's psychrometric equation of Art. 176; the latter should, of course, be employed in the solution of problems.

187. Compression of Air-steam Mixture.—When a mixture of steam and air is compressed, the humidity changes. Let atmospheric air at 80 F dry-bulb, 70 F wet-bulb, and 14.45 psia be compressed *isothermally* to 100 psia, temperature being kept constant at 80 F. In Fig. 143, the state point of the *air*, starting at A, proceeds to the left, crossing progressively higher and higher pressure lines, until it arrives at H, the final partial pressure of the air.

In Fig. 144, the state point of the steam, starting at A , proceeds to the left into the region of higher pressure, until F , on the saturation line, is reached. The pressure can then be no further increased (with temperature constant at 80 F), but some of the steam will be condensed, the final state of the original steam content being represented by some point G . The amount of condensation can be computed.

Before compression, $p_H = 0.3108$ from Eq. (86), Art. 176, $p_a = 14.1392$, and from Eq. (89a), Art. 182, $w_H = 0.622$ ($0.3108/14.1392$) = 0.01367 lb of steam with each pound of air.

After compression, the steam is saturated, and

$$p_s = 0.5069,$$

$$p_a = 100.00 - 0.5069 = 99.4931, \text{ and}$$

$$w_s = 0.622(0.5069/99.4931) = 0.00317 \text{ lb of steam}$$

with each pound of air.

$0.00317/0.01367 = 0.23$, or 23 per cent; *i.e.*, only 23 per cent of the original steam in the air can exist as *steam* in the final condition. The other 77 per cent has been condensed. The same effect would result with any kind of compression, followed by cooling to the original temperature.

The water resulting from condensation can be drained from the lines or receiver, leaving the air more nearly free of moisture, which is usually an advantage.

In an adiabatic compression the steam state progresses away from the saturation line, and the terminal pressures and temperature of steam and air may be calculated by treating the mixture as a gas, after determining necessary constants for the equation.

From Eq. (75), Art. 174,

$$R_m = \frac{\Sigma(\bar{W}R)}{\Sigma\bar{W}} = \frac{\bar{W}_H R_s + \bar{W}_a R_a}{\bar{W}_t}$$

where $\bar{W}_H = 0.01367$ lb and $\bar{W}_a = 1$ lb. Introducing numer-

ical values and solving, R_m (for the mixture) = 53.78. The specific heat of the mixture at constant pressure, c_{pm} , is given by Eq. (85), Art. 175,

$$c_{pm} = \frac{\Sigma(\bar{W}c_p)}{\Sigma \bar{W}} = \frac{\bar{W}_H c_{ps} + \bar{W}_a c_{pa}}{\bar{W}_t}$$

$$c_{pm} = \frac{(0.01367 \times 0.46) + (1.00 \times 0.24)}{1.01367}$$

whence

$$c_{pm} = 0.243$$

$$c_{vm} = c_{pm} - \frac{R}{J} = 0.243 - \frac{53.78}{778} = 0.174$$

and

$$k_m = \frac{c_{pm}}{c_{vm}} = \frac{0.174}{0.243} = 1.40$$

The final temperature for constant-entropy compression can now be computed and is found to be 940 F abs, or 480 F, assuming terminal pressure to be 100 psia.

The final partial pressure of the steam can be determined by Eq. (73a), Art. 174,

$$\frac{p_H}{p_t} = \frac{(\bar{W}_H R_s)}{(\bar{W}_H R_s + \bar{W}_a R_a)}$$

whence

$$p_H = 100 \times \frac{(0.01367 \times 85.7)}{(0.01367 \times 85.7 + 1.00 \times 53.34)} = 2.15 \text{ psia}$$

The saturation temperature corresponding to 2.15 psia is 129 F, and the steam is therefore superheated

$$480 \text{ F} - 129 \text{ F} = 351 \text{ deg}$$

Problems

1. The analysis of a gas shows the following composition by volume: CO = 30.24, CH₄ = 1.75, H₂ = 3.25, CO₂ = 5.93, and N₂ = 58.83 per cent. Determine the composition by weight, molecular weight, and the value of R for the mixture.

2. The analysis of a furnace gas shows the following percentages by volume: CO₂ = 11.2, O₂ = 6.2, H₂O = 3.5, and N₂ = 79.1.

Determine the percentages by weight, molecular weight, and value of R for the mixture; also its specific heat at constant pressure, if the temperature is 100 F.

3. The analysis of a gas shows the following composition by weight: CO = 3 per cent, CO₂ = 13 per cent, H₂ = 1 per cent, SO₂ = 2 per cent, and N₂ = 81 per cent. Determine the composition by volume, molecular weight, and value of R for the mixture.

4. Express the composition of the gas of Prob. 3 in mols per 100 lb of gas.

5. In a mixture of 25 parts of O₂, 65 of N₂, and 10 of CO₂, by weight, what is the partial pressure of each gas in per cent of the total pressure of the mixture? Determine R for the mixture; also its molecular weight.

6. In a mixture of CO₂, O₂, N₂, and H₂O at 16 psia total pressure, it is desired that the partial pressures shall be respectively 10, 20, 30, and 40 per cent of the total. In what proportions, by weight, should the gases be mixed to accomplish this result?

7. The dry-bulb temperature of a room is 74 F, the wet-bulb temperature 55 F, and the barometer 29.45 in.

a. Determine the absolute humidity, or weight of steam per cubic foot in grains.

b. Determine the relative humidity in per cent.

c. Determine the weight per cubic foot or density of this atmospheric air.

8. A fan is handling air at a temperature of 75 F and a barometric pressure of 29.40 in.; the wet-bulb temperature is 65 F. What error, in per cent, will be made in computing density, if it is assumed that the substance is pure dry air (the humidity factor being entirely neglected), instead of a mixture of air and steam, as specified?

9. Which is the heavier, 1 cu ft of dry air (0 per cent humidity) or 1 cu ft of saturated air (100 per cent humidity), at the same dry-bulb temperature? Assume the barometer to be 29.50 in., and the temperature 80 F. Calculate the weights for the two cases. Compare.

10. A compressor handles 400 cu ft of free air per minute, compressing it from atmospheric conditions to 100 psi gage pressure. The barometric pressure is 29.60 in. Hg, 75 F dry-bulb temperature, and 65 F wet-bulb temperature. The water-jacketing of the compressor cylinder together with cooling after compression reduces the temperature to the original atmospheric temperature, *viz.*, 75 F. What weight of water will be condensed per minute due to the compression and cooling in the manner described?

11. Let t_w = temperature of the air in a room, t_c = temperature of cold air outside, t_g = temperature of inside (warm) surface of glass of a window, and H_R = relative humidity of the air in the room. For a given window, let $(t_w - t_g)/(t_w - t_c) = 0.70$. At what outside temperature (t_c) will condensation and frost begin to appear?

a. If $t_w = 75$ F, and $H_R = 50$ per cent.

b. If $t_w = 75$ F, and $H_R = 30$ per cent.

c. If $t_w = 70$ F, and $H_R = 50$ per cent.

12. 12,000 cu ft of atmospheric air at 90 F dry-bulb and 65 F wet-bulb temperature and 14.40 psia barometric pressure are supplied per minute to a spray cooler, the water for which is recirculated.

a. If the air leaves at 50 per cent relative humidity, what will be the temperature of the water, and of the air leaving? What weight of steam will be associated with each pound of dry air, before entering and after leaving the cooler? What total weight of water will be evaporated per hour? What is the enthalpy per pound of dry air before and after traversing the cooler?

b. If the air leaves the cooler at 75 F, what is the temperature of the spray water? What weight of steam is associated with each pound of dry air? What weight of water is evaporated per hour? What is the enthalpy per pound of dry air?

13. Air enters a drying tunnel at 140 F dry-bulb temperature with 60 deg depression of wet bulb and leaves at 100 F dry bulb with 10 deg wet-bulb depression. Barometric pressure is 29.30 in. Hg.

a. What weight of moisture is absorbed by each pound of dry air in the tunnel?

b. If 2100 lb of moisture is to be absorbed per hour, how many cubic feet per minute of atmospheric air must be supplied?

14. Water is cooled in a tower from 100 F to 85 F. Air is supplied to the tower at 95 F dry-bulb and 75 F wet-bulb temperature. Air leaves the tower at 87 F dry-bulb and 82 F wet-bulb temperature. Barometric pressure is 14.50 psia.

a. What is the enthalpy per pound of dry air before and after passing through the tower?

b. What weight of water is evaporated per pound of dry air while passing through the tower?

c. How many pounds of dry air must be moved through the tower for each pound of cooled water delivered?

d. What are the relative humidities before and after passing through the tower?

15. In order to illustrate the limitations of a psychrometric chart constructed for one particular barometric pressure, first determine partial pressure of steam, relative humidity, dew-point temperature, and grains of moisture per pound of dry air, for 85 F dry-bulb temperature and 70 F wet-bulb temperature, using the working psychrometric chart that you have available, and noting the barometric pressure for which the chart is applicable.

Then determine these same quantities by use of appropriate equations.

a. For 29.00 in., illustrating a rather low barometric pressure for localities not much above sea level.

b. For 24.00 in., illustrating a barometric pressure appropriate to a rather high altitude such as that of Denver.

16. The following exercise is given to show the close conformity in behavior of *saturated steam* to that of a perfect gas in the low-pressure range representing partial steam pressures in atmospheric mixtures.

Choosing temperatures of say 40, 60, 80, 100, 120, 140, 170, 200, and 230 F, look up pressures corresponding in Table 2 of the Steam Tables. For each set of values, calculate specific volume by equation $pV = \bar{W}RT$, where $R = 85.7$. Compare calculated value with that given in the Steam

Tables for the saturated vapor, and express deviation as a plus or minus error in per cent.

It will be found advantageous to set up a calculation table, in which each item represents the results of a step in the procedure.

17. In a condenser there is 0.2 lb of air per pound of steam at a temperature of 80 F. Barometric pressure is 29 in. Hg. (a) If the steam is dry and saturated, what is the pressure? (b) If the steam has a quality of 90 per cent, what is the pressure?

18. (a) Dry air at 85 F enters a water spray at 50.5 F. The air leaves the spray with 100 per cent relative humidity. What is the vapor pressure of the steam, grains of moisture per pound of dry air, wet-bulb temperature and dry-bulb temperature of the air leaving the spray? How much heat is lost by the air? How much heat was required to evaporate the water added to the air while passing through the spray? Calculate each heat quantity. (b) Find the quantities of (a) if the air entered the spray with a relative humidity of 34 per cent and a temperature of 85 F.

19. Air at 80 F and 60 per cent relative humidity flows in a duct at a rate of 4000 cfm. Air in another duct at 40 F and 90 per cent relative humidity flows at a rate of 2000 cfm. If the air in the two ducts is mixed, what is the dry-bulb temperature and relative humidity? Standard atmospheric pressure.

20. Air at a dry-bulb temperature of 85 F and a wet-bulb temperature of 70 F is cooled to a dew-point temperature of 40 F. How much heat must be removed per pound of dry air?

CHAPTER XIII

FLOW OF FLUIDS

188. The Transformation of Available Energy into the Kinetic Form.—In the kinetic type of heat engine, such as the steam turbine and the kinetic pump, the over-all transformation of heat into work is accomplished in two distinct steps. Available energy of the working substance is first changed into kinetic energy of fluid flow, and this kinetic energy is then transformed into mechanical work. It is the purpose of this chapter to consider the first of these steps, *viz.*, the transformation of available energy into velocity.

189. The Equation of the Nozzle.—Let Fig. 145 represent the longitudinal section of a passage through which a gas is flowing from a higher pressure p_1 to a lower pressure p_5 . The substance will enter the passage at section 1 with but little velocity and will emerge at section 5 with a higher velocity. In general, there will occur a continual transfer of energy along the length of the nozzle. The general energy equation for a nozzle is Eq. (16), Art. 19.

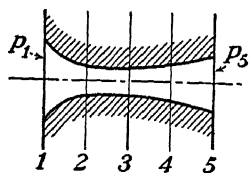


FIG. 145.

$$\frac{\bar{V}_2^2}{2gJ} = (h_1 - h_2) + \frac{\bar{V}_1^2}{2gJ} \quad (16)$$

190. The Equation of the Continuity of Mass.—In Fig. 145, for any section designated, the following relation holds:

$$\bar{w} = \frac{A\bar{V}}{v} \quad (93)$$

where \bar{w} = weight passing the section, lb per sec

A = area of the section, sq ft

\bar{V} = velocity of the gas at the section, fps

v = specific volume of the gas at the section,
cu ft per lb.

Although A , \bar{V} , and v may have different values at various sections along the channel, \bar{w} is the same at every point. Hence,

$$\bar{w} = \frac{A_1 \bar{V}_1}{v_1} = \frac{A_2 \bar{V}_2}{v_2} = \dots = \frac{A_5 \bar{V}_5}{v_5} \quad (93a)$$

This equation may be called the equation of the *continuity of mass*.

191. General Form of Nozzle Passage.—A nozzle is an element whose primary function is to convert enthalpy energy into kinetic energy. The passage of Fig. 145 is a nozzle if its chief function is to generate velocity rather than merely to transfer a substance from one chamber to another.

The equation of the continuity of mass may be written thus:

$$\bar{w} = \left(\frac{\bar{V}_1}{v_1} \right) A_1 = \left(\frac{\bar{V}_2}{v_2} \right) A_2 = \dots = \left(\frac{\bar{V}_5}{v_5} \right) A_5 \quad (93b)$$

As the gas progresses through the nozzle, its pressure falls. The velocity increases, and the specific volume increases. In order to allow the expansion to take place properly, the area at any section must be such that it will accommodate the substance at whatever volume and velocity may prevail at that point; or, to put it the other way around, at any point along the nozzle a certain velocity and specific volume will exist, depending upon the area of the section. The quantity \bar{V}/v may be regarded as a coefficient of A . The manner in which both \bar{V} and v vary depends upon the properties of the substance flowing. Hence, the contour of the passage of a nozzle depends upon the nature of the substance flowing.

For example, consider a liquid—a substance whose specific volume undergoes practically no change with change of pressure. In traversing a nozzle, v will remain constant while \bar{V} increases. The value of the coefficient \bar{V}/v in the equation of the continuity of mass is therefore continually increasing, and necessarily therefore A , the area of cross section, should decrease. Case *B*, Fig. 146, illustrates the proper contour of longitudinal section of a nozzle suitable for a liquid.

Case *C* represents the correct contour for some hypothetical substance for which both velocity and specific volume increase at the same rate, so that their ratio is a constant at all points. The area of cross section should therefore be constant at all points, and the nozzle becomes a plain tube.

Case *D* is that of another hypothetical substance whose peculiarity is that, while both velocity and specific volume increase, the rate of specific-volume increase is greater than that of the velocity, resulting in a divergent contour from the very entrance of the nozzle.

E is the general case of a nozzle suitable for an elastic fluid. It will be shown that for a gas, while velocity and specific volume both increase from the start, the velocity first increases faster than the specific volume, but after a certain critical point, specific volume increases more rapidly than velocity. Hence, the value of \bar{V}/v first increases to a maximum and then diminishes, necessitating a nozzle contour that is convergent-divergent in form.

192. The Critical Pressure of a Gas.—The pressure associated with the maximum value of the ratio \bar{V}/v is called the *critical pressure* and is wholly dependent upon the initial pressure and the properties of any particular gas; i.e., $p_c = r p_1$, where p_c is the critical pressure; p_1 the initial pressure (both in absolute units); and r , a constant depending upon the properties of the particular gas under consider-

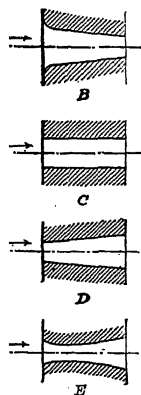


FIG. 146.

ation; or, more exactly, r is itself one of the characteristic constants of the gas, like k and R . The value of r for air is 0.53, and this will first be determined by a specific example taken at random, after which a general expression will be derived by means of which the value of r for *any* gas may be calculated.

Example.—Compressed air is expanded adiabatically and reversibly. (constant entropy) through a nozzle from 120 psia pressure and 140 F to 20 psia. To show that the critical pressure $p_c = 63.6$ psia or 0.53×120 .

As the air progresses through the nozzle, the pressure changes from 120 psia in the entrance chamber to 20 psia at the discharge end. Consequently, *any* pressure between these limits may be selected with the knowledge that that pressure will be assumed by the air at *some* point along the nozzle's length. With initial conditions known and with the specification of constant-entropy adiabatic expansion, the velocity \bar{V} and specific volume v , and thence the ratio \bar{V}/v , can be calculated for the chosen pressure. In Table 13, Column 2, nine different pressures, at 10-psi intervals between the two extremes, have been chosen. The determination of the quantities in the other columns of Table 13 will be illustrated by calculations for one particular pressure, say 50 psia. As this is the eighth value in the pressure column, the subscript 8 will be used to designate quantities associated with the pressure of 50 psia, while the subscript 1 will designate quantities associated with the initial state.

$$h_1 = c_p T_1 = 0.241 \times (140 + 460) = 144.6 \text{ Btu per lb} \quad (\text{Art. 106})$$

For adiabatic expansion,

$$\frac{T_1}{T_8} = \left(\frac{p_1}{p_8} \right)^{\frac{k-1}{k}} = \left(\frac{120}{50} \right)^{\frac{2}{7}} = 1.284 \quad (42)$$

$$T_8 = \frac{600}{1.284} = 467 \text{ F abs.}$$

$$h_8 = c_p T_8 = 0.241 \times 467 = 112.5 \text{ Btu per lb}$$

The available energy from 120 to 50 psia is

$$144.6 - 112.5 = 32.1 \text{ Btu per lb,}$$

which appears at point 8 as kinetic energy.

Neglecting velocity of approach V_1 , by Eq. (16)

$$\bar{V}_2 = \sqrt{-h_2} = 223.7 \sqrt{h_1 - h_2} \quad (17)$$

or

$$\bar{V}_8 = 223.7 \sqrt{h_1 - h_8} = 223.7 \sqrt{32.1} = 1268 \text{ fps}$$

The specific volume at point 8 is determined by the equation

$$\begin{aligned} p_8 v_8 &= RT_8 \\ v_8 &= \frac{(53.34 \times 467)}{(50 \times 144)} = 3.46 \text{ cu ft} \\ \frac{\bar{V}_8}{v_8} &= \frac{1268}{3.46} = 367 \end{aligned}$$

If a flow of 1 lb of air per second is assumed, the cross-sectional area of the nozzle at the point where the pressure is 50 psia is found by the relation $\bar{w} = A_8(\bar{V}_8/v_8)$ or $A_8 = \frac{1}{367} = 0.00272 \text{ sq ft}$. In more convenient units, $a_8 = 0.392 \text{ sq in}$.

In like manner, values for the several items of Table 13 are calculated for the other pressures of Column 2. It is to be noted that some of the quantities may be determined by the use of other equations than those employed in the illustrative calculations. Thus, the available energy is the equivalent of the work of expanding isentropically from 120 to 50 psia pressure and may be calculated by Eq. (53)

$$AE = \frac{k}{(k-1)} \frac{p_1 v_1 - p_8 v_8}{J}$$

The value of v_1 may be determined by the equation $p_1 v_1 = RT_1$, and v_8 by the relation $p_8 v_8^k = p_1 v_1^k$. The available energy calculated in this way will be found to agree with the value 32.1 Btu per lb already found.

TABLE 13.—CALCULATIONS TO DETERMINE CRITICAL PRESSURE RATIO, FOR AIR

Reference number	Pressure, p	Temperature absolute, T	Enthalpy Btu/lb h	Available energy, Btu/lb AE	Velocity, \bar{V}	Specific volume, v	Ratio, \bar{V}/v	Area for 1 lb per second, a , sq in.
1	120	600	144.6	0	0	1.85	0	
2	110	585	141.0	3.6	424	1.97	215	0.67
3	100	569	137.1	7.5	613	2.11	291	0.49
4	90	552	133.0	11.6	762	2.27	336	0.43
5	80	534	128.7	15.9	892	2.47	361	0.399
6	70	514	123.9	20.7	1018	2.72	374	0.385
7	60	492	118.6	26.0	1141	3.03	377	0.382
8	50	467	112.5	32.1	1268	3.46	367	0.392
9	40	438	105.6	39.0	1397	4.05	345	0.42
10	30	403	97.1	47.5	1542	4.97	310	0.46
11	20	360	86.8	57.8	1701	6.66	255	0.56

The values of \bar{V} , v , the ratio \bar{V}/v , and a of Table 13 are plotted against pressure on a diminishing scale in Fig. 147.

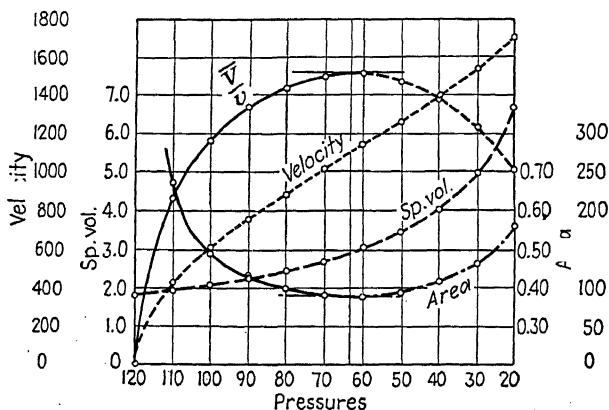


FIG. 147.

A vertical line dropped from the peak of the \bar{V}/v curve shows the pressure at which this maximum occurs to be 63.6 psia, which is therefore the *critical pressure*. This example, selected at random, has verified the statement

that the value of $r (= p_c/p_1)$ is 0.53 for air. Any number of examples with different conditions specified might be worked out, with the common result that the critical pressure p_c would be found to be $0.53p_1$. The same procedure might be followed with any other gas, with similar results, except that the value found for r would in general be different for each different gas. In the following article, it will be shown that the constant r for any gas is definitely determined by the value of k for that gas through the relation

$$r = \frac{p_c}{p_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (94)$$

where p_c and p_1 are the critical and initial pressures, respectively.

193. Derivation of the Relation, $r = \frac{p_c}{p_1} = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}}$.—

The critical or throat pressure occurs when \bar{V}/v is a maximum.

Let p , \bar{V} , and v represent the pressure, velocity, and specific volume at any point of progress along a nozzle's length. It is desired to obtain a suitable expression for \bar{V} and v so that a differentiation can be made to ascertain the value of p when \bar{V}/v is a maximum.

$$\frac{\bar{V}^2}{2g} = J(h_1 - h) = Jc_p(T_1 - T)$$

In terms of pressures and volumes

$$\frac{\bar{V}^2}{2g} = \frac{k}{k-1} (p_1 v_1 - p v) \quad (\text{Art. 125}) \quad (A)$$

where v = specific volume corresponding to p

$$p v^k = p_1 v_1^k \quad (B)$$

$$\left(\frac{v}{v_1} \right)^k = \frac{p_1}{p}$$

$$v = v_1 \left(\frac{p_1}{p} \right)^{\frac{1}{k}} \quad (C)$$

Substituting Eq. (C) in Eq. (A) and transposing $2g$,

$$\bar{V}^2 = \frac{2gk}{k-1} \left[p_1 v_1 - p v_1 \left(\frac{p_1}{p} \right)^{\frac{1}{\gamma}} \right] \quad (D)$$

Multiplying the second term (in the brackets) of Eq. (D) by p_1/p_1 ,

$$p v_1 \left(\frac{p_1}{p} \right)^{\frac{1}{\gamma}} \frac{p_1}{p_1} = p_1 v_1 \left(\frac{p_1}{p} \right)^{\frac{1-k}{k}}$$

Let $r =$ the ratio p/p_1 .

Then

$$p_1 v_1 \left(\frac{p_1}{p} \right)^{\frac{1-k}{k}} = p_1 v_1 \left(\frac{1}{r} \right)^{\frac{1-k}{k}} = p_1 v_1 (r)^{\frac{k-1}{k}} \quad (E)$$

Substituting Eq. (E) in Eq. (D),

$$\bar{V}^2 = p_1 v_1 \frac{2gk}{k-1} \left[1 - (r)^{\frac{k-1}{k}} \right] \quad (F)$$

Let $p_1 v_1 \frac{2gk}{k-1} = C_1$ a constant, then

$$\bar{V}^2 = C_1 \left[1 - (r)^{\frac{k-1}{k}} \right] \quad (G)$$

Again

$$\begin{aligned} v &= v_1 \left(\frac{p_1}{p} \right)^{\frac{1}{k}} \quad (\text{Eq. C.}) \\ v^2 &= v_1^2 \left(\frac{p_1}{p} \right)^{\frac{2}{k}} = v_1^2 \left(\frac{1}{r} \right)^{\frac{2}{k}} \end{aligned} \quad (H)$$

Combining Eq. (G) and Eq. (H)

$$\frac{\bar{V}^2}{v^2} = \frac{C_1 [1 - (r)^{\frac{k-1}{k}}]}{v_1^2 \left(\frac{1}{r} \right)^{\frac{2}{k}}} \quad (I)$$

Let $C_1/v_1^2 = C_2$, a constant, and let $z^2 = \bar{V}^2/v^2$.
Then

$$z^2 = C_2 \left[1 - (r)^{\frac{k-1}{k}} \right] (r)^{\frac{2}{k}} \quad (J)$$

$$= C_2 \left[(r)^{\frac{2}{k}} - (r)^{\frac{k+1}{k}} \right] \quad (K)$$

Differentiating

$$2z \, dz = C_2 \left[\frac{2}{k} (r)^{\frac{2-k}{k}} - \frac{(k+1)}{k} (r)^{\frac{1}{k}} \right] dr$$

To determine the value of r for which $z = \bar{V}/v$ is a maximum, let $dz/dr = 0$.

$$\frac{2}{k} (r)^{\frac{2-k}{k}} - \left(\frac{k+1}{k} \right) (r)^{\frac{1}{k}} = 0 \quad (L)$$

and

$$r = \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \quad (94)$$

194. Critical-pressure Ratio for Various Gases.—In order to determine the value of $p_c/p_1 = r$ for any gas, it is only necessary to substitute the value of k for that gas (see Table 6, Art. 100) into the expression derived in the preceding article and solve for r . Thus, for air, oxygen, nitrogen, hydrogen, etc., for which k has the value 1.40, r is found to be 0.528, or more commonly 0.53. For helium and argon with the unusually high value of $k = 1.66$, $r = 0.488$; while for the unusually low value of $k = 1.20$ for methyl chloride, $r = 0.565$.

For steam (and the same is probably true of other vaporous substances), the value of k for states bordering on saturation is somewhat uncertain, as would be expected because of the considerable departure from behavior as a perfect gas. A value of $k = 1.135$, determined by Zeuner, for *steam* at or near the *saturated state initially*, is commonly used. In the critical-pressure relation, for $k = 1.135$, $r = 0.578$, or more commonly 0.58. For *steam initially superheated*, $k = 1.30$ and the corresponding value of the critical pressure, r , is 0.546. We are left in uncertainty regarding transitional values between $r = 0.578$ and $r = 0.546$ for intermediate initial states of steam between wet or saturated and some definite degree of superheat, at which we would expect the value of $k = 1.30$ to begin to

apply. The situation here is associated with the peculiar phenomenon of supersaturation described in Art. 204.

195. The Pressure at the Throat of a Nozzle.—The critical pressure of a gas flowing through a nozzle is that which exists when the ratio \bar{V}/v (velocity to specific volume) attains its maximum value. This is also the necessary condition for minimum cross-sectional area of passage (Art. 191). Hence the pressure at the throat of a nozzle is the critical pressure, provided *the back pressure in the region into which the nozzle discharges is lower than the critical value*. For example, if compressed air at 100 psia pressure is supplied to a nozzle that discharges into a chamber in which the pressure is anything less than 52.8 psia, the throat pressure will be 52.8 psia, regardless of what particular back pressure may prevail between zero and 52.8 psia. On the other hand, if the back pressure is higher than the critical value, the critical pressure is not reached at all, and the pressure at the least section, whether the nozzle is convergent only (which is the appropriate form for this back-pressure condition), or convergent-divergent, will be identical with the back pressure. As a convenient summarization of the situation at the throat of a nozzle, we may say that the throat pressure is determined by *either* the initial pressure alone or the back pressure alone, but *not* by a combination of the two. When the back pressure is less than the critical, the critical pressure is itself the throat pressure ($p_t = p_c = rp_1$); when the back pressure is greater than the critical, the pressure in the throat is the back pressure itself.

196. Convergent vs. Divergent Nozzle.—It has been shown (Art. 191) that the generally appropriate form of passage in which to effect the transformation of available heat of a *gas* into kinetic energy is the convergent-divergent nozzle. The convergent part of the passage is always necessary no matter through what range of pressure expansion occurs. However, a divergent part is necessary only *when the back pressure is less than the critical*.

The convergent nozzle can expand a gas only from the initial pressure to the throat pressure. If the back pressure exceeds the critical, then the throat and back pressures become coincident, and expansion is therefore completely effected in a convergent nozzle. If the back pressure is less than the critical (which then becomes the throat pressure), expansion cannot be completed in a convergent nozzle alone, and a divergent part is essential to utilize the remainder of the available heat from throat to back pressure. Expansion beyond the critical pressure means that volume must grow more rapidly than velocity, and this requires a widening passage.

The ratio of end area a_e , of a divergent nozzle to throat area a_t , is determined by the amount of further expansion that can take place from throat to back pressure. As an illustration, refer to Fig. 147, where, for a flow of 1 lb of air per second, the throat area $a_t = 0.382$ sq in., and the discharge-end area a_e (for 20 psia back pressure) is 0.56 sq in. The ratio $a_e/a_t = 0.56/0.382 = 1.47$. The actual cross-sectional areas of the nozzle are determined by the amount of flow to be provided for. Thus, for 2 lb per sec, all areas must be twice those for 1 lb. But for the pressure conditions specified for the example of Fig. 147, the expansion ratio a_e/a_t must be 1.47 no matter what the actual value of a_e and a_t .

If, in the case represented by Fig. 147, the back pressure is made 40 psia, then a_e would need to be only 0.42 sq in. (for 1 lb flow per second) and the expansion ratio required is $0.42/0.382 = 1.10$. If the back pressure is made 63.6, expansion is completed in the convergent part of the nozzle, and no divergent part is needed.

197. Practical Forms of Nozzles.—If, for any reason, it is desired to make a nozzle to expand air from 120 psia pressure and 140 F temperature to 20 psia back pressure, *with uniform pressure drop along its length*, the area curve of Fig. 147 would correctly represent the variation of cross-sectional area with the base line representing length. A

nozzle of this kind, of circular section, is shown in Fig. 148. The manner of variation of \bar{V} , v , and the ratio \bar{V}/v in such a nozzle is correctly represented by the curves of Fig. 147. Experience proves that a nozzle giving uniform pressure drop has no particular merit. It is, in fact, a rather poor design because of the long convergent part which offers a large surface for skin friction. From the constructional

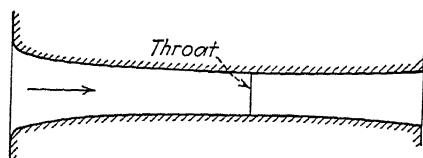


FIG. 148.

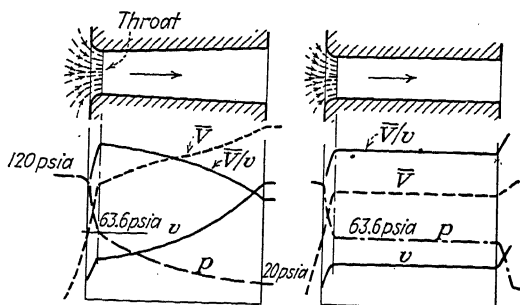


FIG. 149.

FIG. 150.

standpoint, the curved contours make for difficulties in manufacture, while the great length would increase the dimensions of a structure, such as a steam turbine, in which the nozzle is to be used.

A nozzle of more practical form, to accomplish the same service, is illustrated in Fig. 149. Here the convergent part is shortened to a mere well-rounded entrance, while the divergent part (in this sketch) is made of the same length as that of Fig. 148 but is conical in form. The nozzle of Fig. 149 will perform as efficiently as, or more so than, that of Fig. 148. The convergent part virtually extends back into the chamber to which the nozzle is attached, being formed naturally by the movement of fluid toward the nozzle entrance proper. The manner of variation of pressure,

velocity, volume, and the ratio of velocity to volume is indicated by the curves drawn below the nozzle sketch. All the changes accompanying expansion from initial to throat pressure occur in a very short length of travel and a very brief length of time. Completion of expansion from throat to back pressure takes place more leisurely in the divergent part of the nozzle, in a manner similar to but not exactly like that in the divergent part of the nozzle of Fig. 148, because the variation of area from throat to end is somewhat different in the two cases. The areas of the two nozzles at throats and discharge ends are made alike, so both will expand air to 20 psia back pressure and deliver the same amount per unit of time.

In the *nondivergent or straight nozzle* of Fig. 150, the divergent part of Fig. 149 has been replaced by a tube of uniform area. The curves show the variations in p , \bar{V} , v , and \bar{V}/v for the same initial (120 psia) and back pressure (20 psia) assumed for the two other nozzles. Expansion in the convergent part is exactly like that in the convergent part of Fig. 149. But in the straight tube, further expansion is completely suppressed because there is no increase of area. In the equation $\bar{w} = A(\bar{V}/v)$, \bar{w} and A are constant; hence \bar{V}/v is constant, and therefore no change in either \bar{V} or v individually can take place and no drop of pressure can occur. Expansion from 63.6 to 20 psia is completed after the air emerges from the nozzle, but since the stream is then no longer confined in a passage, lateral acceleration (which is useless) takes place.

The cylindrical tube serves no useful purpose in the transformation of available heat into kinetic energy and has no advantage in this respect over a nozzle with a convergent part only. However, a short straight length (much shorter than that indicated in Fig. 150) is useful in directing the jet upon buckets or other elements utilizing the kinetic energy.

Theoretically, a convergent or straight nozzle should be employed only when the back pressure is greater than the

critical pressure; but *after-expansion* occurring outside of the nozzle does result in some useful axial acceleration of the jet (as suggested by the velocity curve of Fig. 150), and, for this reason, the straight or convergent nozzle may be used for back pressures considerably lower than the critical pressure, without appreciable loss in efficiency.

The *length* of the divergent part of a nozzle where the expansion ratio a_e/a_t is large is determined by the practical consideration of angle of flare. The total angle included between the walls of the nozzle of Fig. 149 should not very much exceed 12 degrees. Otherwise, the high velocity of fluid may carry it out of contact with the walls of the passage, and the nozzle ceases to exert control of the transformations taking place, with resultant loss of efficiency.

198. Acoustic Velocity in a Nozzle.—The velocity in the throat of a nozzle, assuming adiabatic expansion, may be expressed by the equation

$$\bar{V}_t^2 = \frac{2gk(p_1v_1 - p_tv_t)}{(k-1)} \quad (\text{Eq. A}) \quad \text{Art. 193} \quad (95)$$

where \bar{V}_t = velocity in throat, fps

$g = 32.2$; and for values of k , see Table 6,
Art. 100

p_1 and p_t = initial and throat pressures, in lb per sq ft abs.

v_1 and v_t = specific volumes, cu ft

When the back pressure is less than the critical, then
= p_cv_c , where p_c = critical pressure, and v_c = specific volume corresponding thereto.

By means of the relations $p_1v_1^k = p_tv_t^k$, and

$$p_t = p_c = rp_1 = [2/(k+1)]^{\frac{k}{k-1}} p_1$$

(Art. 193), either p_1v_1 or p_tv_t may be eliminated from the right-hand side of Eq. (95). Eliminating p_1v_1 (by intermediate steps omitted here), the equation takes the form

$$\bar{V}_t^2 = p_tv_tgk \quad \text{or} \quad RT_tgk \quad (96)$$

Now this is the equation for the *velocity of sound* in a gaseous medium whose pressure and specific volume are p_t and v_t . Hence, the velocity of a gas in the throat of a nozzle, when the back pressure is less than the critical value, is frequently referred to as the *acoustic velocity*.

199. Rate of Flow through a Nozzle—General Equation. The general equation of flow of any fluid through a passage is

$$\bar{w} = \frac{A \bar{V}}{v} \quad (93)$$

where \bar{w} = rate of flow, lb per sec

A = area of any designated section, sq ft

\bar{V} = velocity at that section, fps

v = specific volume, cu ft

In the case of a nozzle, by far the most convenient section to consider is that at the throat, because here the area can be most readily and accurately determined and also because the pressure is known to be either the critical pressure or the back pressure (Art. 195). Hence the general equation above becomes, practically,

$$\bar{w} = \frac{A_t \bar{V}_t}{v_t} \quad (93c)$$

where the subscript alludes to conditions at the throat section. Equation (93c) is applicable to the determination of flow of *any gas* and against *any back pressure*. With initial conditions, back pressure, throat area, and kind of gas specified the pressure in the throat will be known, whence \bar{V}_t and v_t can be determined and \bar{w} calculated.

200. Flow through a Nozzle—Back Pressure Less than Critical Pressure.—For that wide range of back pressures below the critical value, the pressure in the throat of a nozzle is itself the critical pressure, which in turn is wholly dependent upon the initial pressure. Hence it should be possible to express \bar{w} in terms of initial conditions only.

In Art. 198, it was shown that either $p_1 v_1$ or $p_t v_t$ of Eq. (95) may be eliminated, provided the critical pressure

prevails at the throat. Eliminating $p_1 v_1$ (by steps here omitted but not difficult to follow),

$$\bar{V}_t = \sqrt{\frac{2gkp_1 v_1}{(k+1)}} \quad (97)$$

Also, for the critical pressure condition, it can be shown that

$$v_t = v_1 \left(\frac{k+1}{2} \right)^{\frac{1}{k-1}} \quad (98)$$

Introducing Eq. (97) and Eq. (98) into Eq. (93c) and simplifying

$$\bar{w} = A_t \sqrt{\frac{p_1}{v_1}} \sqrt{\frac{gk}{[(k+1)/2]^{\frac{k+1}{k-1}}}} \quad (99)$$

Equation (99) is applicable to *any gas* but is limited to conditions where the back pressure is less than the critical. By substituting the value of k for any particular gas, a very simple expression for flow results. In the following equations (100a), (100b), and (100c), this has been done for air and steam with the further change of *area, units to square inches and pressure to pounds per square inch.*

For *air*, for which $k = 1.40$,

$$\bar{w} = 0.3238a_t \sqrt{\frac{p_1}{v_1}} \quad (100a)$$

For *superheated steam*, for which $k = 1.30$,

$$\bar{w} = 0.3155a_t \sqrt{\frac{p_1}{v_1}} \quad (100b)$$

For *saturated steam*, for which $k = 1.135$,

$$\bar{w} = 0.3005a_t \sqrt{\frac{p_1}{v_1}} \quad (100c)$$

In these equations,

\bar{w} = flow, lb per sec

a_t = area of throat of nozzle, sq in.

p_1 = initial pressure, psia

v_1 = initial specific volume, cu ft

The equations of this article are applicable only *when the back pressure is below the critical value*.

In the case of air, Eq. (100a) can be still further simplified by replacing the specific volume v_1 by its equivalent, $(RT_1)/(144p_1)$, which results in the following:

$$\bar{w} = 0.53 \frac{a_1 p_1}{\sqrt{T_1}} \quad (100a1)$$

This is the familiar *Fliegner formula* for the flow of air, when the back pressure is below the critical.

Napier's empirical equation (101), commonly called *Napier's rule*, for the flow of steam when the back pressure is below the critical value is widely used because of its simplicity and gives results that agree closely with those of Eqs. (100b) and (100c).

$$w = \frac{(a_1 p_1)}{70} \quad (101)$$

Symbols and units are the same as those used in Eq. (100). The rule as stated is for steam initially dry and saturated. For superheated steam, the \bar{w} calculated by Eq. (101) should be multiplied by the factor $1/(1 + 0.00065D)$, where D = degrees of superheat. For steam initially wet, multiply by the factor $1/(1 - 0.012m)$, where m is the per cent of moisture.

201. Flow through a Nozzle—Back Pressure Greater than Critical Pressure.—Where the back pressure is greater than the critical value, the throat pressure or, more accurately in this case, the pressure in the nozzle is no longer associated with the initial pressure but assumes the value of the back pressure. Consequently, any flow equation applicable to this situation must include the back pressure or some quantity derived from it. No strictly rational equation of simplified form comparable to Eqs. (100a), (100b), and (100c) can be derived, but approximate results will be given by Eqs. (102) and (103) below.

For *air*,

$$\bar{w} = 1.06a_t \sqrt{p_e \frac{(p_1 - p_e)}{T_1}} \quad (102)$$

For *initially saturated steam*,

$$\bar{w} = 0.029a_t \sqrt{p_e(p_1 - p_e)^*} \quad (103)$$

where \bar{w} = flow, lb per sec

a_t = area of nozzle at smallest section, sq in.

p_1 = initial pressure, psia

p_e = back pressure, psia

Equation (102) is attributed to Fliegner, and Eq. (103) to Napier.

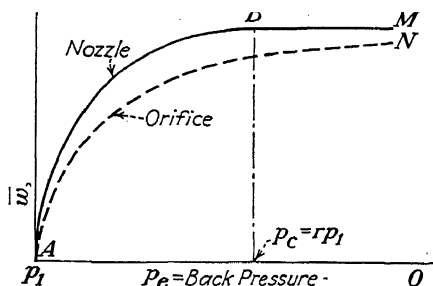


FIG. 151.

These two equations are really those for a quadrant of an ellipse similar in form to AB , Fig. 151, which is a curve of flow plotted against a descending scale of back pressure for constant initial pressure. In both equations, the simplifying assumption is made that maximum flow is attained when $p_e = 0.50p_1$. Notwithstanding this approximation, and the fact that the actual curve is not strictly elliptical in form, the equations give results of sufficient accuracy in tentative calculations.

* The usual form in which this equation is expressed is

$$\bar{w} = \frac{ap_e}{42} \sqrt{\frac{3(p_1 - p_e)}{2p_e}}$$

It has been changed to the form of Eq. (103), in order to emphasize the similarity to Eq. (102).

The rate of flow through a nozzle is independent of the shape or character of the section that lies beyond the throat. In Fig. 152, *A* represents a simple convergent nozzle, while *B*, *C*, *D*, and *E* represent four nozzles all having their entrance ends identical with *A*, but the sections beyond are different in each case. *B* shows a widely divergent addition beyond the throat;

C, a moderately divergent addition; *D*, a straight section; and *E*, a convergent section continuous with the entrance part. For the same conditions of pressure, the weight of flow through *A*, *B*, *C*, and *D* is the same. The different sections attached beyond the throat do not affect the weight of the flow. In the case of *E*, the

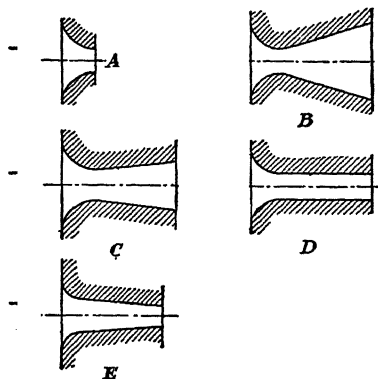


FIG. 152.

convergent section attached to the part *A* does not constitute an addition beyond the throat. The instant the additional convergent piece is attached, the small end of the new piece becomes the throat. The area of the throat has been altered, and therefore, as a matter of course, the rate of flow is changed.

It must not be forgotten that a nozzle's primary function is to generate velocity from available energy, and in the performance of this function the proportioning of the divergent section or portion beyond the throat is all important to efficient energy transformation.

202. Example of Steam-nozzle Calculation.—A nozzle is to expand 2700 lb of steam per hour from 140 psia pressure and 100 degrees superheat to a back pressure of 10 psia. What must be the areas at throat and discharge end?

Using Eq. 100*b*, Art. 200, $\bar{w} = 0.3155a_t \sqrt{p_1/v_1}$, $\bar{w} = 2700/3600 = 0.75$ lb per second, $p_1 = 140$, and v_1 (from Steam Tables) = 3.727. Whence

$$a_t = 0.388 \text{ sq in.}$$

[From Napier's rule, Eq. (101), Art. 200, $a_t = 0.399$.]

From the continuity equation, $A_t \bar{V}_t / v_t = A_e \bar{V}_e / v_e$.

Expansion ratio,

$$\frac{a_e}{a_t} = \frac{\bar{V}_t v_e}{\bar{V}_e v_t}$$

where a_e and a_t = areas, sq in.

\bar{V}_e and \bar{V}_t = velocities, fps.

v_e and v_t = specific volumes at discharge end and throat, respectively.

Since the back pressure is lower than the critical values,

$$p_t = 0.546 p_1 \text{ (Art. 194)} = 0.546 \times 140 = 76.4 \text{ psia}$$

From the Mollier diagram:

h_1 at 140 psia and 100 degrees superheat = 1250.3 (1.6417 entropy).

h_t at 76.4 psia at 1.6417 entropy = 1195.8 (333 F temperature).

h_e at 10.0 psia at 1.6417 entropy = 1048.3 (9.8 per cent moisture).

Available heat from entrance to throat = 1250.3 - 1195.8 = 54.5 Btu per lb

$$\bar{V}_t = \sqrt{2gJ \times 54.5} = 1652. \quad v_t \text{ at 76.4 psia and 333 F} = 5.947.$$

Available heat from entrance to end = 1250.3 - 1048.3 = 202.0 Btu per lb

$$\bar{V}_e = \sqrt{2gJ \times 202} = 3181. \quad v_e \text{ at 10 psia, 9.8 per cent moisture} = 34.68.$$

Introducing numerical values,

$$\begin{aligned} \frac{a_e}{a_t} &= \frac{1652 \times 34.68}{3181 \times 5.947} = 3.029 \\ a_e &= 3.029 \times 0.388 = 1.175 \text{ sq in.} \end{aligned}$$

If the efficiency of the nozzle is taken as 96 per cent, the 4 per cent loss occurs principally in the relatively long divergent part, reducing the velocity of discharge \bar{V}_e somewhat and increasing the volume v_e at discharge by reheating. Then

$$\bar{V}_e = \sqrt{2gJ \times 202 \times 0.96} = 3117$$

$$\text{Waste heat} = 202 \times 0.04 = 8.1 \text{ Btu per lb}$$

Enthalpy of steam at discharge end = 1048.3 (enthalpy at isentropic end point) + 8.1 (waste) = 1056.4 Btu per lb. At 10 psia pressure and 1056.4 enthalpy, the moisture content is 8.9 per cent (Mollier diagram), and specific volume v_e = 35.03.

$$\frac{a_e}{a_t} = \frac{1652 \times 35.03}{3117 \times 5.947} = 3.122$$

and

$$a_e = 3.122 \times 0.388 = 1.211 \text{ sq in.}$$

The expansion ratio of the nozzle must be made somewhat greater because of friction loss, which operates both to reduce velocity and to increase volume.

203. Flow through Orifices.—An orifice is a passage for flowing fluids in which the area is large with respect to the length, in contradistinction to a nozzle, wherein the length is large in comparison to the lateral dimensions of the passage. In general, the equations of flow for a nozzle will not apply to an orifice.

In Fig. 153, *A* and *B* represent orifices in thin plates. *A* has sharp edges, while *B* has a rounded entrance. When flow takes place the streamlines of the fluid really constitute a convergent nozzle, the true area of the throat of

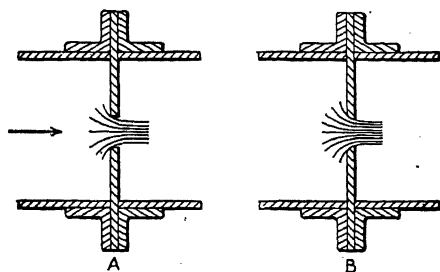


FIG. 153.

which is the *vena contracta* of the jet instead of the area of the orifice. If it were possible to measure the area of the *vena contracta* in a given case, it is probable that the substitution of that value for the term a_t in the flow formulas for nozzles would give fairly accurate results. In practice, rates of flow from orifices are determined by equations similar to those of Arts. 200 and 201 but are multiplied by coefficients which correct for the area at the *vena contracta* and account for losses such as that caused by turbulence. The coefficients are determined experimentally.¹

For an orifice there appears to be no strictly definite critical value of the back pressure, below which the latter ceases to manifest an influence upon the weight of flow. The situation is represented by the curve *AN*, Fig. 151,

¹ A.S.M.E. Power Test Code PTC 19.5.4, Instruments and Apparatus, Part 5, Chap. 4 contains coefficients for orifices and nozzles and an extensive list of formulas for flow of various fluids through nozzles and orifices under various conditions of flow.

in which it is seen that reduction of back pressure, even down into the lower ranges, results in a slight increase in flow.

204. Supersaturation.—If a droplet of a pure substance such as water is small enough, it will evaporate in a saturated atmosphere of the substance. Consider a molecule in the surface of a small drop of radius r , Fig. 154. The molecule is held in the droplet by attraction of other molecules within a certain sphere of influence having a radius ρ and shown by the shaded area of Fig. 154. If the

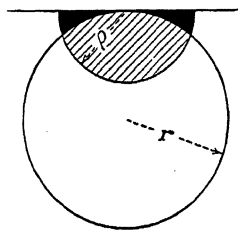


FIG. 154.

molecule is in a plane surface, or in a drop having a large radius r , it is held there by the molecules in the shaded area and those in the dark area, *i.e.*, a molecule is held more firmly in a plane surface than in a curved surface. During equilibrium in a saturated vapor containing drops having a radius larger than some given value, molecules are constantly leaving and entering the surface at equal rates, *i.e.*, molecules are evaporating from the drops at the same rate others are condensing on the drop. The rate of molecular transfer depends on the temperature and vapor density or vapor pressure. If a drop having a radius less than some small critical value is present (less than the wave lengths of visible light), the tendency for molecules to leave the surface of the small droplets will be greater than the tendency for vapor molecules to enter, thus the small droplet will evaporate in a saturated vapor. This is equivalent to saying that the vapor pressure of a liquid at a given temperature must be greater outside the curved surface of a droplet than above a plane surface if small droplets are to exist. The larger vapor pressure counteracts the greater tendency of the molecules in the curved surface to leave.

When steam, slightly superheated, expands isentropically through a nozzle, the state of the steam crosses the satura-

tion line. If thermal equilibrium is maintained, a fog of water droplets appears as soon as the state enters the wet region. The temperature at any state in the wet region is the saturation temperature corresponding to the pressure. However, in a dust-free atmosphere, expansion may continue into the wet region without condensation and a metastable state is present. Such a state is called *supersaturation*. The temperature of the supersaturated steam falls below the saturation temperature corresponding to the existing pressure. Experiments¹ show that supersaturation

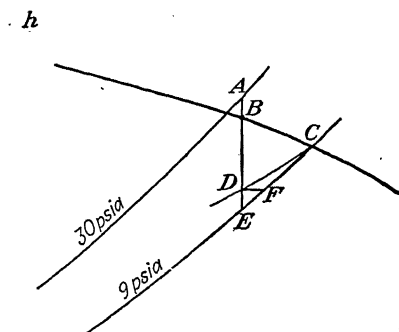


FIG. 155.

may exist until the enthalpy drops about 55 Btu per lb below the enthalpy of dry and saturated steam. After this enthalpy drop, fog suddenly appears and the temperature rises. Also the pressure rises a slight amount. The ratio of the vapor pressure, p_c , measured at condensation to the vapor pressure corresponding to the temperature of the steam, p_s , is the supersaturation ratio.

Consider isentropic expansion of superheated steam as illustrated on the Mollier diagram, Fig. 155, in a dust-free atmosphere. The initial state is represented by point A, $p_A = 30$ psia, $t_A = 290$ F, 40 degrees superheat. At point B under equilibrium conditions, condensation would occur, but if no nuclei of large enough diameter are present

¹ YELLOTT, *Trans. A.S.M.E.*, vol. 56, pp. 411-430, 1934.

YELLOTT and HOLLAND, *ibid.*, vol. 59, pp. 171-183, 1937.

on which droplets may grow the pressure may drop to $p_D = 9$ psia, before condensation occurs and the temperature falls below the saturation temperature for the pressure p_D . Point D represents the state of supersaturated steam, and the 9-psia pressure line for supersaturated steam passes through C and D . The 9-psia pressure line for wet steam is the line through E and F . If condensation is considered to be a constant enthalpy process,¹ the state point is F after fog has formed and thermal equilibrium has been established. The temperature of the steam at D may be found by assuming isentropic expansion from A to E which is on the 9-psia pressure line at which condensation takes place. Supersaturated steam has very nearly the same value of k as superheated steam. Using $k = 1.315$

$$\begin{aligned} T_E &= T_A \left(\frac{p_E}{p_A} \right)^{\frac{k-1}{k}} = 750 \times \left(\frac{9}{30} \right)^{\frac{1.315-1}{1.315}} \\ &= 562 \text{ F abs} = 102 \text{ F} \end{aligned}$$

From the steam tables, the saturation pressure corresponding to 102 F is 1.01 psia. The supersaturation ratio is $9/1.01 = 8.9$, and the steam is undercooled

$$188 \text{ F (9 psia)} - 102 \text{ F} = 86 \text{ deg.}$$

As steam expands in a nozzle, enthalpy is being converted into kinetic energy. In the wet region under equilibrium conditions, kinetic energy is being produced at the expense of the enthalpy of evaporation as condensation takes place. Under supersaturation conditions, the kinetic energy is produced at the expense of the enthalpy of the dry vapor, and therefore the temperature drops much more in the latter case. This energy transfer at a lower temperature represents a loss of availability and an increase of entropy. In Fig. 155, the loss of available energy is $h_F - h_E$ and the entropy has increased from s_E to s_F .

¹ There is evidence that condensation is not a constant-enthalpy process but perhaps more nearly a constant-volume process.

Losses in a nozzle may be the result of supersaturation, friction between fluid and nozzle, turbulence, and shock. Shock is a loss accompanied by sudden changes of velocity with attendant change of pressure.

Nuclei are necessary for the formation of droplets. Dust particles, ions, and groups of molecules, *i.e.*, exceedingly small droplets, are suitable for drop formation. When dust particles are present, their diameter is usually large enough so that molecules condensing on them form a droplet having a curvature large enough to prevent supersaturation. Fog forms in a normal manner. When the only nuclei present are groups of a few molecules, the radii are so small that droplets are unable to form until a sufficiently large supersaturation ratio exists.¹

Problems

1. Steam is supplied to a nozzle at 150 psia and 100 per cent quality; the back pressure is 15 psia. Considering constant-entropy adiabatic expansion, calculate and tabulate quantities of Table 13, Art. 192, except that in the third column "quality" will replace temperature. Plot the curves of Fig. 147, and ascertain the pressure at the throat of the nozzle and its relation to the initial pressure.

2. Compressed air at 89.5 psia pressure and 90 F temperature is discharged through a nozzle into the atmosphere. The diameter of the nozzle at the throat is 0.443 in. What weight of air in pounds per minute will be discharged?

3. What diameter of throat must a nozzle have in order to deliver 1 ton of steam per hour from a pressure of 200 psia and 150 degrees superheat against a back pressure of 15 psia?

4. What weight of *air*, in pounds per hour, will be delivered by a nozzle, 0.50 in. in diameter at smallest section, when discharging into the atmosphere from a pressure 5 psi above atmospheric? Barometric pressure is 29.60 in.; initial temperature, 80 F. Calculate by both, Eq. (93c), Art. 199, and Eq. (102), Art. 201.

5. What weight of *saturated steam*, in pounds per hour, will be delivered by a nozzle, 0.50 in. in diameter at smallest section, when discharging into the atmosphere from a pressure 5 psi above atmospheric? Barometric pressure is 29.60 in. Calculate by both Eq. (93c), Art. 199, and Eq. (103), Art. 201.

6. The nozzles in the first stage of a certain multipressure-stage impulse turbine are to expand steam from 450 psia pressure and 700 F to 300 psia.

¹ For an extensive discussion of supersaturation refer to the references in footnote p. 307 and Keenan, "Thermodynamics," John Wiley & Sons, Inc., New York, 1941.

a. Assuming constant-entropy adiabatic expansion, determine velocity, specific volume, and moisture (or superheat) of the steam delivered upon the buckets.

b. What is the critical pressure for the initial conditions specified?

c. Should these nozzles have a divergent part? Explain.

d. If the nozzles are each 1 sq in. in area at the place of least section, what weight of steam, in pounds per hour, will each nozzle deliver?

7. A nozzle for a steam-jet air pump for a condenser is supplied with steam at 160 psia pressure (p_1) at 100 per cent quality and must expand the steam to 0.80 psia pressure. Assuming constant-entropy adiabatic expansion, no supersaturation, and critical pressure as $0.578p_1$, determine the following:

a. Velocity and specific volume at throat.

b. Velocity and specific volume at discharge end of nozzle.

c. Expansion ratio a_e/a_t (a_t and a_e = area at throat and end, respectively).

d. a_t and a_e , if the nozzle is to discharge 60 lb of steam per hour. [Use Eq. (100c), Art. 200, and check by Napier's rule, Eq. (101), Art. 200.]

e. Length of divergent part of nozzle.

8. Referring to Art. 198, derive Eq. (96) for acoustic velocity from the general equation, Eq. (95).

9. Steam at 60 psia pressure and 20 degrees of superheat expands adiabatically in a perfect nozzle to 14.7 psia back pressure. Supersaturation persists until the enthalpy drops 55 Btu per lb below the enthalpy of dry and saturated steam.

a. What is the temperature of the steam at the throat and at the discharge end of the nozzle?

b. At approximately what pressure does condensation occur?

10. A nozzle without a divergent part is arranged to discharge from a compressed-air tank into the atmosphere (say 14.5 psia). The pressure in the tank (p_1) is varied step by step from 14.5 psia to 100 psia, and the rate of flow \dot{w} is determined for each value of p_1 . The temperature of the compressed air in the tank is kept constant. With p_1 represented by horizontal scale and \dot{w} by vertical, make a sketch of a curve to show the relation between \dot{w} and p_1 . Explain.

11. A nozzle is fitted in a compressed-air tank to discharge into the atmosphere. If the temperature of the compressed air is kept constant, and if the pressure (p_1) ranges from 30 to 300 psia.

a. Prove that the temperature at the throat is constant through all values of p_1 .

b. Prove that the velocity at the throat is constant for all values of p_1 .

c. Prove that the specific volume at the throat is inversely proportional to p_1 .

12. Steam at 250 psia and 460 F expands to atmospheric pressure in a perfect nozzle. Calculate velocity and specific volume at the throat:

a. Using Steam Table data.

b. Using gas-law equations with $k = 1.30$.

13. What is the function of the divergent part of a nozzle for a gaseous substance, as in Fig. 149? Is the same function served by the divergent part of a venturi meter? Explain.

CHAPTER XIV

KINETIC ENGINES. THE STEAM TURBINE AND THE INJECTOR

205. Kinetic vs. Direct-pressure Engines.—The steam turbine is the most familiar example of the kinetic engine, and the distinction between the kinetic and direct-pressure principles will be brought out by a comparison between the steam turbine and the steam reciprocating engine.

In the reciprocating steam engine, force is the result of the direct application of the pressure resulting from the thermal condition of the steam to the yielding resistance of the piston.

In the steam turbine, the force is the result of the pressure that is set up when a mass is hurled against a surface that can move in response to the impulse. The mass is the working fluid, the steam itself; the yielding resistance is the bucket.

The distinction between the two principles is clear cut. The piston of the steam reciprocating engine receives no push from any stream of steam playing upon it; neither is the bucket of a turbine propelled in any degree directly by the elastic pressure of the working substance.

The turbine heat engine necessitates two distinct operations in the transformation of stored energy into mechanical work:

a. The conversion of as much as possible of the enthalpy of the working substance into kinetic energy of the working substance.

b. The communication of the kinetic energy of the working substance to the mechanism of the turbine whereby it may appear in useful form.

The first operation is performed by the nozzle of the turbine. It is strictly a thermodynamic one and has been discussed, in a general way, in the preceding chapter. The second operation is, in its theoretical aspect, a purely mechanical problem, involving the transformation of mechanical energy of one kind into another. However, on account of unavoidable losses, some of the work is transformed into heat within the turbine and as such is reabsorbed by the flowing steam; hence the second operation is, practically, a thermodynamic one also.

206. The Impulse vs. the Reaction Principle.—Whenever a fluid jet is deflected in direction or decelerated in velocity, by any sort of obstacle in its path, the force exerted upon the obstacle is called an *impulse force*. In a turbine, the obstacle takes the form of a series of curved buckets on the rim of a wheel. The curvature at entrance must be such that the stream or jet enters tangentially and is then gradually deflected to the exit tip. The deflection of the jet results in an impulse force upon the bucket. If this could be accomplished without friction or turbulence, with the bucket wheel held stationary, the jet would leave the buckets with the same velocity with which it entered. An impulse force would be produced, but no useful work would be done by it, because there is no motion. If the bucket is moving, the steam will be deflected, the jet will leave with less absolute velocity than it had before entering, and (in the case of the frictionless bucket) the loss of kinetic energy is exactly equal to the work represented by the impulse force multiplied by the bucket speed.

In the impulse turbine, the jet is produced by the transformation of available heat into velocity in *stationary nozzles*, which expand the fluid from the supply pressure to that in the chamber containing the bucket wheel. The steam emerges from the nozzle at the pressure of the fluid in the chamber and, hence, has no further inherent tendency to expand or change its own velocity. Whatever

effects it then produces are accomplished entirely by its previously acquired momentum.

Velocity is imparted to a jet in a nozzle by the application of a continuing force sufficient to accelerate its component particles from a low speed at entrance to the maximum value at the discharge end. The force of acceleration is in the direction of the jet; its reaction is in the opposite direction. There is a backward push on the nozzle, whether the nozzle is permitted to move or not. If the nozzle can move, work is done and the *reaction principle* is employed to turn the rotor. The useful force or push on the rotor is exerted simultaneously with the creation of the velocity. The nozzle performs the functions of both the velocity-generating element and the buckets, of the impulse turbine.

207. Classes of Impulse Turbines.—The impulse turbine is capable of exemplification in a variety of combinations. It may be single- or multiple-pressure stage, and each pressure stage may have a single velocity stage, or several velocity stages.

In a *single-pressure-stage turbine*, all the available energy possible is transformed into kinetic energy of the jet in one step. The kinetic energy thus created is then communicated to the rotor by means of one or more velocity stages.

In a *multiple-pressure-stage turbine*, only a portion of the available energy is converted into kinetic energy by the first set of nozzles. The resulting velocity is absorbed in one or more velocity stages. A portion of the remaining available energy is then converted into kinetic energy by another set of nozzles, and the velocity is absorbed in corresponding velocity stages, and so on, until the entire available energy is utilized. A multiple-pressure-stage turbine is in effect a group of single-pressure-stage turbines operated upon serially by the working substance.

208. Energy Changes in a Single-pressure-stage Turbine.—For illustration, assume a *single-stage turbine* to

operate between the initial condition of 150 psia pressure and 98 per cent quality, and an exhaust pressure of 1 psia, which corresponds to a vacuum of about 28 in. Assume also that the turbine is to have three velocity stages.

Figure 156 represents the enthalpy-entropy chart, or Mollier diagram. The initial state of the steam is found at *A*, the intersection of the 150 psia pressure line, and the 98 per cent quality line, at which point the enthalpy, as read

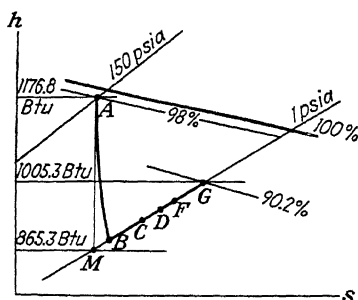


FIG. 156.

from the Mollier diagram accompanying the Steam Tables, is 1176.8 Btu per lb.

If the nozzles were perfect, the steam would expand in them at constant entropy to *M*. The enthalpy at *M* is 865.3 Btu and the quality 76.8 per cent. The available energy of the steam is $1176.8 - 865.3 = 311.5$ Btu,

which would appear as kinetic energy of the jet emerging from a perfect nozzle. The nozzle is not perfect, however, and its losses are returned as heat to the steam, which leaves the actual nozzle at some state *B*, on the 1 psia pressure line, but with a higher enthalpy than at *M*. The first of the two steps in the transformation of heat energy into useful work has been consummated. The available energy has been transformed into kinetic energy of the steam with whatever efficiency the nozzles were capable.

The kinetic energy of the jet is now to be absorbed, and in passing through the bucket channels of the several velocity stages the steam is subjected to further losses which result in the formation of heat that is returned to the steam as fast as generated. Thus in the first velocity stage, the steam is reheated to *C*, in the second to *D*, and in the third to *F*. After the steam emerges from the confinement of the bucket passages of the last velocity stage, it still has some "residual" velocity unclaimed by the buckets.

Furthermore the steam, or the particular pound of it whose history is being traced, spreads and momentarily fills the shell or chamber in which the rotor is revolving. The windage or rotation loss of the wheels cutting through the steam or whirling it about introduces a braking effect which contributes to the enthalpy of the steam before it passes on to the condenser. The combined reheating effect of the residual velocity and windage losses is represented by the vertical distance from F to G , Fig. 156. The steam enters the condenser at the state G .

Assume that the reheating losses, including nozzle loss, bucket friction,¹ residual velocity, and windage, amount to 140 Btu per lb of steam. The enthalpy at G is the sum of the originally unavailable heat, 865.3 Btu and the waste, 140 Btu, or 1005.3 Btu. The utilized energy per pound of steam is $1176.8 - 1005.3 = 171.5$ Btu. The whole history of the steam while passing through the turbine is a continual depletion of the available energy by the successive losses encountered.

209. Energy Changes in a Multiple-pressure-stage Turbine.—The energy change in 1 lb of steam traversing a multiple-pressure-stage turbine is much the same as that which occurs in the single-stage turbine, with one important exception. In the single-stage turbine, the heat that reverts to the steam as a result of frictional effects is irrevocably unavailable; in the multiple-stage turbine, the heat that reverts to the steam as a result of frictional effects in any given stage becomes in a slight degree available in succeeding stages.

For illustration, assume a *two-pressure-stage turbine* to operate between 150 psia pressure and 98 per cent quality, and 1 psia exhaust pressure. Let the entire work of the turbine be divided nearly equally between the stages. The points A and M are located on the Mollier diagram, illustrated in Fig. 157, and, as the specifications are the same as for the illustrative example of the single-

¹ This includes both moving and stationary buckets.

pressure-stage turbine, the available energy of the steam is necessarily the same, *viz.*, 311.5 Btu. The duty of handling this available energy is to be divided approximately equally between two stages. Hence, each stage will be assigned about $311.5 \div 2 = 155.7$ Btu. Theoretically, the enthalpy of the steam, as it leaves the first stage to enter the second, will be $1176.8 - 155.7 = 1021.1$ Btu, and the state point *N* will lie on the constant-entropy line *AM*. The enthalpy and entropy being known, the

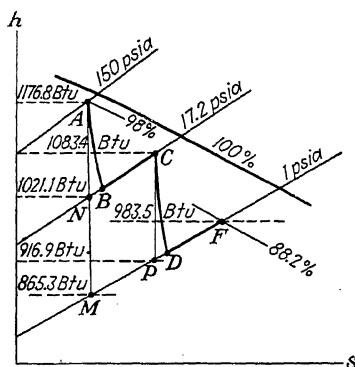


FIG. 157.

point *N* is located. From the Mollier diagram, the pressure corresponding is found to be 17.2 psia. The first-stage nozzles will, therefore, expand the steam from 150 psia pressure to 17.2, and the velocity thus created will be absorbed by an appropriate number of velocity stages. The second-stage

nozzles will then complete the expansion from 17.2 psia to 1 psia, and the velocity generated will be absorbed by suitable velocity stages. The problem incidentally illustrates the disproportionality between the pressure range of an expansion and the resulting available energy. In the first stage of the turbine, the steam expands from 150 psia pressure to 17.2 psia (which is nearly atmospheric)—a range of 132.8 psi. In the second stage, the range is from 17.2 psia to 1 psia, or 16.2 psi. The available energy in both cases is the same.

Assume the total reheating losses of each stage to amount to 40 per cent of its available energy. In the first stage the reheating losses = $155.7 \times 0.40 = 62.3$ Btu. The enthalpy of the steam, as it leaves the first stage, is 62.3 Btu higher than the theoretical enthalpy at *N*. The path of the state point of the steam, in the first stage, is *ABC*—complete expansion through the pressure range allotted,

represented by AB , and constant-pressure reheating along BC .

At the entrance to the second stage, the steam pressure is 17.2 psia and the quality 92.6 per cent, represented by the point C , Fig. 157. In the second stage, the steam is to expand from this initial condition to 1 psia. From the chart, the available energy is found to be 166.5 Btu. The reheating losses total up to 40 per cent of the available energy, or $166.5 \times 0.40 = 66.6$ Btu. The enthalpy

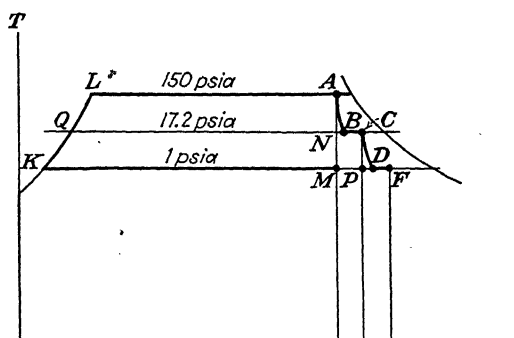


FIG. 158.

of the steam as it leaves the turbine—represented by point F —is 983.5 Btu. The final quality is 88.2 per cent. The state of the steam while passing through the entire turbine is represented by the path $ABCDF$, Fig. 157. Such a curve is called a *condition curve*.

It will have been noted above that the available energy of the second stage is 166.5 Btu, whereas that of the first stage is 155.7 Btu, which was taken as one-half the total available energy between the initial and exhaust conditions specified for the turbine. Why should not the available energy be the same for the second stage as for the first? The answer is that some part of the available energy that became unavailable in the first stage is still available to the second. This fact can best be explained by reference to a temperature-entropy sketch, Fig. 158. A is the initial state at 150 psia pressure and 98 per cent

quality. The total available energy, resulting from adiabatic expansion to 1 psia exhaust pressure, is 311.5 Btu, represented by area $KLAM$. The line QN is drawn so that $QLAN = KQNM = 155.7$ Btu. In the first stage, the actual expansion is represented by ABC ; 40 per cent of the area $QLAN$ is rendered unavailable above the pressure of 17.2 psia; 62.3 Btu are taken from above the 17.2-psia line and placed below it, thus increasing the unavailable energy to the first stage by the area A_1NCC_1 .

By virtue of the lower pressure range of the second stage, a part of the area A_1NCC_1 , representing waste from the first stage, falls above the lower limiting condition of the second stage. Area $MNCP$ represents the portion of the 62.3 Btu waste energy of the first stage that becomes available in the second. The value of $MNCP$ can be easily computed. The height of the strip A_1NCC_1 is 680 F abs. The width is $62.3 \div 680 = 0.0916$ units of entropy. The height of the section $MNCP$ is 118 degrees. Hence, the area is $0.0916 \times 118 = 10.8$ Btu. The total energy available in the second stage is

$$KQNM + MNCP = 155.7 + 10.8 = 166.5 \text{ Btu}$$

which agrees with the results of the computation from the Mollier diagram.

On the Mollier diagram the constant-pressure lines in the wet region are straight lines that diverge slightly as they proceed toward the right or drier region, Art. 68. The effect of the losses in the first stage of the two-stage turbine is to displace the state point from N , Fig. 157, its position if no first-stage losses occurred, to C , the actual state at entrance to the second stage. Whether the initial state of the steam with respect to the second stage is at N or C , the pressure range of the expansion is the same. But the available energy represented by CP is greater than that represented by NM .

The sum of the available energies in the separate stages is referred to as the *cumulative enthalpy drop* and will be

greater than the available energy of a single-stage turbine operating between the same initial and exhaust pressures and having the same initial enthalpy. The ratio of the cumulative enthalpy drop to the direct available energy is called the *reheat factor*. In Fig. 157, the reheat factor is $(AN + CP)/AM$, or $(155.7 + 166.5)/311.5 = 322.2/311.5 = 1.034$

210. The Reaction Turbine.—Whenever there is an acceleration of speed, as in the generation of velocity when a fluid passes through a nozzle, there is a backward push upon the nozzle itself. In the impulse turbine, the nozzle is held rigidly in the frame, and consequently this reaction force does not become active in doing mechanical work. In the reaction turbine, it is this backward push or thrust that is employed as the force element of the machine. An outstanding problem in the design of a reaction turbine is that of keeping the moving or traveling nozzle supplied with the steam or other working substance, under pressure, so that continuous expansion can occur and a steady push be applied to the rotor.

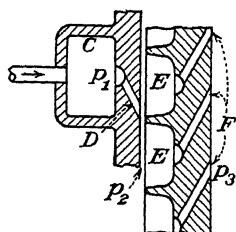


FIG. 159.

In the Parsons type of steam turbine, which is an example of a commercially built steam turbine that embodies the reaction principle, the steam is *thrown* into the entrances of the traveling nozzles by stationary nozzles. The idea is illustrated in Fig. 159. *C* is the stationary steam chest supplied with steam from the boiler. *D* is a stationary nozzle whose function it is to expand the steam from the initial pressure p_1 to some intermediate pressure p_2 , such that the available energy of the expansion results in a velocity of the steam high enough to cause it to throw itself into the series of moving steam chests *E* on the rotor. The steam can then expand through the moving nozzles *F*, from the intermediate pressure p_2 to the final pressure p_3 .

The force of reaction pushes the rotor around. Since the steam is not fully expanded in the stationary nozzle, its elasticity or pressure will cause it to try to escape from the moving chests *E* through any opening offered. A reaction turbine must depend upon small clearance spaces between stator and rotor, to prevent excessive loss by leakage.

If the velocity generated in the stationary nozzle *D* is higher than the speed at which the moving chests *E* are traveling, the moving element will experience a driving force when the jet from *D* strikes it. As a consequence, the entire driving force acting will be made up partly of impulse and partly of reaction, and the turbine will operate upon a combination of the two principles.

The picture of Fig. 159 is intended to illustrate the idea of the reaction turbine and does not at all represent the appearance of the actual steam elements of a Parsons turbine. The steam passages are made to resemble the conventional nozzle in order to emphasize their function of velocity generation. In the actual machine the elements have more the appearance of blades, and are called blades, in fact; but they perform the functions of both nozzles and blades.

211. The Steam Injector.—The steam injector is a familiar example of a kinetic machine, wherein the mechanical effect sought is that of a pump. The available energy of the steam is first transformed into kinetic energy. The momentum of the steam jet is then imparted to the water that is to be pumped, thus generating a high velocity in the column of water. The kinetic energy of the water jet is finally converted into flow-work energy of sufficient head to enable the water to be delivered into the boiler or other high-pressure vessel into which it is to be pumped.

Figure 160 represents the elements of an injector. Steam enters the chamber *F* through the pipe *A* and is expanded by the nozzle *N*. The steam jet is thrown into the space *E*, where it commingles with water that enters around the jet

from the supply pipe *B*. Usually, the water enters because of a vacuum existing in *E* resulting from the condensation of the steam. From the mouth of the nozzle *N* to the point *C*, the steam imparts momentum and enthalpy to the water. Or, more properly speaking, there is an equalization of the enthalpies of the steam and water, because at *C* the water consists of the original water that entered through *B* and the condensed steam. At the point *C*, the

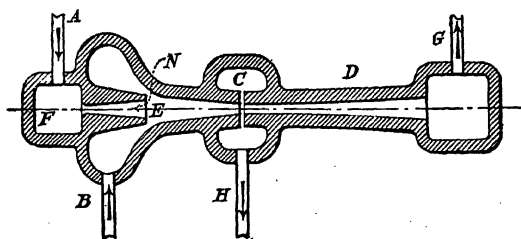


FIG. 160.

water column has its maximum velocity. In the divergent tube *D*, the water is gradually slowed down, thus converting its kinetic energy into a pressure sufficient to impel it into the boiler through the pipe *G* against the resisting head. In starting the injector, when the steam is turned on and before the injector begins to function, the discharged and uncondensed steam finds a way of escape at *C*, out through the overflow pipe *H*.

The first operation of the injector, that of transforming available energy of steam into velocity energy, is purely a steam-nozzle function and can be performed with very good efficiency. When the initial pressure and quality and the approximate amount of steam required by the injector are known, the area at the throat can be determined. Further, if the back pressure on the nozzle in the space *E* can be predicted, the relation of mouth to throat area can be determined. The length of nozzle is then set by the restriction that the total angle of divergence between the sides of the nozzle shall not exceed 12 degrees.

The steam nozzle can thus be satisfactorily designed for good efficiency by the nozzle laws already established.

The third operation is that of a reversed nozzle. The function of the tube *D* is gradually to slow down the jet of water and so convert kinetic energy into flow-work. This is a process that has been found to be far less capable of efficient accomplishment than its reverse, *viz.*, the generation of velocity from pressure. Turbulence of flow, which is a frictional effect, resulting in the generation of heat from work energy that should have been conserved during the operation, is much more difficult to prevent.

212. Impact.—Of all the three energy transformations performed in an injector, however, the second is by far the most wasteful. Into a relatively slow moving column of water, the jet of steam is hurled at the terrific velocity of 3000 fps or more. The momentum of the steam jet is imparted to the water, but the operation is extremely wasteful of energy. For an example, assume 1 lb of steam per second emerging from the nozzle with a velocity of 3000 fps. Assume that for each pound of steam there is supplied to the injector 15 lb of water, through the pipe *B*, Fig. 160, and that the water has a velocity of its own of 20 fps, just as it makes contact with the steam jet. Assume further that the original momenta of both the steam and water jets are completely conserved in the commingled stream at *C*. Then the velocity of the jet at *C* can be computed:

$$\frac{(1 \times 3000) + (15 \times 20)}{16} =$$

Now compare the kinetic energy possessed by the mediums before and after impact.

$$\text{Kinetic energy of steam jet} = 1 \times 3000^2 \frac{9,000,000}{2g}$$

$$\text{Kinetic energy of water before mixing} = \frac{15 \times 20^2}{2g} \frac{6000}{2g}$$

$$\begin{aligned}\text{Total before mixing} &= \frac{9,006,000}{2g} \\ \text{Kinetic energy of the combined jet at } C & \\ &= \underline{16 \times 206^2} - \underline{679,000}\end{aligned}$$

Only 7.5 per cent of the original kinetic energy has been conserved. The rest has been lost by impact.

Impact is, of course, another frictional effect—an irreversible process. The 92.5 per cent of kinetic energy lost in the example above appears as enthalpy in the combined jet. Impact is an effect that the steam-turbine designer endeavors to eliminate by curving the buckets so that the steam enters tangentially to the surface. In fact, next to the proper proportioning of the area of section to the existent values of velocity and specific volume, the most important problem of all those that have to deal with the question of fluids in motion is the avoidance of impact. The turbine's bucket must receive the stream of steam, or air, or water, and change its direction in the gentlest possible manner. The pipe must change the direction of its column of contents without whirls or eddies, because, if any kinetic energy is lost in impact, it will result in a reduction of the work output. Impact is the essence of an irreversible process—one that is chaotic and disordered.

213. Efficiency of the Injector.—With the second energy-transforming operation so remarkably low in efficiency as demonstrated in the preceding article, it must follow that the efficiency of the entire apparatus, as a heat-engine pump, is exceedingly low. It is so low, in fact, that the injector cannot compete in economy with the very poorest of mechanical pumps. However, in the capacity in which it finds its widest use, *viz.*, that of feeding boilers, the injector offers the advantage of perfectly conserving the enthalpy supplied to it in the steam, returning practically 100 per cent to the boiler. But no matter how perfect a conserver of enthalpy the injector is, it is a waster of

available energy and from a purely economical standpoint is inferior to a feed-pump system in combination with a feed-water heater.

On the other hand, the remarkable simplicity of the injector, together with its small size and low cost, recommends it for certain situations where economy is not the ruling consideration. Outside of the use of the injector for feeding boilers, it is frequently applied to the evacuation of spaces of water, or air, or any other fluid. When so used, the instrument is generally called an *ejector* or *aspirator*, but the difference in name relates rather to the service performed than to any difference in principle. The motive fluid may be anything of which there is an available supply under pressure; and the substance handled may be anything that will flow.

The steam-jet air pump for maintaining the vacuum in a condenser and the steam-jet vapor pump for maintaining the low pressure in a water chamber to produce the corresponding low temperature for refrigeration (see Fig. 136) are further examples of the wide application of the kinetic pump. In these, unlike the steam injector, the steam jet is not condensed by the medium pumped, but the motivating effect is produced entirely by entrainment or the "drag" of a high-velocity stream upon the substance to be moved.

The energy efficiency of the injector or ejector is much higher when the motive fluid is relatively heavy or dense in comparison with the substance to be pumped, as, for example, in the case of an ejector using water to evacuate a chamber of air.

Problems

1. A one-pressure-stage two-velocity-stage turbine is supplied with steam at 120 psia and containing 2 per cent moisture; exhaust pressure is 16 psia. Assume loss in nozzle to be 6 per cent; in buckets, due to frictional effects upon the steam jet, 18 per cent; in residual velocity of steam leaving the last row of buckets, 8 per cent; and in windage loss, 13 per cent; all percentages being based upon available energy.

a. Make a diagrammatic sketch of the turbine showing nozzle and bucket passages, casing enclosing the wheel, and exhaust outlet. Letter the

points along the path of steam flow from steam chest to exhaust for which information is given above, concerning the state.

b. Make an enthalpy-entropy sketch, similar to Fig. 156, showing state-point path of steam, lettering in correspondence with sketch of part (a).

c. Determine enthalpy and moisture content at each point (using Mollier diagram) and place these values, together with entropy values, upon sketch of part (b).

2. A one-pressure-stage one-velocity-stage turbine is supplied with compressed air at 200 psia and 400 F temperature. The loss in the nozzle is 5 per cent, and the sum of bucket-friction losses, residual velocity, and windage loss is 45 per cent of the available energy.

a. Make a diagrammatic sketch of the turbine, showing nozzle and bucket passages, casing enclosing wheel, and exhaust outlet. Letter points along the path of air flow for which information is given above.

b. Make a temperature-entropy sketch showing state-point path of air, lettering to agree with sketch of part (a).

c. Determine the temperature, and the entropy *change* from the initial state of the air, for each point, and place values on sketch of part (b).

3. A two-pressure-stage steam turbine receives steam at 160 psia pressure and 100 degrees superheat, and exhausts at 16 psia. Assuming that the available energy is about equally divided between the two stages and that 40 per cent of the available energy in each stage is returned to the steam in that stage by reheating losses:

a. Make an enthalpy-entropy sketch, similar to Fig. 157, to represent, in a general way, the state-point path of the steam, and on which to indicate the quantities determined below.

b. Determine the pressure at the point *N* that divides the available energy equally between the two stages on a constant-entropy line.

c. Determine the enthalpy, moisture, and entropy of the steam as it enters the second stage.

d. Determine the available energy to second stage on the new entropy line; also enthalpy and moisture at exhaust.

e. Calculate what number of Btu out of the 40 per cent available energy wasted in the first stage becomes available in the second stage. Use sketch of temperature diagram, similar to Fig. 158.

f. The efficiency of each of the stages in respect to its utilization of the available energy furnished it is 60 per cent. What is the efficiency of the whole turbine in respect to the available energy of the steam supplied?

4. An injector is supplied with steam at 100 psia and 98 per cent quality. The pressure in the chamber *E*, Fig. 160, is 9 psia, or about 12 in. vacuum. The injector handles 15 lb of water per pound of steam. The water enters at 70 F and has a velocity of its own in the direction of the steam of 20 fps.

a. Determine the velocity of the steam if the nozzle efficiency is 94 per cent.

b. Determine the loss of energy by impact per pound of steam.

c. What is the temperature of the water delivered by the injector?

5. An impulse turbine having seven pressure stages develops 15,000 kw when supplied with steam at 250 psia and 800 F and exhausting at 0.75 in. Hg abs. Generator efficiency is 97 per cent.

a. Determine the pressures in the stages that will make the division of the total available energy among them approximately equal.

b. If 20 per cent of the available energy in each stage is lost by friction, windage, etc., and thus reheats the steam, find the heat equivalent of the work done per pound of steam in each stage.

c. What is the condition of the steam as it leaves each stage in part *b*?

d. If 10 per cent of the total work developed at the shaft is used to overcome bearing friction, operate oil pumps, governor, etc., find the steam consumed per delivered kilowatt-hour.

e. Determine thermal efficiency of turbogenerator unit.

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